An Example to Particle Settling in a non-Newtonian Fluid

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1. Introduction

As an example to the application of CEF fluid, we can consider the settling of small particles in a non-Newtonian fluid medium. The geometry of the problem is shown in Fig. 1.

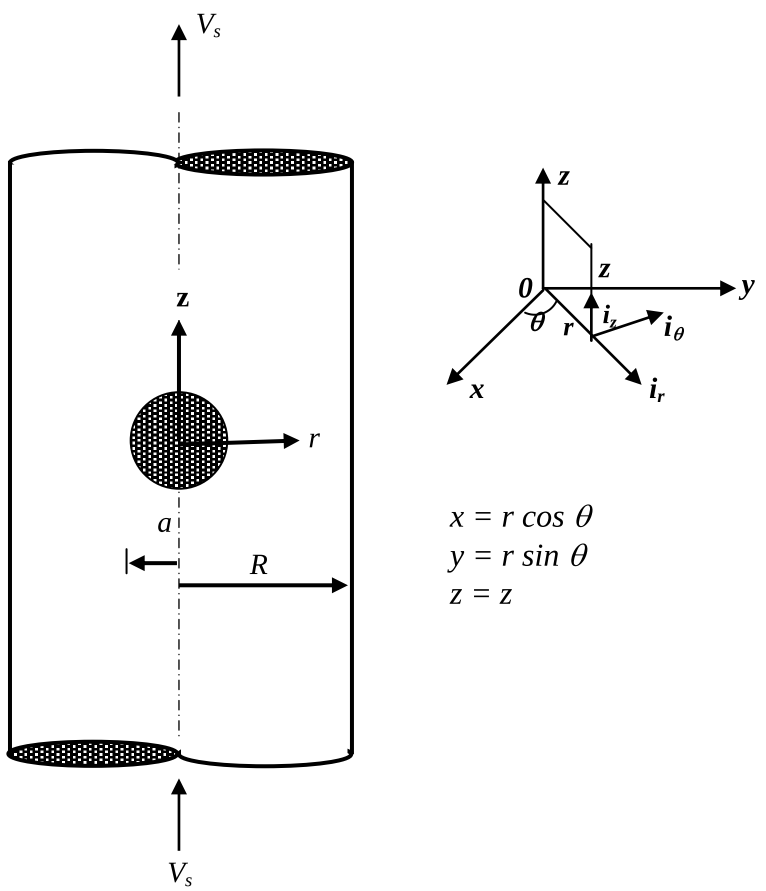

Fig. 1 Schematic diagram of a sphere falling through a fluid in a cylinder

2. The well-known equations

2.1 The CEF equation

The constitutive equation of the CEF fluid is:

$$\boldsymbol{\tau} = -p\,\boldsymbol{I} + \eta\,\boldsymbol{A_1} + (\upsilon_1 + \upsilon_2)\,\boldsymbol{A_1}^2 - \frac{1}{2}\upsilon_1\,\boldsymbol{A}_2$$

2.2 Governing equations

Continuity equation

$$\nabla.\boldsymbol{V} = \frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} + \frac{v_r}{r} = 0$$

Motion equations

$$\rho\frac{D\boldsymbol{V}}{Dt} = \nabla.\boldsymbol{\tau}$$

Supposing a creeping flow

$$\nabla.\boldsymbol{\tau} = 0$$

3. Boundary conditions (dimensionless)

At the inlet of the flow:	$v_r = 0$	$v_z = 1$
Along the cylindrical tube ($r = R$):	$v_r = 0$	$v_z = 1$
Along the centreline of the cylindrical tube (r=0):	$v_r = 0$	$\frac{\partial v_z}{\partial r} = 0$
On the surface of the sphere:	$v_r = v_z = 0$	
At the outlet of the flow:	$v_r = 0$	$\frac{\partial v_z}{\partial r} = 0$
	p=0 (atmospheric pressure)	

4. Dimensionless stress components

$$\tau_{rr} = -p + \eta(A_1)_{rr} + K\left[0.85\upsilon_1(A_1^2)_{rr} - \frac{1}{2}\upsilon_1(A_2)_{rr}\right]$$

$$\tau_{rz} = \eta(A_1)_{rz} + K\left[0.85\upsilon_1(A_1^2)_{rz} - \frac{1}{2}\upsilon_1(A_2)_{rz}\right]$$

$$\tau_{\theta\theta} = -p + \eta(A_1)_{\theta\theta} + K\left[0.85\upsilon_1(A_1^2)_{\theta\theta} - \frac{1}{2}\upsilon_1(A_2)_{\theta\theta}\right]$$

$$\tau_{zz} = -p + \eta(A_1)_{zz} + K\left[0.85\upsilon_1(A_1^2)_{zz} - \frac{1}{2}\upsilon_1(A_2)_{zz}\right]$$

5. Application of the Finite Element Method to fluids

Inside each element an interpolation function is assumed for the variables. The flow domain is meshed using linear and quadratic triangular elements. Here we have chosen linear pressure and quadratic velocity fields over the element.

Independent unknown variables are due to axisymmetry v_r, v_z and p.

6. Conclusion

Resolving the global equation system obtained while taking into account the boundary conditions and integrating numerically by means of Gauss quadrature over the effective area around each node, the values of the variables are found. Then we computed stresses with summing them up on the effective area.

Table 1 Normal and shear stresses

Stresses	τ_{rr}		$\tau_{\theta\theta}$		τ_{zz}		τ_{rz}	
Extreme values	Min	Max	Min	Max	Min	Max	Min	Max
	-6.0019	3.3792	-6.0115	3.400	-6.1339	3.7165	-0.3701	0.8390

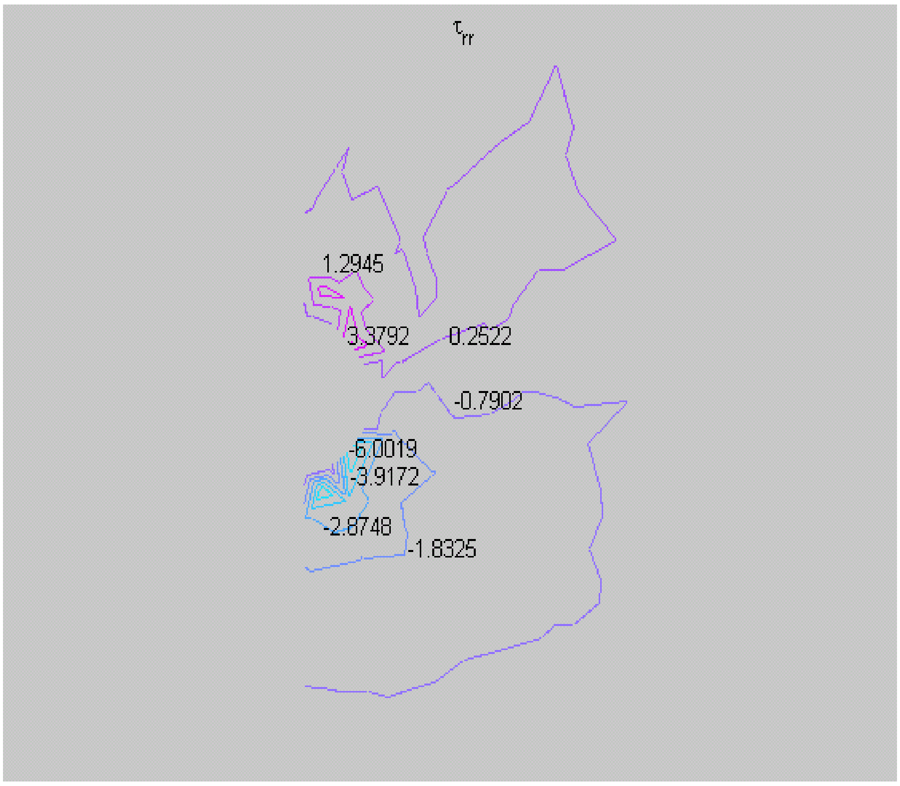

Fig. 2 Normal stress (τ_{rr})
[-6.0019 -4.9595 -3.9172 -2.8748 -1.8325 -0.7902 0.2522 1.2945 2.3369 3.3792]

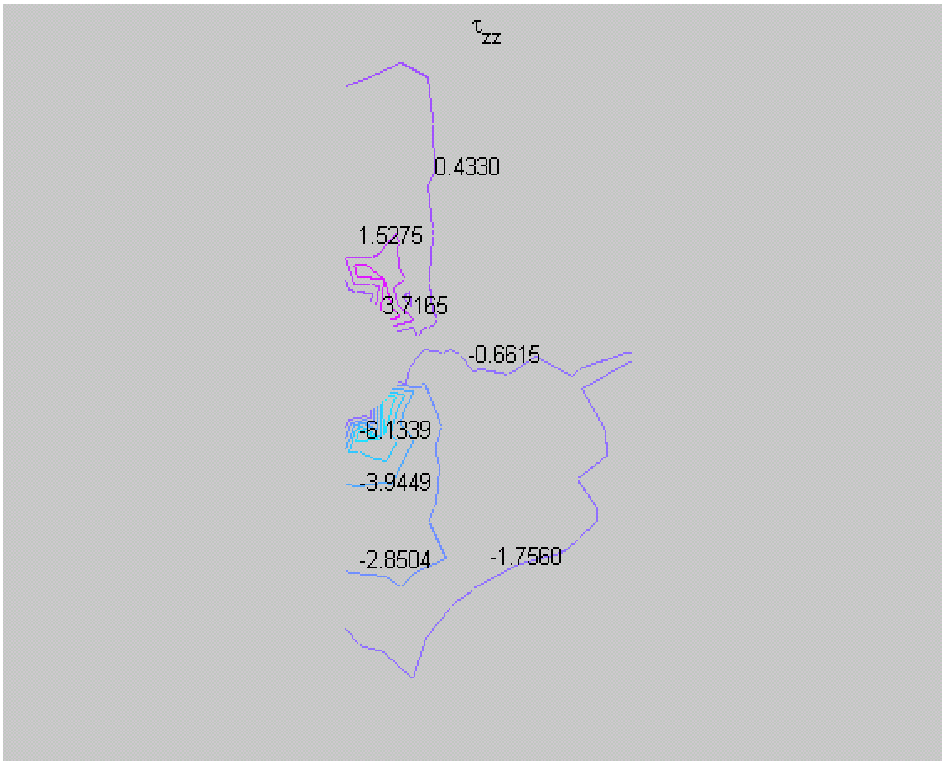

Fig. 3 Axial stress (τ_{zz})
[-6.1339 -5.0394 -3.9449 -2.8504 -1.7560 -0.6615 0.4330 1.5275 2.6220 3.7165]

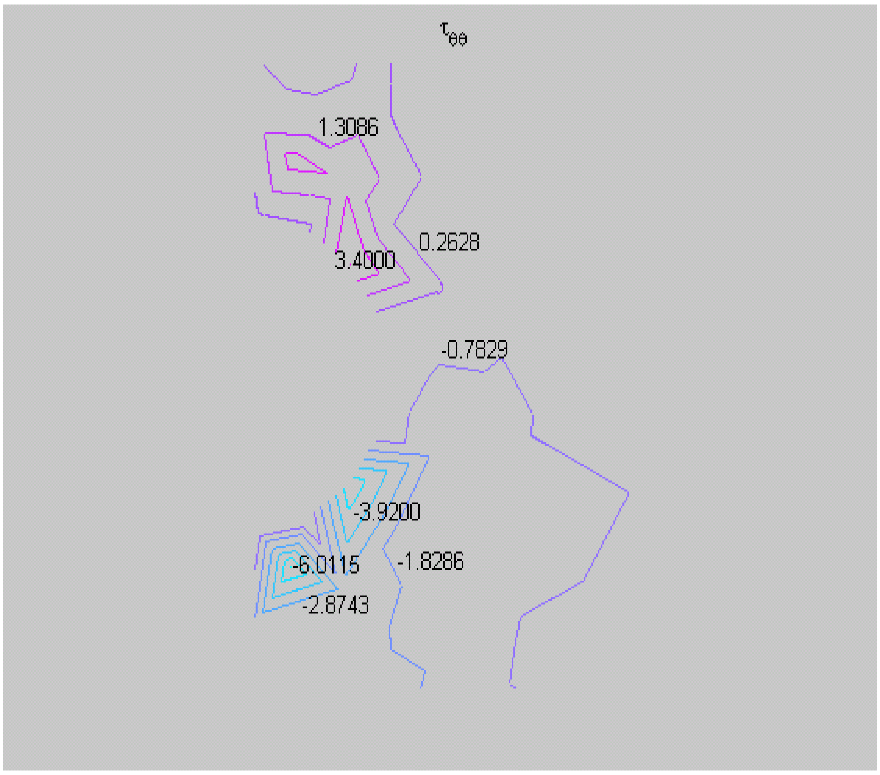

Fig. 4 Azimuthal stress ($\tau_{\theta\theta}$)
[-6.0115 -4.9658 -3.9200 -2.8743 -1.8286 -0.7829 0.2628 1.3086 2.3543 3.4000]

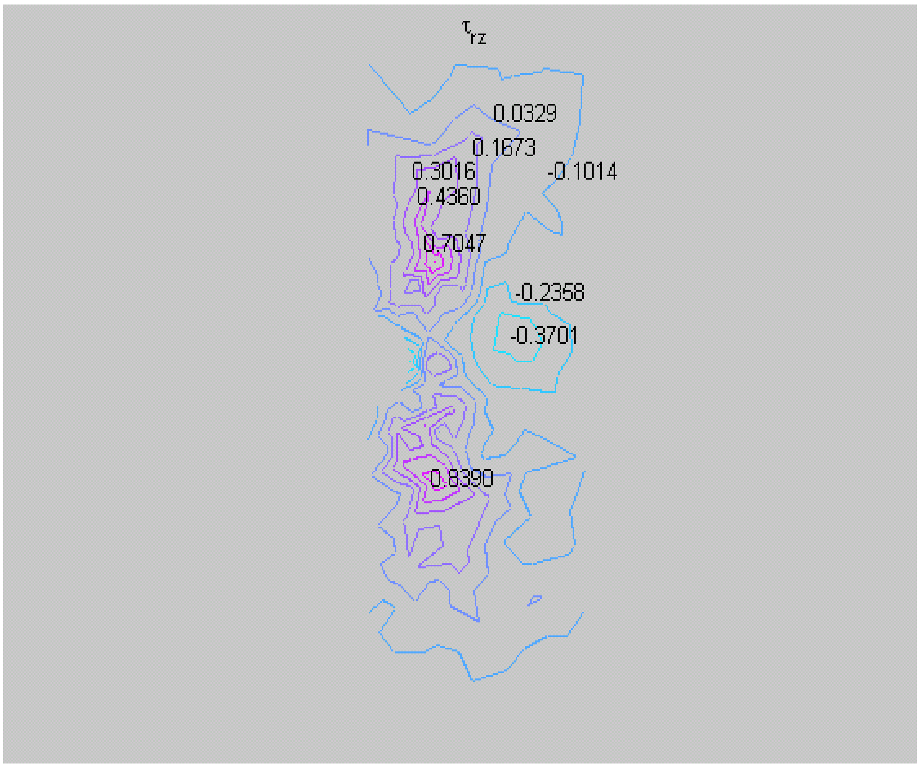

Fig. 5 Shear stress (τ_{rz})
[-0.3701 -0.2358 -0.1014 0.0329 0.1673 0.3016 0.4360 0.5703 0.7047 0.8390]

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