Energy Harvester from Unmanned Airplane Wing Fluttering: Coupling of Aeroelastic and Piezoelectric Models

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Abstract
Electrical energy can be harvested from aeroelastic vibrations of lifting and dynamically oscillating surfaces of aircrafts such as wings, elevators, fins, etc. Moderately sized wings (i.e. ~2-3m span), particularly unmanned air vehicles, can use this energy for onboard operation thereby saving the carry weight of energy storage (i.e. heavy batteries) and rendering the vehicle more efficient. In this work, an energy harvester is modeled using an electromechanically coupled system using a lumped wing/cantilever model and unsteady aerodynamic model. They are combined to develop a piezoaeroelastic model for a small airplane. The electromechanical system is solved using Adams-Bashforth-Moulton’s (ABM) method and the aerodynamic model is solved using the Unsteady Vortex Lattice Method (UVLM). The harnessed electrical power and conditions from the piezoaeroelastic model of the oscillating wing are investigated at several airflow speeds until harvested energy is obtained. While maintaining structural integrity, proposed work has shown that energy harvested from the system is optimized by proper choice of structural and geometrical configurations along with airspeeds where heave is maximized and coupled bending-torsional vibration is minimized.

Keywords: Piezoaeroelastic; Vibrational energy harvesting; Adam-Bashforth-Moulton
1. Introduction
The scale of growth of low power electronic devices and their growing ubiquity has given rise to increased interest in developing energy harvesting techniques. The energy harvesters collect power, albeit low, from the ambience of the device to make it self-sufficient in its power resources. Collecting energy from aircraft wings is considered a future breakthrough technology for unmanned air vehicle (UAV) design where load-bearing wings can generate inexhaustible electrical energy used in self-powered devices such as microelectromechanical and actuator systems [1].

The vibrational motion of aircraft wings proves to be a good source of energy to power the small electronics that constitutes the gadgetry on the aircraft that include microelectromechanical systems, sensors and communication devices. The goal in harnessing this energy is to convert ambient or aeroelastic vibrations to electrical power. The transduction mechanisms used for transforming vibrations to electric power include electromagnetic, electrostatic, and piezoelectric mechanisms [2,3,4]. Among these mechanisms, piezoelectric energy harvesters have piqued most interest owing to their larger frequency band, simplicity, compactness, and lower cost [5,6].

In this work, a coupled piezoaeroelastic system is modeled and is composed of two main component modules:

1) A lumped-parameter electromechanical model that only accounts for rigid body heave and pitch movements of the wing as two-degree of freedom system (2DOF). This electromechanical system models the wing as a cantilever beam coupled with a piezoelectric electric potential generation system via a simplified first-order electromechanical coupling model.

2) An aerodynamic model that generates the aerodynamic loads on the wing as a function of the incident air speed and the angle of attack.

The two modules are coupled via a predictor-corrector numerical solution that predicts the system response as far as heave and pitch history, in addition to the generated voltage and power subjected to the actual aerodynamic loading.

Most research work on energy harvesters is directed towards better designs to enhance the output performance of the harvesters both in efficiency and the width of frequency band covered. Much like other design and manufacturing tasks, the piezoaeroelastic energy harvesters design also has a lot to gain from mathematical and computational models that help explain the operating point of the device. Different researchers have proposed various models to represent the electromechanical behavior of piezoelectric energy harvesters [7]. Analytical distributed parameter solutions for unimorph and bimorph piezoelectric energy harvesters are presented in [7,8]. De Marqui et al [9] formulates the electromechanically coupled model using FEM and couples it with the aerodynamic model using UVLM. They attempt to better model the inherited unsteadiness of the problem by employing the doublet lattice method [10]. More recently, Abdelkefi et al [11] explored global nonlinear distributed-parameter models for piezoelectric energy harvesters under direct and parametric excitations. More work on global nonlinear distributed-parameter models for piezoelectric energy harvesters was conducted earlier by the same group [12,13]. It is known that distributed-parameter models give a clearer picture of the stress distributions and the related generated output. Lumped-parameter wing-section models, however, are appealing due to their physical simplicity and the added insight they can provide [14]. In this work, a coupled fluid and lumped-parameter model for the electromechanical system is analyzed for output trends under various input and load conditions. The novelty of this research work lies in its ability to determine the range of airspeeds and structural configurations that maximize the energy harvested while maintaining structural integrity. This work also
consider the fluid-structure coupling to provide insight and guidance into the type of vibration needed to optimize energy harvested. It shows the necessity of a holistic look at the vibration system when designing for energy harvesting and the optimal conditions need to achieve such goal.

2. Model Description
The most common vibration-based energy harvester consists of a composite cantilever beam with single or double piezoelectric layers. Electrodes spanning these piezoelectric layers harvest the electrical energy and use it to drive loads or to store in small batteries or capacitors. The electric circuit that the electrodes connect to is generally modeled as a load resistor. Along with the piezoelectric system, the piezoaeroelastic problem studied here consists of a mechanical and aerodynamic component. The piezoelectric and mechanical system are lumped together in a piezoaerelastic system. It is composed of two main subsystems: The aerodynamic and the piezoelastic subsystem. The inputs to the system are the incident air speed \( V_{\alpha} \) and the angle of attack \( \alpha \). The two subsystems are coupled since the piezoelastic system requires aerodynamic loads (lift \( L \) and Moment \( M_{L} \)) evaluated by the aerodynamic system and applied on the structure to solicit its electrodynamic response. This coupling is illustrated in Fig.1.

**Fig. 1: Fluid-Piezoaeroelastic subsystem coupling**

On the structural side, the wing shown in fig 2 is modeled as a cantilever beam which is allowed to move in the heave \( h \) and the pitch directions \( \theta \). It is subjected to linear \( k_{h} \) and torsional \( k_{\theta} \) springs vibrations as a result of the lift and moment profile coupled with the structural characteristics of the beam, i.e. wingspan. The span of the wing is aligned along the \( y \)-axis and the chord is along the \( x \)-axis. The unswept wing has a span and cord length of \( Le \) and \( c \), respectively. Hereafter more distributions of the submodels and their couplings are presented.

**Fig. 2: A cantilever wing subjected to point load due to aerodynamic flow**
2.1 Aerodynamic Load: Incident wind causes a pressure distribution on the surface of an object in its path. Under certain assumptions, the calculations of this pressure distribution can be simplified and obtained without the need to resolve the velocity fields. In the case of an airfoil wing, the pressure distribution on the wing surface provides a lift and moment loads which are required and used as input in solving the piezoeaerodynamic problem. When the wing is subjected to aerodynamic loads, it responds in various modes based on its structural properties and geometry [15]. These are observed as vibrational modes including vortex-induced vibrations (VIV), flutter, buffeting, and less so for galloping mode. From a structural viewpoint and for large scale systems (bridges, aircraft, pipelines, etc.), these types of vibrations are avoided in order to reduce possible structural damage [16]. However, the aforementioned aerodynamic instability phenomena can be exploited to harvest energy.

Flutter is an aerodynamic instability phenomenon and is a self-excited vibratory motion caused by coupled aerodynamic effects, i.e. the increase of wind speed and the absence of sufficient structural damping. VIV and galloping are those present by Von-Karman vortex street and large-amplitude aeroelastic oscillation in-plane normal to the incoming flow, respectively, which are of lesser interest than fluttering mode as far as the application of the unmanned air vehicle. The modeling caters analyzing the system output at different speeds of the incident wind and identifying the onset of flutter which depends also on the structural properties and geometry of the wing system. A sequence of temporal and frequency analyses will render the efficient use of flutter.

2.2 Structural Analogy: The wing structure analogy is made of a cantilever beam per Fig 2. It is clamped at one end and free at the other end. The aerodynamic loads act transversely to the longitudinal/span wing axis causing shear and bending moments, thereby deforming the axis of the wingspan into a curve or deflection line in the x-z plane. Fig. 2 shows the wing/beam setup. Above the deflection curve it undergoes tensile, while below the curve goes into compressive stresses. This stress is responsible for generating electrical potential in the piezoelectric layer impeded or coating the wing upper and lower surfaces through piezoelectric effect.

2.3 Piezoelectric Effect: This effect explains the coupling between electric potential and mechanical strain in specific “piezoelectric” materials. This coupling could exist in both directions, i.e. mechanical strain to electric potential or the contrary. Quartz, tourmaline and barium titanate are examples of piezoelectric materials with. Lead zirconate titanate (PZT) is widely used for energy harvesting. The brittleness of PZT, however, limits the level of applied strain and compromise its structural integrity at higher frequencies. Polyvinylindene fluoride (PVDF) is another polymeric piezoelectric candidate with greater flexibility. Piezoelectric energy harvesters can work in the 3-1 coupling mode (bimorphs) and the 3-3 coupling mode (stack actuators), as shown in Fig. 3. The voltage is obtained from a direction perpendicular to the applied stress in the former while from the same applied direction for the latter.

Fig. 3: The two coupling modes of PEH [15] and the uniform clamped free wing
2.4 The Aerodynamic Model: Fig. 4 shows a lumped model of the vibrational cantilever including the linear \((k_h)\) and torsional \((k_\theta)\) springs. This common model is taken from the work of Hodges et al [14]. Additionally, linear and angular dampers are modeled with damping coefficients \(d_h\) and \(d_\theta\). \(C\) denotes the center of mass, \(Q\) is the location of the quarter chord, and \(T\) is the location of the three-quarter chord point on the chord line. The heave and plunge movement is denoted by \(h\), and \(\theta\) is the pitch around the reference point \(P\). Notice that \(e\) and \(a\) encode the shift of \(C\) and \(P\) from the center chord point. All lengths are dimensionless and are measured relative to the semi-chord length, i.e. \(b\). Notice that the geometry and structural model of the system constrains the wing displacement into rigid body heave and pitch movements under stiffness and damping constraints defined by the respective parameters. This is done to simplify the model and perform useful temporal and frequency analyses of the system.

![Fig. 4: Lumped model of wing section showing equivalent pitch and plunge spring restraints [10]](image)

The aerodynamic model takes in airspeed \((V_\infty)\) and angle of attack \(\alpha\) as its inputs and computes lift \((L)\) on the wing surface and moment \((M_L)\) about the elastic axis defined by the reference point \(P\). This is a fluid dynamics problem and would traditionally involve solving the Navier-Stokes system of equations over and around the airfoil surface. The solution would require a dense mesh and a solution space extending several wingspans around the wing to incorporate the wake and other aerodynamic features. This is expected to be a complex task with long computational time. Instead, the Unsteady Vortex Lattice Method (UVLM) greatly reduces the problem complexity and provides results that are well within the required tolerance to complete the system, particularly for aerodynamic shapes such as wings. UVLM is an efficient computational technique to solve 3-D potential flow problems about lifting surfaces where viscosity and compressibility of the flow can be ignored and for a reamined bodies under small angles of attack and relatively thin.

3. System Setup and Solution

3.1 The Unsteady Vortex Lattice Method: UVLM solves for low subsonic speeds, 3-D unsteady potential-flow which incorporate 3-D effects such as interference and at low computational costs. The flow is governed by the Laplace’s equation for the flow velocity potential, \(\nabla^2 \phi = 0\). This Laplace’s equation allows a 3-D flow-field problem to be modeled as 2-D, by distributing elementary singularity solutions over the surface and in the wake of the wing. As the flow potential \((\phi)\) is solicited subjected to a no-penetration velocity condition, a boundary value problem is raised. The UVLM is used to solve for the aerodynamic load on the wing. Following the geometrical definition of the lifting object, the algorithm proceeds as follows and per Fig. 5 [17, 18].

1. Divide the platform/wing into a lattice of quadrilateral panels.
2. Place bound vortex (horseshoe or ring of unknown strength), \((T_n)\) on quarter chord element line of each panel.
3. Place control point on the three-quarter chord at the spanwise midpoint.
4. Assume flat wake that is trailing edge at the end of the airfoil.
5. Set up the linear equations that satisfy no penetration condition for each panel.
6. Determine $f_q$ for each panel by solving the algebraic system of equations.

It should be emphasized that no-penetration boundary condition is applied at a selected control point on each of the panels to arrive at an algebraic system of equations. The lifting surface is discretized in vortices of a prescribed shape. Horseshoe vortices are popular but unsteady problems are usually solved using rectilinear vortex rings. The leading vortex segment is placed on the panel’s quarter-chord line. The control point where the velocity is computed is placed at the three-quarter chord line, which falls at the center of the vortex ring. The vortex rings are continued past the wing surface into the wake to incorporate the effect of wake vortices and induce velocity at the control point of the panels on the wing.

![Diagram](image)

**Fig. 5:** UVLM wing discretization scheme [4]

### 3.2 The UVLM Solution and validation:

The 2-D vortex singularity, a point vortex, satisfies Laplace’s equation when $v_\theta = \frac{r}{2\pi r} \tilde{\theta}$ where $v_\theta$ is the irrotational flow angular velocity component. Now, in order to extend the solution considering a point vortex to the case of a 3-D vortex filament, a 3-D vortex filament is considered and the Biot-Savart law is used to obtain the increment in velocity induced by a filament $d\ell$ [17,19].

$$d\mathbf{V}_p = \frac{r}{4\pi} \cdot \frac{d\mathbf{x}_{pq}}{|r_{pq}|^3}$$

(1)

The velocity induced by the entire vortex filament length is then obtained by integrating over the entire filament.

$$\mathbf{V}_p = \frac{r}{4\pi} \cdot \frac{d\mathbf{x}_{pq}}{|r_{pq}|^3}$$

(2)

Expanding the integral for a finite length vortex filament leads to the following simplified expression that can be directly computed from the given wing geometry as:

$$\mathbf{V}_p = \frac{r}{4\pi} \cdot \frac{d\mathbf{x}_{pq}}{|r_{pq}|^3} \cdot \left( \frac{r_1}{|r_{1}|} - \frac{r_2}{|r_{2}|} \right)$$

(3)

Referring to Fig. , we have $r_1 = AC$ and $r_2 = BC$. Factoring the $l_q$ out of this equation, the remaining value is termed $C_{pq}$, the influence coefficient of vortex $q$ on control point $p$. The velocity is then expressed as $\mathbf{V}_p = C_{pq}l_q$. 

67
The total induced velocity at \( p \) owing to \( N \) vortices is written as:

\[
V_{pi} = \sum_{q=1}^{N} C_{pq} \Gamma_q
\]  

(4)

The freestream velocity considering the angle of attack (\( \alpha \)), and a side-slip (\( \beta \)) is:

\[
V_{\infty} = V_{\infty}\cos\alpha \cos\beta \mathbf{i} - V_{\infty}\sin\alpha \sin\beta \mathbf{j} + V_{\infty}\sin\alpha \cos\beta \mathbf{k}
\]  

(5)

Add the contribution from the wake (\( V_w \)) here as well as the structural motion of the wing itself (\( V_m \)) to obtain the total velocity:

\[
V_p = V_{pi} + V_{\infty} + V_w + V_m
\]  

(6)

Apply then the no penetration condition, i.e. the component of velocity normal to the surface of the platform at the point \( p \) is zero.

\[
V_p \cdot \mathbf{n}_p = 0
\]  

(7)

Where \( \mathbf{n}_p \) is the surface unit normal vector at the panel \( p \). Considering \( a_{kl} = V_{kl} \cdot \mathbf{n}_k \) and applying the no penetration condition given above we get the following linear system:

\[
\begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1N} \\
a_{21} & a_{22} & \cdots & a_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
a_{N1} & a_{N2} & \cdots & a_{NN}
\end{bmatrix}
\begin{bmatrix}
\Gamma_1 \\
\Gamma_2 \\
\vdots \\
\Gamma_N
\end{bmatrix}
+
\begin{bmatrix}
V_{m1} + V_{\infty} + V_{w1} \\
V_{m2} + V_{\infty} + V_{w2} \\
\vdots \\
V_{mN} + V_{\infty} + V_{wN}
\end{bmatrix}
\cdot
\begin{bmatrix}
n_1 \\
n_2 \\
\vdots \\
n_N
\end{bmatrix}
= 0
\]

where \( V_{m1} \) is the wing motion due to structural deformations and \( V_{wi} \) are the wake induced velocities. We solve the above system for the circulations \( \Gamma_q \) and evaluate \( V_p \) from \( V_p = C_{pq} \Gamma_q \). This leads to the evaluation of the pressure coefficient distribution using the relationship \( C_p = 1 - \left( \frac{V}{V_{\infty}} \right)^2 \). Once one obtains the \( C_p \) distribution, evaluating the aerodynamic loads, i.e. the lift (\( L \)) and the moment (\( M_L \)) is trivial. The obtained overall UVLM lift coefficient (\( C_L \)) values for the NACA0012 airfoil are compared to the solution sought by higher fidelity which based on Navier-stokes quations of incompressible viscous flow (CFD) as presented in Fig. 7. The CFD geometry is a 2D surfaces made of boolean subtraction of a centered NACA0012 airfoil and a large circle that extends 20 chords radius. The CFD domain is made of an O-ring mesh with clustered/boundary-layer near the earofoil walls. Fig. 7 shows the contours of the resulted CFD solution of the pressure coefficient (\( C_p \)) at different angle of Attack, i.e. 5, 7.5 and 10°. It also shows good agreements in the evaluated \( C_L \) values of high fidelity CFD and those of the UVLM. These results serves as a good validation to the Tornado code which is used in the subsequent analyses.
3.3 Piezoelastic System: The structural model is based on the simple cantilever beam lumped-parameter model. The lumped-parameter model works with a mass-spring-damper system. These are cantilever equivalent values where the mass, spring stiffness and damping coefficients are all lumped parameters of the cantilever beam, its structure, and geometry. The mass/spring/damper system is modeled according to the following system equation:

\[ M \ddot{z} + C \dot{z} + Kz = F \]  

(9)

Where \( M \) is the inertia matrix, \( C \) is the damping coefficient and \( K \) is the stiffness of the spring in the lumped-parameter model and \( F \) is the loading matrix. Also, \( z \) is the net displacement of the mass. Here the dynamics of the system can be analyzed to obtain the resonance frequency, \( \omega_R = \sqrt{\frac{K}{M}} \) and the damping ratio, \( \zeta = \frac{C}{2\sqrt{MK}} \). The stiffness is \( K = \frac{3EI}{L^3} \), where \( E \) is the elasticity modulus of the material, \( I \) is the moment of inertia of the cross-section of the beam and \( L \) is the length of the beam. Extending the system above for piezoelasticity the system of piezoelastic coupled equations is derived using Lagrangian mechanics as in [14, 22, 25] as:

\[ \ddot{h} + m_w x_\theta b \dot{\theta} + d_h \dot{h} + k_h(h)\dot{h} - \frac{C_p}{l} \dot{v} = -L \]  

(10.a)

\[ m_w x_\theta b \ddot{\theta} + I_p \dddot{\theta} + d_\theta \dot{\theta} + k_\theta(\theta)\dot{\theta} = M_L \]  

(10.b)

\[ C_p \ddot{v} + \frac{1}{l} \dot{v} + \gamma \dot{h} = 0 \]  

(10.c)

Here \( h \) is the plunge deflection, \( \theta \) is the pitch angle and \( v \) is the generated voltage. Also, \( m_e \) is the total mass of the wing apparatus that is permitted to heave, \( m_w \) is the mass that is permitted to pitch, \( b \) is the semi-chord length, and \( x_\theta = \frac{r_{c0}}{b} \) is the normalized distance between the center of mass and the elastic axis. \( I_p \) is the mass moment of inertia about the elastic axis at the reference point \( P \). \( L \) and \( M_L \) are the aerodynamic lift and moment about the elastic axis. The coefficients \( d_h \) and \( d_\theta \) are the plunge and pitch viscous damping coefficients and \( k_h \) and \( k_\theta \) are the structural stiffnesses for the plunge and pitch motions. If desired, nonlinear stiffness can be also considered and approximated as a polynomial as:

\[ k_h(h) = k_{h_0} + k_{h_1} h + k_{h_2} h^2 + \ldots \]  

(11.a)

\[ k_\theta(\theta) = k_{\theta_0} + k_{\theta_1} \theta + k_{\theta_2} \theta^2 + \ldots \]  

(11.b)

In this work, the effects of non-linear stiffness is not explored and hence stiffness values are considered fixed values.

3.3.2 The Piezoelastic Solution: Considering piezoelastic equations in 10, we obtain the following structural system of three variables \( h, \theta \) and \( v \):
\[
\begin{pmatrix}
    m_t & m_w x_\theta b & 0 & 0 \\
    m_w x_\theta b & I_p & 0 & 0 \\
    0 & 0 & C_p^{eq} & \frac{1}{\gamma} \frac{dy}{dt} \\
\end{pmatrix}
\begin{pmatrix}
    \dot{h} \\
    \dot{\theta} \\
    \dot{v} \\
\end{pmatrix}
+ \begin{pmatrix}
    d_h & 0 & 0 & -\frac{1}{l} \\
    0 & d_\theta & 0 & \frac{1}{\gamma} \\
    0 & 0 & k_\theta (\theta) & 0 \\
\end{pmatrix}
\begin{pmatrix}
    \dot{h} \\
    \dot{\theta} \\
    \dot{v} \\
\end{pmatrix}
= \begin{pmatrix}
    -L \\
    M_L \\
    0 \\
\end{pmatrix}
\] (12)

Now, taking \( M_{3 \times 3} = \begin{pmatrix} m_t & m_w x_\theta b & 0 \\ m_w x_\theta b & I_p & 0 \\ 0 & 0 & C_p^{eq} \end{pmatrix} \) and \( C_{3 \times 3} = \begin{pmatrix} d_h & 0 & -\frac{1}{l} \\ 0 & d_\theta & 0 \\ \frac{1}{\gamma} & 0 & \frac{1}{k_\theta (\theta)} \end{pmatrix} \) and \( K_{2 \times 2} = \begin{pmatrix} k_\theta (\hat{h}) & 0 \\ 0 & k_\theta (\theta) \end{pmatrix} \)

we write the system as:

\[
\begin{pmatrix}
    \ddot{h} \\
    \ddot{\theta} \\
    \ddot{v} \\
\end{pmatrix}
= -M_{3 \times 3}^{-1} C_{3 \times 3} \begin{pmatrix} \dot{h} \\ \dot{\theta} \\ \dot{v} \end{pmatrix}
- M_{3 \times 2}^{-1} K_{3 \times 2} \begin{pmatrix} \dot{h} \\ \dot{v} \end{pmatrix}
+ M_{3 \times 3}^{-1} \begin{pmatrix} -L \\ M_L \\ 0 \end{pmatrix}
\] (13)

To set up the state space system, the states are chosen as:

\[
\begin{pmatrix}
    \dot{h} \\
    \dot{\theta} \\
    \dot{v} \\
\end{pmatrix}
= A \begin{pmatrix}
    h \\
    \theta \\
    v \\
\end{pmatrix}
+ B \begin{pmatrix}
    0 \\
    0 \\
    -L \\
\end{pmatrix}
\] (14)

where the matrices \( A \) and \( B \) are written in the following block matrix form:

\[
A = \begin{bmatrix}
    [0_{2 \times 2}] & [I_{2 \times 2} & 0_{2 \times 1}] \\
    [-M_{3 \times 3}^{-1} K_{3 \times 2}] & [-M_{3 \times 3}^{-1} C_{3 \times 3}] \\
\end{bmatrix}
\] (15)

\[
B = \begin{bmatrix}
    [0_{2 \times 2}] & [0_{3 \times 3}] \\
    [0_{3 \times 2}] & [M_{3 \times 3}^{-1}] \\
\end{bmatrix}
\] (16)

The state vector \( \mathbf{x} \) is defined as \( \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} h \\ \theta \end{pmatrix} \) and the input to the system is \( \mathbf{u} = \begin{pmatrix} 0 \\ 0 \\ -L \\
M_L \\
0 \end{pmatrix} \). The problem now is set up as the derivative equation, \( \dot{\mathbf{x}} = A \mathbf{x} + B \mathbf{u} = f(\mathbf{x}) \) and the predictor-corrector method is used to solve this equation.

### 3.4. Overall System Solution:

The aerodynamic model and the piezoelastic model are coupled using the Adams-Bashworth-Moulton predictor-corrector method as described below. Predictor-corrector methods attempt to leverage the benefits of explicit and implicit methods for the predictor-corrector steps, respectively. The predictor-corrector methods for integrating ordinary differential equations have been widely used for the various advantages and chiefly the low computational complexity since only one or two derivative evaluations are necessary [20]. Also, the difference between the predicted and corrected values provides a relative error view of the solution and allows for the tweaking of the algorithm accordingly. The algorithm considers a differential equation with a time derivative represented as:

\[
\dot{y} = f(t, y)
\] (17)

Adams-Bashworth-Moulton (ABM) method is used to solve for \( y \). ABM method is a fourth-order method where the solution for \( y_{k+1} \) requires \( y_k \cdots y_{k-3} \). This requires an initialization step that computes the first three points \( y_0, y_1 \) and \( y_2 \). This is usually done using the Runge Kutta method. There onwards at each timestep the next value is predicted using the predictor and the resultant is used to implicitly correct using the corrector formula given as:

\[
p_{k+1} = y_k + \frac{h}{24} \left( -9f_{k-3} + 37f_{k-2} - 59f_{k-1} + 55f_k \right)
\] (18.a)

70
\[ y_{k+1} = y_k + \frac{h}{24}(f_{k-2} - 5f_{k-1} + 19f_k + 9f_{k+1}) \] 

(18.b)

The piezoelastic and aerodynamic systems are coupled by supplying the wing motion computed by the piezoelastic system to the aerodynamic system and supplying the aerodynamic loads computed by the aerodynamic system to the piezoelastic system. This coupling is illustrated in Fig. 8.

![Illustration of Piezoelectric-aero-elastic coupling system](image)

**Fig. 8: Illustration of Piezoelectric-aero-elastic coupling system**

UVLM can be used to compute lift, \( L \) and moment, \( M_z \) about the reference point \( P \). These values can be directly fed into the piezoelastic system and the ABM method can be used to solve the piezoelastic equation. The aerodynamic system requires wing surface velocities at each of the panels, \( V_{m,i} \). However, the solution to the piezoelastic system produces values for the heave velocity, \( \dot{h} \) and pitch angular velocity, \( \dot{\theta} \). While heave velocity, \( \dot{h} \) directly contributes to the \( z \)-component of \( V_{m,i} \), additional contribution to the various components of \( V_{m,i} \) are computed by applying simple trigonometry explained in Fig. 9.

![Computing \( V_{m,i} \) component from \( \dot{h} \) and \( \dot{\theta} \)](image)

**Fig. 9: Computing \( V_{m,i} \) component from \( \dot{h} \) and \( \dot{\theta} \)**

5. Results and discussion

The code to simulate the integrated system is implemented using MATLAB. For the UVLM code, the open-source vortex lattice method library of Tornado developed at Royal Institute of Technology (KTH) [21] is used. The code was modified to make it compatible with the unsteady problem and to couple it with the piezoelastic solution using ABM. The ABM implementation along with its Runge-Kutta initialization was also developed using MATLAB. The geometry of the wing is illustrated in Fig. 9. The geometry (geo) and the loading status files are used by the UVLM code to compute the aerodynamic loads. The wing geometry consists of the cambered NACA2412 airfoil of 1.0 m root chord length and a 2.0 m span length. The middle axis (MAC), reference point, and the centre of gravity located at \( \frac{1}{4} \) chord are per the depiction in Fig. 9.
Fig. 10: Wing geometry showing the reference point, center of gravity and middle chord

The values for the structural system parameters used in this work were chosen based on the empirical work of O’Neil et al [22]. The UVLM-ABM coupling was implemented following the scheme presented in Fig. 11. An intertwined coupling scheme was devised that follows a predict cycle between the ABM and UVLM codes before going through a correct cycle. During the predict step, ABM generates a prediction for wing motion along with other state variables. This is fed into the UVLM code that generates a prediction for aerodynamic loads which is in turn fed into the ABM corrector that updates the states before passing them on to UVLM. These corrected values then start the next round of predict and correct cycles for the next time step.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_w$</td>
<td>1.5 kg</td>
</tr>
<tr>
<td>$k_h$</td>
<td>2860 N/m</td>
</tr>
<tr>
<td>$d_h$</td>
<td>7.5 kg/s</td>
</tr>
<tr>
<td>$m_t$</td>
<td>10 kg</td>
</tr>
<tr>
<td>$k_\theta$</td>
<td>2</td>
</tr>
<tr>
<td>$d_\theta$</td>
<td>0.01 kgm$^2$/s</td>
</tr>
<tr>
<td>$R_t$</td>
<td>10kΩ</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.55</td>
</tr>
<tr>
<td>$C_p^{eq}$</td>
<td>1μF</td>
</tr>
</tbody>
</table>

Fig. 11: UVLM-ABM coupling scheme

5.1 Flow results of the UVLM: Fig. 12(a) shows the panels generated by the UVLM code on the airfoil while the Fig.12(b) shows the panels on the airfoil and wake. The surfaces are discretized to 10 by 20 panels chordwise and spanwise, respectively with vortices distributed at $\frac{1}{4}$ chord and the control points distributed
\[
\text{at } \frac{3}{4} \text{ chord of each of the panels. The figure also shows the trail of vortices distributed over the wake and collocation points are only placed on the panels bound to the wing surface where the loads are to be computed.}
\]

\[\text{Fig. 12: 3-D panels, collocation points on the mean surface and surface normals, (a) airfoil, (b) airfoil and wake}\]

\[\text{Fig. 13 shows computed pressure coefficient distribution on the wing as obtained from the UVLM code. Results are shown at two values of } \alpha \text{ i.e. } 0^\circ \text{ and } 5^\circ. \text{ This pressure distribution is translated to lift, } L \text{ and moment, } M_L \text{ and fed into the ABM piezoelectric system solution code. The results show an appropriate trend where the pressure coefficient (Cp) depicts values of 0.3 and -0.1 near the leading edge at the normal incidence (}\alpha=0^\circ\text{) and } \alpha=5^\circ \text{ respectively and proceeds to decrease downstream.}\]

\[\text{Fig. 13: dCp distribution over the wing, (a) at } \alpha=0^\circ, \text{ (b) at } \alpha=5^\circ\]

\[\text{5.2 Results of the Coupled Piezoelectric System: In this analysis the incident airspeed (} V_\infty \text{), the angle of attack (} \alpha \text{) and the reference point offset (} a \text{) are considered the inputs to the system and looped over various values of these parameters while the response data was collected. Fig. 14 shows the advent of flutter as the incident air velocity reaches } 40 \text{ m/s at } \alpha = 5 \text{ deg. The heave velocity and the output voltage grow with time creating an unstable condition. Evidently, this operating point does not favor the structural integrity of the aircraft; however it provides opportunity to quickly harness a good amount of electric energy.}\]
As incident airspeed increases to $55 \text{ m/s}$, the heave velocity and consequently the generated output voltage stabilizes as shown in Fig. 15.

Extensive analyses can be performed on the rich data that has been obtained from the simulation as per the incoming wind velocity sweep and their corresponding temporal and frequency response in Fig. 16 through Fig. 23. The states for a range of incident airspeeds are evaluated. The frequency spectra of voltage, heave velocity, and pitch angular velocity show clear peaks at around $1.1 \text{ kHz}$. The lift and its frequency response are characterized by an additional harmonic. The 1st harmonic frequency is a function of the structural, material and geometric parameters of the system. It is also interesting to note that the lift spectrum clearly shows two harmonics of this resonance frequency, one at around $1.1 \text{ kHz}$ and the other at around $2.2 \text{ kHz}$ as depicted in Fig. 23.
Fig. 16: Output voltage transient response

Fig. 17: Output voltage frequency response
Fig. 18: Heave velocity transient response

Fig. 19: Heave velocity frequency response
Fig. 20: Pitch transient response

Fig. 21: Pitch frequency response
At low air speeds, i.e., $V_\infty < 45$ m/s, Fig. 19 shows an exponentially growing heave velocity, i.e. unstable flutter. Inspection of Fig. 23 shows that at these low speeds a second harmonic at 2.2 kHz appears in the frequency response of the Lift force ($L$), but does not appear in the heave (structural) frequency response. This 2nd harmonic appears to be the cause of unstable flutter. Furthermore, inspection of Fig. 17 indicates...
that, during unstable flutter, the piezo material generates low voltage compared to the case with stable flutter. These observations can be interpreted as follows:

1. The 2nd harmonic of lift occurring at 2.2 kHz does not appear in the lumped mass structural vibration, hence it seems not to be structurally related; rather it is associated with the aerodynamic part of the system. Since it causes unstable flutter, it implies negative damping. This negative damping appears in the form of structural instability at 1.1 kHz which is the structural resonant frequency. This could be attributed to certain flow characteristics that take place at low airspeed leading to self-induced vibration of the structure at 1.1 kHz resonant frequency. This can also be explained by certain flow characteristics at 2.2 kHz, that have the effect of negative damping in the structure at a wide band of frequencies, including the structural own resonant frequency of 1.1 kHz. Furthermore, the 2.2 kHz is the 2nd harmonic of 1.1 kHz indicating that it is intimately related to the Lift, i.e. flow characteristics.

2. The fact that unstable flutter produces low voltage, and knowing that piezo voltage depends on the direction of piezo material strain, indicates that the instability of the structure at 1.1 kHz is caused by the vibration of the airfoil at a coupled bending-torsional mode. This mode does not produce large bending vibration (heave), i.e. high voltage, but affects the flow such that the 2.2 kHz harmonic appears in the lift spectrum. The reason for this interpretation is that the structural model is 2-DOF lumped mass model, while the aerodynamic model which generates the Lift force, is based on the actual wing (airfoil) in the UVLM. Therefore, the aerodynamic model is affected by the 2.2 kHz Lift resonance, while the lumped mass structural model will not exhibit any vibration at that frequency because it is 2-DOF lumped mass model that is characterized by only two resonant frequencies.

Therefore, more fidelity in the model can be achieved by the adaptation of the time-steps to reduce solver error and using other predictor-corrector algorithms such as Hamming’s 4th order method. Additionally, including cross-span heave variations and large beam deflections as well as incorporating a distributed beam model instead of the lumped-model to accommodate tapered cantilevers as per the recent work and reconditions or elsewhere in the literature [23, 24].

6. Conclusion
The Unsteady Vortex Lattice Method (UVLM) coupled with Adams-Bashworth-Moulton method forms a viable solution for the lumped-parameter piezoaeroelastic system. The system solves efficiently and allows for transient and frequency analyses. The heave velocity and the output voltage is shown to grow with time creating an unstable condition at specific and fixed wind velocity. At these conditions of airspeed ($V_a$), the angle of attack ($\alpha$), and the reference point offset ($\delta$), the advent of flutter was provoked and the system energy was harnessed. A sweep of incident airspeeds was attempted and resulted in a clear peak in the frequency spectrum which is a function of the structural, material and geometric parameters of the system. It is also interesting to note that the lift spectrum clearly shows two harmonics of resonance frequency in which the 2nd harmonic suggested to be non-structurally related, but rather aerodynamically induced. Also as unstable flutter produces low voltage and knowing that piezo voltage depends on the direction of piezo material strain, this indicates that the instability of the structure is caused by the vibration of the airfoil at a coupled bending-torsional mode. This mode does not produce large bending vibration (heave), i.e. high voltage, but affects the flow that attributed to the appearance of the 2nd harmonic in the lift spectrum.
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References


