BERNOULLI LOOPS WITH NO FRICTION

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Published: 26 November 2019

ABSTRACT
Examples of closed fluid flow streamlines, along which Bernoulli's law applies, are collected together. To be in the collection the flow must be steady and a cross-stream force balance maintained, one of which is always a pressure gradient. The equal but opposite force in the cross-stream balance is either the Coriolis force or the centrifugal force. Solutions for the velocity or velocity shear are obtained and it is shown that in all cases the friction term is zero according to the Navier-Stokes equations. Applications of some of the model solutions to geophysical and laboratory phenomena are discussed in view of the lack of friction and how adding in friction might change the situation.

Keywords: Bernoulli Loops, No Friction

1. Introduction

A Bernoulli loop is a closed streamline along which Bernoulli's law appliesin a steady flow [1]. Steadiness is provided by a cross-stream force balance involving a pressure gradient and an equal but opposite force, such as the centrifugal or Coriolis one. For simplicity the loop is taken to be circular. There are two classes of such Bernoulli loops: one for which the centrifugal force exceeds the Coriolis force and the other is the reverse of that. Examples of the second class, where the Coriolis force is dominant, include the Hurricane [2],and certain large-scale outbreaks of cold dry air moving south in the North Pacific [3]. Solid body rotation of a contained fluidis in the first class [1] along with smoke rings [4] and possibly the tornado.

In order to maintain a steady state, when Bernoulli's law is combined with the cross-stream force balance, in which a pressure force balances either the Coriolis or the centrifugal force, an equation for

Cite this article: Kenyon, K.E. Bernoulli Loops With No Friction. *European International Journal of Science and Technology*, 8(10), 25-28.

the velocity itself, or the radial shear in the velocity, is the main result. For the large-scale flows the shear equals a constant, the Coriolis parameter, and for the small-scales the flow speed decreases inversely as the radius increases, except for solid body rotation where the flow speed varies linearly with the radius. Remarkably in all these theoretical cases the friction force vanishes according to the Navier-Stokes equations in plane polar coordinates [5].
If any of the models apply to a geophysical phenomenon, then the absence of friction would tend to make it last longer without constantly having to be maintained by a generating mechanism. In other words, the particular fluid velocity structures which ideally have zero friction might be expected to be more commonly found in nature. And under such circumstances wind speeds of 100 mph or more could last an amazingly long time.

2. Method

Common to both models is Bernoulli's law

$$p = const - \frac{1}{2}\rho V^2 \tag{1}$$

Valid along a streamline in steady flow, where *p* is the pressure and *V*is the flow speed tangent to the streamline. It is assumed that the *const* is the same for all streamlines and ρ is the constant fluid density.
Normal to the flow the force balance for the large-scale is

$$\frac{\delta p}{\delta r} = \rho f V \tag{2}$$

The geostrophic relation, where *f* is the Coriolis parameter and *r* is the outward distance along a radius in the circular geometry.
Eliminating the pressure between (1) and (2) gives

$$-\frac{\delta V}{\delta r} = f \tag{3}$$

Showing that the velocity shear is a constant for a given latitude.
The smaller-scale cross-stream force balance is

$$\frac{\delta p}{\delta r} = \rho \frac{V^2}{r} \tag{4}$$

And when *p* is eliminated between (1) and (4) the result is

$$-\frac{\delta V}{\delta r} = \frac{V}{r} \tag{5}$$

Which has the solution for *V*

$$V = \frac{const}{r} \tag{6}$$

It can be seen almost immediately from [Ref. 5] that there is no friction force connected with Equations (3) or (6).

That the velocity structure (6) has no associated friction might be a surprise, but see [7] for a different derivation not involving Bernoulli's equation and a different suggested application involving flow outside a rotating cylinder. If (6) were to be applied to the circular fluid particle orbits of surface gravity waves, that would provide a physical explanation for why these waves have been observed to travel so far without decaying, such as half way around the world on occasion.

3. Discussion

Based on a typical maximum wind speed of 100 mph just outside the eye of a hurricane and a typical radius of 500 km, a shear equal to the Coriolis parameter appears to be a possibility, as was suggested recently [2]. Then in the literature [6] a specific example was found whereby a hurricane went through a small town on the edge of the Gulf of Mexico and the weather station there kept on working, producing a continuous record of wind speed as a function of time (Gregory, TX, August 3, 1970, Hurricane Celia). Before the eye arrived the wind speed increased very nearly linearly, and after the eye passed by it decreased approximately linearly also. If the translational speed of the hurricane, which was not reported, was constant, these linear relations are what would be expected for a hurricane with constant radial wind shear. Whether or not the shear equaled f cannot be ascertained from the information given.

Although Equation (6) was put forward as a way to understand the dynamics of a smoke ring [5], no observational material has yet been uncovered that could confirm this idea. If Equation (6) is in any way relevant to the workings of a tornado, it is a potential project for developing in the future as well.

Friction is involved with the growth of the motion. For example, in solid body rotation growth starts at the side of the rotating container and penetrates radially inward. When the linear velocity profile is reached, friction vanishes. Steady state is complete.

Just the opposite happens in a hurricane. A spurt of growth takes place near the eye. All the details are not spelled out but one component is the release of heat due to condensation of moisture which increases the updraft just outside the eye. Somehow the horizontal rotating component of the flow is increased also. Then friction communicates (diffuses) this increased speed radially outward until the constant radial shear is attained and friction ceases. Now the size of the hurricane is a bit larger.

4. Conclusions

Bernoulli loops with a cross-stream force balance can be frictionless. There are two classes of such dynamic steady flows in circular geometry, and both involve a radial pressure gradient: 1) in the large-scale case the Coriolis force dominates over the centrifugal force, and 2) just the other way around in the small-scale case. A few applications to geophysics and laboratory fluid mechanics have been proposed recently, such as the hurricane in the atmosphere and solid body rotation of a contained fluid in the lab. Future studies may include the smoke ring and the tornado. More observations are needed for all the projects.

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