A NOTE ON PERFECTLY CONTINUOUS FUNCTIONS

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ABSTRACT
The concept of perfectly continuous functions was introduced by Noiri [4]. In this paper some more properties of perfectly continuous functions are obtained.

INTRODUCTION

In 1984, Noiri [4] introduced the class of perfectly continuous functions and investigate its relationship with other strong forms of continuity such as complete continuity [1], strong continuity [2]. In this paper we give new results on perfectly continuous functions.

Definition 1. A function $f: X \to Y$ is said to be perfectly continuous [4] if the inverse image of every open set in Y is both open and closed in X.

Definition 2. A function $f: X \to Y$ is said to be strongly continuous [2] iff the inverse image of every open subset in Y is open and closed in X.

Definition 3. A function $f: X \to Y$ is said to be completely continuous [1] if the inverse image of every open set in Y is regular open in X, where A is regular open iff $A = \mathrm{Int}(clA)$.

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Noiri proved that:
Strong continuity $\Rightarrow$ Perfect continuity $\Rightarrow$ Complete continuity $\Rightarrow$ continuous.
He proved also by means of examples that the converse implications are not true in general.

Theorem 1. Every perfectly continuous function into a T_1 space is strongly continuous.

Proof: Let $f : X \to Y$ be perfectly continuous and Y is a T_1 space. Let A be any subset of Y. Then $f^{-1}(A)$ is open and closed in X, since singletons are closed in a T_1-space.

Theorem 2. If $f : X \to Y$ is a perfectly continuous function from a connected space X on to any space Y, then Y is an indiscrete space.

Proof: If possibl, suppose that Y is not indscrete . Let A be a proper non-empty open subset of Y. Then $f^{-1}(A)$ is a proper non-empty clopen subset of X, which is a contradiction to the fact that X is connected.

Theorem 3. Every perfectly continuous function from an extremally disconnected space is perfectly continuous.

Proof: The proof is obvious since every regular open subset in an extremally disconnected space is clopen.

Theorem 4. Let $f : X \to Y$ be perfectly continuous and one-to-one. If Y is T_0, then X is Urysohn.

Proof: Let a and b be any pair of distinct points of X. Then $f(a) \neq f(b)$. Since Y is T_0, there exists an open set U containing one of them say f(a) but not f(b). Then $a \in f^{-1}(U)$ and $b \notin f^{-1}(U)$ is clopen. f is perfectly continuous implies $f^{-1}(U)$ is clopen. Also, $a \in f^{-1}(U)$ and $b \in X - f^{-1}(U)$. Thus a and b are separated by disjoint closed sets. Hence X is Urysohn.

Theorem 5. If X is connected and Y is T_0, then the only perfectly continuous functions $f : X \to Y$ are constant functions.

Proof: If possible, let f(X) consist of more than one point. But then f(X) is T_0 also, which is a contradiction.

Definition 4. A function $f : X \to Y$ is said to be almost open [3] if the image of every regular open set is open.

Theorem 6. The image of a locally connected space under a perfectly continuous almost open function is locally connected.

Proof: Let $f : X \to Y$ be perfectly continuous and almost open. Let X be locally connected. Let U be any open subset of Y and $f(x) \in U$. Then $x \in f^{-1}(U)$. Now, $f^{-1}(U)$ is open and closed in X. Since X is locally connected, there exists a connected open subset V of X such the

$x \in V \subseteq f^{-1}(V)$. But $f^{-1}(U)$ is closed, therefore, $x \in V \subseteq cl(V) \subseteq f^{-1}(U)$. Hence V and cl(V) are connected. Also, int(clV) is connected. Clearly, $x \in int(clV) \subseteq f^{-1}(U)$. Hence $f(x) \in f(int(clV)) \subseteq U$. Now, f(int(clV)) is connected and f(int(clV)) is open since f is open. Hence Y is locally connected.

***<u>Theorem 7</u>*.** If $f : X \to Y$ is perfectly continuous and $A \subseteq X$, then $f_{|A} : A \to Y$ is perfectly continuous.

***<u>Proof</u>*:** Let V be any open subset of Y. Then $f^{-1}(V)$ is clopen in X. Then $f^{-1}(V) \cap A = (f_{|A})^{-1}(V)$ is clopen in A. Hence $f_{|A}$ is perfectly continuous.

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