BERNOULLI LOOP

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ABSTRACT
Applying Bernoulli's Law to the closed streamlines in a steady fluid flow appears not to have been done before. Here the method is used to facilitate the understanding of solid body rotation of a contained fluid, which has caused confusion among physicists for a long time.

Keywords: Bernoulli, Solid Body Rotation

1. Introduction

Many configurations involving Bernoulli's law have been put down in print [1], some constructive and some destructive. By constructive is meant additions of terms to the law that improve the understanding of existing circumstances, and by destructive is meant shredding of the law by a mathematical method such that the normal terms are separated or scattered away from each other.

Here is presented an application that appears to be new: Bernoulli's law along a streamline that is a closed circuit or a loop. Is there any reason a priori why such an extension should not be attempted? In most presentations of the law along a streamline, the beginning and end of the streamline are never mentioned, as if they were of no particular importance.

To start the ball rolling one concrete example is examined: solid body rotation of a contained fluid. In the field of physics this has been a very controversial subject for several hundred years, and still is, the culprit being the centrifugal force [for a non-reference, which speaks more volumes that a real one, consult the index of a typical contemporary high-school physics text; centrifugal force will not be listed there in all probability]. Never mind, push on.

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2. Model

Consider a vertical solid cylinder, hollow with a flat bottom and open top, containing water that has been rotating at a constant rate for a long enough time that the water has reached a steady state. Along all streamlines of the flow Bernoulli's law applies

$$p = const - \frac{1}{2}\rho V^2 \tag{1}$$

Where p is the pressure, ρ the constant density and V the tangential speed. For simplicity the constant is taken to be the same for every streamline.

For flow along a curved streamline there is a centrifugal force that points radially outward. During the steady state there must be an equal but opposite force to balance the centrifugal force, and in the present situation it can only be a pressure gradient. Consequently the cross-stream force balance is

$$-\frac{dp}{dr} = \frac{\rho V^2}{r} \tag{2}$$

Where r is distance along a radius.

Equations (1) and (2) are two equations in the two unknowns *p,V* from which one variable can be eliminated to get one equation in one unknown. Taking the derivative of (1) with respect to r and combining with (2) produces one equation for V

$$\frac{\mathrm{dV}}{\mathrm{dr}} = \frac{\mathrm{V}}{\mathrm{r}} \tag{3}$$

Which has the solution *V=const r*. The speed varies linearly with the radius, and inserting that solution into (1) shows that the pressure varies quadratically with the radius or parabolically.

A velocity that is a linear function of the radius is in complete agreement with the concept of solid body rotation. Although a convex parabolic shape of the water surface is noticed qualitatively, no relevant quantitative measurements have come to my attention.

Reference

[1] Kenyon, K. E. (2018) Believing Bernoulli. Natural Science, 10, No. 4.