SHALLOW WATER WAVE PHYSICS

Kern E. Kenyon

4632 North Lane
Del Mar, USA
Email: kernken@aol.com

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ABSTRACT
Einstein's elementary water wave model [1] is adapted to surface gravity waves propagating in shallow water, and it is carried to the point where the classical phase speed formula is derived: $c = \sqrt{gh}$*, where c is the phase speed, g the acceleration of gravity, and h the mean depth of water, which is assumed to be much smaller than the wavelength. Thus a shorter way to explain why tsunamis travel so quickly in the deep ocean depths is presented in contrast to the lengthy mathematical discussions found in the literature.*

Keywords: Tsunami, phase speed

1. Introduction

How can the majority of people understand the incredibly fast speed of a tsunami in the open ocean? Since the normal mathematical theory is very long and basically not accessible to them, a short explanation is given here that is more physical. Mastery of Bernoulli's law is all it really takes: where the speed is greatest, the pressure is least, and vice versa. This law applies along the streamlines of a steady fluid flow.

Either the phase speed formula of a shallow water surface gravity wave is accepted and used, or the text book discussion is attempted after the prerequisites have been learned. First, potential flow methods and assumptions need to be swallowed. [Irrotational flow is the main assumption, but it has no purpose here.] Then perturbation expansions of the nonlinear equations of fluid motion are required. Finally the results for an arbitrary constant mean depth of water are reduced to the limit of a shallow water depth, whereby

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the wavelength is much greater than the mean depth. After all that complicated manipulating, it is rather amazing that the simple phase speed relation comes out.
When ocean depths far from land (4,000 m) are inserted into wave speed formula, speed equals the square root of mean depth time the acceleration of gravity, it is easily seen how the high speeds arise. But understanding it takes a little bit of work anyway.

2. Model

Here follows an adaptation of Einstein's 1916 nutshell physical explanation of the surface gravity wave [1]. This adaptation to shallow water waves has not ever happened, as far as I am aware, and it is very suitable for pushing the model to the point of obtaining the speed formula of the waves.
Adopt the so-called steady reference frame in which the wave shape does not move but the water underneath it does. Assume two things: 1) under the crest and trough, in particular, the horizontal flow speeds do not vary with the distance below the wave surface, and 2) the square of the wave amplitude *a* is small compared to the square of the mean depth *h*.
Following Einstein, straight out of Bernoulli's law, the static pressure difference between crest and trough [LHS] is set equal to the dynamic pressure difference between crest and trough[RHS] in (1).

$$2\rho g a = \frac{1}{2}\rho u_1^2 - \frac{1}{2}\rho g u_2^2 \tag{1}$$

Where ρ is the constant fluid density, g is the acceleration of gravity, $a$is the wave amplitude ($2a$ is the wave height), and the flow speeds under trough and crest are respectively:u_1, u_2.
Conservation of mass between crest and trough (leaving out the density) is

$$u_1(h-a) = u_2(h+a) \tag{2}$$

Equations (1) and (2) are two algebraic equations in the two unknowns u_1 and u_2; where *h,a,* and *g* are given. Normally a solution is expected in such a situation.
First, convert the reference frame from the steady one to the fixed one where the wave shape passes by the person watching with phase speed *c*. The transfer relations are

$$u_1 = c + \Delta u$$
$$u_2 = c - \Delta u \tag{3}$$

Where Δu is small compared to *c* and does not need to be specified. Everything is in place and with no further assumptions or approximations the speed formula is readily obtained from the above three equations.

$$c = \sqrt{(gh)} \tag{4}$$

3. Discussion

One thing Einstein said in his short paper that might be helpful to some readers is to initially imagine the surface is a rigid sinusoidal lid. Adjust the mean flow speed until the static and dynamic pressure differences between crest and trough are balanced on the lid. Then the lid can be removed without disturbing the wave shaped free surface. That adjusted mean flow speed turns out to be the phase speed of the wave after converting to the fixed reference frame.

4. Conclusion

Very fast speeds of a tsunami across the deep ocean are made understandable by using an elementary shallow water wave model containing more physics than mathematics to derive the classical phase speed formula: speed equals the square root of the acceleration of gravity times the mean water depth. As a result a considerably shorter explanation is presented than can be found in the literature.

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This study resulted from a question Phil Schneider asked me recently.

REFERENCE

Einstein, A. (1916) ElementareTheorie der Wasserwellen und des Fluges. Die Naturwissenschaften, **4**, pp. 509-510.
For an English translation see Kenyon, K. E. and D. Sheres (1997) Einstein's gravity wave method applied to a two-layer fluid with current shear. Phys. Essays, **10**, 60-62.