TRAPPED CAPILLARY WAVES

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ABSTRACT
Observations suggest that capillary waves can be trapped at a vertical wall and propagate along it. A physical model is proposed to explain how that might happen.

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1. Introduction

For several years my attention has been drawn to a curious wiggling of the water surface going on along the far wall of the swimming pool, which I can see through the window while sitting inside the house. It happens every time a gentle breeze blows through the back yard. When I go up close, bend down and put my face near the surface of the water, the phenomenon does not present itself as well, and I do not learn anything more.

Two unusual features appear to be happening at the same time: a trapping of the wiggling surface at the wall as well as a rather rapid movement either to the right or to the left. Light winds can only create small-scale capillary waves on the surface of a body of water the size of a swimming pool. But none of my six or seven fluid dynamics text books have anything to say about capillary waves being trapped at a vertical wall. Also the speed of the wiggling to the right or the left is faster than that expected from that observed when the usual capillary waves progress across the middle of the pool.

A model is proposed below in order to attempt to explain the strange wiggling of the water surface which is apparently stuck to the wall.

If a trapped capillary wave can exist, then that distinguishes it further by comparison with the surface gravity, which has not ever been found to be able to become trapped at a vertical wall.

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2. Model

Suppose the wind pushes some relatively small pieces of water against the wall so that they rise up a short distance against gravity and stick there a short time by the effect of surface tension. A shape that might work is a series of equally spaced pointed crests at the wall with rounded troughs between each pair of crests. Surface tension could hold that shape to the wall if the thickness in the direction normal to the wall increases outward and downward from the crest in a curve with the concave side pointing up and away from the wall. Small spatial scales of the wiggling surface, within the surface tension range, are what observations indicate, although this needs to be made quantitative by photography or some other means. Also the particular wave shape envisioned here needs to be verified.

What the wind creates in an instant cannot remain that way because of two instabilities at work. First, the pointed crests will try to be pulled down by gravity. Second, the rounded troughs want to flatten out and move up due to surface tension. But evidently there is one way the unusual wave shape can be maintained if it moves horizontally along the wall at just the right speed. What might that speed be?

Freeze the wave shape and consider the flow underneath it. Under a crest the static pressure due to gravity must be balanced by the dynamic pressure of the flow. At the same time the upper pressure of the trough must be balanced by the same downward dynamic pressure, assuming the flow speed is the same under crest and trough. Converting to the reference frame fixed to the wall, this flow speed transforms into the phase speed of the wave.

To estimate the phase speed, the simplest way is to use the crest height h for the static pressure due to the acceleration of gravity and then figure out what flow speed u there would need to be to produce a dynamic pressure that equals it. Leaving out the constant fluid density, the result is

$$\frac{1}{2}u^2 = gh \text{ or } u = \sqrt{2gh} \qquad (1)$$

Notice that the phase speed $c=u$ for this type of wave does not depend on the wavelength or the surface tension coefficient like the standard capillary dispersion formula does. Thus the possibility opens up for the trapped capillary wave to travel faster than the traditionally propagating one.

Next, given the value of the surface tension coefficient between air and water, can the radius of curvature of a trough be found such that the upward pressure due to surface tension balances the downward pressure of the flow in (1)? A priori it would seem that the answer is yes, by starting with Young's formula for the difference in pressure Δp across a curved surface between air and water

$$\Delta p = \frac{\gamma}{R} \qquad (2)$$

Where γ equals the force per unit length due to surface tension, and R is the principle radius of curvature.

3. Conclusions

Evidence is gathering to support the idea that capillary waves can be trapped at a smooth vertical wall, and propagate along it, and that they may propagate faster along the wall than they do normally through open water. More observations are needed to better delineate the wave shape and speed of these waves. Also the generation process by the wind is not understood at all. Then modifications of the above proposed model might be required even though it seems physically plausible at present.