ATMOSPHERE'S SCALE-HEIGHT: A COMMENT

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ABSTRACT
An attempt is made to clear up some of the confusion surrounding the meteorological concept: scale-height. First, there is no exponential decay from the ground upward by pressure, temperature or density that applies to the lower atmosphere (troposphere). Second, no two variables have the same decay rate. These points are illustrated by a short calculation that combines the perfect gas law, the hydrostatic balance of forces in the vertical and a constant temperature gradient based on observations.

Keywords: Scale-Height, Lower Atmosphere

1. Introduction

Scale-height, a dynamical concept in meteorology [1], has some confusion attached to it as two questions will bring out (see also Wikipedia's discussion). Does scale-height imply an upward exponential decrease of a variable (temperature, pressure, density) with increase of altitude from the ground? Second, does it apply to just one variable or to more than one? In magnitude the scale height is estimated to be about 8 km. But what does that mean?

Several textbooks of meteorology [2,3,4] exhibit schematic vertical profiles of temperature and pressure as functions of altitude although curiously they almost never occur on the same page. In the troposphere the temperature on these diagrams appears to decrease upward linearly whereas the pressure looks like it decreases exponentially. Then the wonder is: are these two profiles selfconsistent and do they fit in with the known laws of physics and chemistry of the lower atmosphere? In the brief calculation below the answers will both turn out to be no.

What is a fundamental fact is that all three variables of the troposphere, temperature, pressure and density, decrease with increase of altitude. Surprisingly each one is predicted to decrease at a different rate, none of which is exponential.

2. Calculation

A normal way to begin is to assume that the air in the lower atmosphere is a perfect gas. Then the equation of state from chemistry is [5]

$$p = \rho RT \tag{1}$$

Where p, ρ, T are variables that are functions of altitude, z, and R is the gas constant.

From physics comes a relation apparently so basic that it is not always explicitly displayed, the hydrostatic balance of forces in the vertical direction between gravity and the pressure force on each fluid particle of air

$$\frac{dp}{dz} = -\rho g \tag{2}$$

Where z is positive upward and g is a constant. Two equations in three unknowns.

Since a number of meteorology textbooks show schematically that the air temperature in the tropopause apparently deceases linearly with increasing altitude from the ground, then that function is selected for treatment here because also it is easily handled analytically. Then there are two equations in two unknowns. Therefore choose *T(z)* to be

$$T = T_0(1 - bz) \tag{3}$$

Where the constants T_0 and b are to be obtained from observations.

Combine (1) and (2) by eliminating the density and rearrange

$$\frac{dp}{p} = \frac{gdz}{RT_0(1 - bz)} \tag{4}$$

Each side of (4) can be integrated separately and then raised to the power of the exponential *e* to get

$$\frac{p}{p_0} = [1 - bz]^n \tag{5}$$

Where $n = g/bRT_0$ and p_0 is a constant. Before evaluating the exponent n of the bracket in (5) it can be seen that the pressure will not decay exponentially and it will not turn out to be exactly like the temperature variation in (3). Consequentially the density will have a different altitude decay than either the pressure or the temperature does.

Taking the temperature gradient to b 1 C per 1 km, T_0=300 K, the normal value for g and the appropriate value of R for dry air results in n=0.57; thus RHS of (5) is nearly the square root of the quantity in brackets.

3. Discussion

So where does the notion of an exponential scale-height of the atmosphere come from? Probably the origin is theoretical, an idealization: isothermal atmosphere, although such a characteristic does not come close to observations in the troposphere anywhere around the world. But if that model is put into equations (1) and (2), the pressure variation comes out exponential, and the density is exponential also from (1).

Of the three meteorological unknowns temperature is most familiar from human experience. Anyone who has climbed a mountain anywhere in the world knows that the air temperature gets colder the higher he or she climbs. And if the pilot announces or displays the air temperature outside the plane it will be very much lower than on the ground at the cruising altitude everywhere planes fly. No evidence for an isothermal troposphere exist to my knowledge.

4. Conclusion

Scale-height is not a useful term because each of the three meteorological variables, temperature, pressure and density, has its own altitude decay rate in the troposphere. Also an exponential decay rate does not apply to the troposphere. These conclusions stem from a model combining the perfect gas law, hydrostatic balance and a constant temperature gradient consistent with observations.

References

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