

THERMAL CONVECTION WITH LEAST FRICTION

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ABSTRACT

In Benard's thermal convection problem the fluid has several possible patterns that the cells can take in order to accomplish the flux of heat. Among the three regular polygon shapes the hexagonal one has the least overall amount of friction according to the elementary model presented, and this shape is the one most often observed in laboratory experiments. So far it appears that the standard linear instability theory has not come up with this prediction.

Keywords: Thermal Convection, Least Friction

1. Introduction

In the theory of thermal instability inspired by Benard's experiments with a very thin horizontal layer of fluid heated from below, there are different plan forms available for the circulation to take [1,2]. The regular cell shapes are: triangle, square and hexagon. According to the linear theory the mode with the fastest growth rate is the one the observed disturbance should take. That is the reigning philosophy.

Although existing observations tend to favor hexagonal cells, apparently the classic theory has not yet been able to show that this circulation shape has the fastest growth rate. What is

offered here is a different philosophy that is qualitatively consistent with the majority of experimental results. It can be designated as the principle of least friction. Recently this principle has been applied to a very much larger-scale phenomenon: the tornado [3].

2. Method

Consider the following elementary model. Known from experiments is that no matter what the cell plan form is, relatively warm fluid rises in the cell's center and relatively cool fluid sinks along the edges between neighboring cells. For regular polygons of n sides the area A is

$$A = \frac{1}{4}na^2\cot\left(\frac{180^\circ}{n}\right) \quad (1)$$

Where a is the length of a side. The circumference C is na . In terms of the circumference (1) can be written as

$$A = \frac{1}{4}C^2\left(\frac{\cot\left(\frac{180^\circ}{n}\right)}{n}\right) \quad (2)$$

3. Interpretation

By means of the vertical coordinate the area A is related to the volume of relatively warm fluid rising up and therefore to the heat flux. Also the circumference C is related to the side surface of the cell along which the cooler water flows down and where the greatest amount of friction occurs.

Assume the heat flux is constant independent of the plan form of the cells, so the left side of Equation (2) is constant. Within the outer brackets on the right side of (2) the values 0.19, 0.25, and 0.29 correspond to the following values of: n 3, 4, 6.

Consequently Equation (2) predicts that the amount of friction will decrease as the number of cell sides increases.

4. Conclusion

Using the principle of least friction in fluid flows an explanation is provided for the appearance of hexagonal cells in Benard's thermal convection more times than the other polygonal shapes.

References

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