LIFT ON A CIRCULAR ARC WING II

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ABSTRACT
Lift force formula on a circular arc wing equals fluid density times the square of the uniform flow speed far from the wing divided by the radius of curvature of the wing. It is derived from Bernoulli's law along all streamlines and the cross-stream force balances on all fluid particles between the upward centrifugal force and downward pressure force. Incorporated in the lift as part of the derivation is the novel velocity function above the wing which is inversely proportional to the radius from the center of the circle of which the arc is part. This radial variation needs to be verified by observations. A couple of approximations have been used in the lift equation.

Keywords: Lift Force, Circular Arc Wing

1. Introduction

This calculation of the lift force on a circular arc wing essentially confirms an earlier one [1], but it has the advantage of being shorter and easier to understand. Also a new insight is involved. If high quality streak photographs were available for the steady flow over a circular arc wing, then the previous computation would be the more appropriate model to work with. In the likely absence of

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such observations however any novel approach to increasing knowledge of the lift force on a wing should be welcomed. Awareness of the great importance of flying people and cargo can urge such studies forward. Consider the vast amount of theoretical material on flight, printed since about 1900, based on one foundation: potential flow in fluid dynamics. Ever so gradually a few fluid mechanical problems that have been solved by the potential flow method have started to acquire some criticisms [2]. That is a second motivation for new approaches.

2. Model

For all streamlines of steady flow that are above (or below) the circular arc wing, Bernoulli's law applies

$$p = const - \frac{1}{2}\rho v^2 \tag{1}$$

Where p is the pressure, $\rho$is the fluid density, taken constant, and v is the speed of the flow tangent to the streamline. For ease of presentation the constant on the RHS of (1) is assumed to be the same for all streamlines, and this is not considered to have any significant influence on the dynamics.

For steady state conditions every fluid particle on all streamlines going over the crest of the wing must experience a balance of two forces: an upward centrifugal force on the RHS of (2) and a downward pressure gradient on the LHS of (2).

$$\frac{dp}{dr} = \frac{\rho v^2}{r} \tag{2}$$

Where r measures distance from the center of the circle of which the wing's arc is a part.

Eliminating the pressure between (1) and (2) by taking the derivative of (1) with respect to r gives

$$\frac{dv}{dr} = -\frac{v}{r} \tag{3}$$

Which has the solution

$$v = \frac{const}{r} \tag{4}$$

What remains is to evaluate the constant in (4). First, an important note is that the functional form of (4) with respect to r produces zero friction according to the Navier-Stokes equations in polar coordinates [3]. Application of (4) to the lift on a wing is the new insight mentioned above.

Also for comparison, recall the solution from potential flow theory for the velocity passing by the top of a solid cylinder: it decays at the rate of the inverse square of the distance from the cylinder's center, i. e. as:$const/r^2$. Apparently the inverse square law of velocity decay away from a cylinder has never been held up to measurements in order to see if there is agreement or not.

Conserving mass between two vertical cross-sections, one at a distance before or after the wing, where the constant flow speed everywhere is U, and the other at the top of the wing, is a way to evaluate the constant in (4). Let the greatest thickness of the wing be $h$and its radius of curvature be R_0. Take the height of the cross-section from the top of the wing be H. Then conservation of mass produces

$$const = \frac{U(H - R_0 + h)}{\ln(\frac{H}{R_0})} \tag{5}$$

Where ln is the natural log, which illustrates a problem of the current method when the ratio H/R_0 becomes large. There is a possible way around what seems to be a mostly mathematical dilemma here if the fluid is air. In that case compressibility can be introduced producing more mass per unit time to pass through the vertical cross-section above the wing and closer to its top. Working that out is left for another time.

But for further progress here a choice is made to select a finite value of the height of the vertical cross-section, H, in order to get an example of the lift force. Therefore, select $H = 2R_0$, a reasonable value. Then from (5)

$$const = \frac{U(H + h)}{\ln(2)} \tag{6}$$

And at $r= R_0$, the velocity v at the top of the wing is

$$v = \frac{U(H + h)}{R_0 \ln(2)} \tag{7}$$

At the bottom of the wing the velocity is U.

The lift force L at the wing's center is

$$L = \frac{p_b - p_t}{h} \tag{8}$$

Where p_b is the pressure on the wing's bottom and p_t is the pressure at the top of the wing. By inserting U into Bernoulli's law for p_b and v from (7) into Bernoulli's law for p_t evaluation of (8) gives

$$L = \frac{\rho U^2}{R_0} \tag{9}$$

For simplicity ln(2) in (7) has been taken to be 1.0 and also the approximation was made that $(\frac{h}{R_0})^2 \ll 1$, a thin circular arc wing.

Except for a factor of 1.3 Equation (9) agrees with the lift force calculated by a different method in Reference [1]. There it was mentioned by examples that the more closely bound the flow is to the top of the wing, the stronger the lift is, which makes sense, based on Bernoulli's law, because the faster the flow speed at the top of the wing, the lower the pressure for constant density.

3. Conclusion

An algebraic formulation is developed for the lift force on a circular arc wing. Central to the effort is the idea that the flow speed at the top of the wing dies away with increasing height like a constant times $1/r$, where r measures distance from the center of the circle of which the arc is a part. This suggestion is new and founded on physics that is solid: Bernoulli's law along streamlines plus the cross-stream force balance between the upward centrifugal force and a downward pressure force. One consequence is the implication that with the resulting $1/r$ velocity structure there is no friction in the flow, which may be a reason why such a flow persists in the first place. A limitation of the present particular application of the method to the wing is that the constant involved so far cannot be satisfactorily nailed down. Perhaps observations would be helpful, particularly to verify the $1/r$ dependence of the flow speed above the top of the wing.

References

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