ON SEMI-Di SPACES

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ABSTRACT

A subset S of a topological space X is said to be SD-set if $S = U \setminus V$, where U,V are semi-open sets and $U \neq X$. Using SD-sets, semi-D_i spaces is introduced. A space X said to be semi-D_1 if for any pair of distinct points x, y of X, there exists an SD-set containing x but not y.

INTRODUCTION

In 1963, Levine introduced semi-open sets. A subset A of a space X is said to be semi-open if there exists an open set U such that $U \subseteq A \subseteq \overline{A}$. The complement of a semi-open set is called semi-closed [2]. Using semi-open sets, Maheshwari and Prasad [4] introduced semi-T_0, semi-T_1 and semi-T_2 spaces. Semi-T_2 were also studied by Noiri [5]. In this paper, I introduce semi-D_i spaces and study its relations with other known spaces.

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Definition 1. A subset S of a topological space X is to be an SD-set if $S = U \setminus V$, where U, V are semi-open sets and $U \neq X$.

A semi-open set $U \neq X$ is an SD-set since $U = U \setminus \phi$. But an SD-set is not necessarily semi-open.

If we replace semi-open sets in the definitions of semi-T_0, semi T_1, semi-T_2 spaces with SD-sets, we get semi-D_0, semi-D_1, semi-D_2 spaces, respectively.

Definition 2. A space X is said to be semi-D_1 if for any pair of distinct points x ,y of X, there exists an SD-set containing x but not y.

Theorem 1. Semi-$D_0 \Leftrightarrow$ semi-T_0.

Proof: We need only to show that semi-$D_0 \Rightarrow$ semi-T_0.

Let X be a semi-D_0 space. Let x , y be two distinct points of X. Then there exists an SD-set S containing one of them but not the other. Suppose S contains x, say, Let $S = U \setminus V$, where $U \neq X$ and U, V are semi-open sets in X. Then $x \in U$. For $y \notin S$ we have two cases:

(i) $y \notin U$

(ii) $y \in U$ and $x \in U$.

In case (i), U contains x but not y. In case (ii), V contains y but not x. Hence X is a semi-T_0 space.

Theorem 2. Semi-$D_1 \Leftrightarrow$ semi-D_2.

Proof: We need only to show that semi-$D_1 \Rightarrow$ semi-D_2.

Let X be a semi-D_1 space. Let x, y be two distinct points of X. There exist two SD-sets S_1, S_2 such that $s \in S_1$, $y \notin S_1$ and , $y \in S_2$, $x \notin S_2$. Let $S_1 = U_1 \setminus U_2$ and $S_2 = V_1 \setminus V_2$. From $x \notin S_2$ we have either $x \notin V_1$ or $x \in V_1$ and $x \in V_2$. Now, for $x \notin V_1$, we have two cases from $y \notin S_1$:

(i) $y \notin U_1$ from $x \in S_1 = U_1 \setminus U_2$, we have

$x \in U_1 \setminus (U_2 \cup V_1)$ from $y \in V_1 \setminus V_2$ we have $y \in V_1 \setminus (U_1 \cup V_2)$

Clearly, $\left[U_1 \setminus (U_2 \cup V_1)\right] \cap \left[V_1 \setminus (U_1 \cup V_2)\right] = \phi$.

(ii) $y \in U_1$ and $y \in U_2$ we have $x \in U_1 \setminus U_2$, $y \in U_2$ and $(U_1 \setminus U_2) \cap U_2 = \phi$. For $x \in V_1$ and $x \in V_2$. We have $y \in V_1 \setminus V_2$, $x \in V_2$ and $(V_1 \setminus V_2) \cap V_2 = \phi$. Thus X is a semi-$D_2$ space.

Theorem 3. Semi-$T_1 \Rightarrow$ semi-$D_1 \Rightarrow$ semi-T_0.

Proof: Obvious.

Converse implications in the above Theorem are not true. Following are examples:

Example 1. Let X = {a, b, c} and T = {x, ϕ, {a},{a, b}}. Then (X, T) is semi-T_0 but not semi-D_1, since there is no SD-set containing c.

Example 2. Let X = {a, b, c, d} and T = {X, ϕ, {a},{a, b}}.
Then (X, T) is semi-D_1 but not semi-T_1.

Definition 3. [1] let X be a topological space and $x \in X$. A set S is said to be a semi-neighbourhood of x if there exists a semi-open set M such that $x \in M \subseteq S$.

Definition 4. Let X be a topological space and $x \in X$. If x has no semi-open neighbourhood other than X, then call x a C.SC. point (common to all semi-closed sets).

Theorem 4. A semi-T_0 space X is semi-D_1 if and only if X has no C.SC. point.

Proof: Necessity. Suppose X is a semi-D_1 space. Then every point x of X belongs to an SD-set.

Let S= U\V, where U,V are semi-open sets and $U \neq X$. Then $x \in U$ and $U \neq X$. Hence x is not a C.SC. point.

Sufficiency: Let X be a semi-T_0 space. Then for any pair of distinct points x , y of X, there is a semi-neighbourhood U of one of then, say x, such that $x \in U$ and $y \notin U$, $U \neq X$ is an SD-set.

Since X has no C.SC. points, therefore y is not a C.SC.point and so there is a semi-neighbourhood V of y such that $V \neq X$. Now, $y \in V \setminus U$ and V\U is an SD-set. Thus X is semi-D_1.

Theorem 5. A semi-T_0 space is semi-D_1 if and only if X has a unique C.SC. point.

Proof: We need only to show the uniqueness of the C.SC. point. Let x , y be two C.SC. points in X. Since X is semi-T_0, therefore one of them say x has a semi-neighbourhood U such that $x \in U$, $y \notin U$. Clearly, $U \neq X$ and x is not a C.SC. point, a contradiction.

Following diagram indicates relationships among different spaces considerad in thin paper.

$$\begin{array}{ccccc} \text{semi-}T_0 & \Leftarrow & \text{semi-}D_1 & \Leftarrow & \text{semi-}T_1 \\ \Updownarrow & & \Updownarrow & & \Updownarrow \\ \text{semi-}D_0 & \Leftarrow & \text{semi-}D_2 & \Leftarrow & \text{semi-}T_2 \end{array}$$

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