where a_i are real constants with $a_M \neq 0$ to be determined, while $G(\xi)$ satisfies the following linear ordinary differential equation

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0 \tag{5}$$

where λ and μ are real constants to be determined.
Step 3. Determine the positive integer M in (4) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (4).
If $M = \frac{q}{p}$ (where $\frac{q}{p}$ be a fraction in the lowest term), we let:

$$U(\xi) = V^{\frac{q}{p}} \tag{6}$$

and substitute Eq. (7) into Eq. (3)and then determine a new equation in terms of the function $V(\xi)$.
If M is a negative number, we let

$$U(\xi) = V^M \tag{7}$$

and substitute Eq. (8) into Eq. (4) and then determine a new equation in terms of the function $V(\xi)$.
Step 4. Substituting Eq. (4) along with Eq. (5) into Eq. (3), collecting all terms of the coefficients of $(G'/G)^i, (i = 0, 1, 2, ..., M)$ and setting them to zero, we get a set of algebraic equations which can be solved by using the Matlab Software Package. Equating each coefficient of this polynomial to zero yields a system of algebraic equations for $a_i (i = 0, 1, 2, ..., M), \lambda, \mu$ and c.
Step 5. Solving the system in Step 5, and then substituting $a_i(i = 0, 1, 2, ..., M), \lambda$,
μ and c and also solutions of Eq.(5) into Eq.(4), we can obtain the exact solutions of the given Eq.(1).

3 Applications

In this section, the exact solutions of the Lin-Reissner-Tsien (LRT) equation [14–18] and Kármán-Fal'kovich-Guderley (KFG)[14] equation are obtained by using the (G'/G)-expansion method.

3.1 Lin-Reissner-Tsien (LRT) equation

Consider

$$w_{xt} + aw_x w_{xx} - cw_{xx} - bw_{yy} = 0 \tag{8}$$

For $a = \frac{1}{2}(\gamma + 1), b = \frac{1}{2}, c = \frac{1}{2}(1 - M_\infty^2)$, Eq. (8) is the complete Lin-Reissner-Tsien (LRT) equation, where γ is the adiabatic exponent of the gas and M_∞ is the Mach number.
The transformation

$$w = u(z, y, t), \quad z = x + ct \tag{9}$$

leads to an equation of the form

$$u_{xt} + au_x u_{xx} - bu_{yy} = 0 \tag{10}$$

For $a = b = \frac{1}{2}$, this is the LRT for an unsteady transonic gas flow . Applying the (G'/G)-expansion method to the following LRT

$$2u_{xt} + (\gamma + 1)u_x u_{xx} - u_{yy} = 0 \tag{11}$$

and using traveling wave variable

$$u(x,t) = U(\xi), \xi = x + y - ct \tag{12}$$

where c is nonzero arbitrary constant, when substituting Eq.(11) with Eq. (12) can be turned into an ODE

$$(-2c-1)U'' + (\gamma+1)U'U'' = 0 \tag{13}$$

where $U' = \frac{dU}{d\xi}$.
Integrating Eq.(13) term by term, we obtain

$$(-4c-2)U' + (\gamma+1)U'^2 + 2\xi_0 = 0 \tag{14}$$

and we let

$$U(\xi) = V^{-1}(\xi) \tag{15}$$

Substituting Eq. (15) into Eq. (14), we have

$$(4c+2)V^2V' + (\gamma+1)V'^2 + 2\xi_0 V^4 = 0 \tag{16}$$

By using the ansatz, balancing V^2V' with V^4 in Eq.(16) gives $M = 1$. Suppose that the solution of Eq.(16) can be expressed by a polynomial in (G'/G) as follows:

$$V(\xi) = a_0 + a_1\Big(\frac{G'}{G}\Big), a_1 \neq 0 \tag{17}$$

and we have

$$V'(\xi) = -a_1\Big(\frac{G'}{G}\Big)^2 - a_1\lambda\Big(\frac{G'}{G}\Big) - a_1\mu \tag{18}$$

$$V'^2(\xi) = a_1^2\Big(\frac{G'}{G}\Big)^4 + 2\lambda a_1^2\Big(\frac{G'}{G}\Big)^3 + (\lambda^2 a_1^2 + 2\mu a_1^2)\Big(\frac{G'}{G}\Big)^2 + 2\lambda\mu a_1^2\Big(\frac{G'}{G}\Big) + \mu^2 a_1^2 \tag{19}$$

$$V^2(\xi) = a_1^2\Big(\frac{G'}{G}\Big)^2 + 2a_0a_1\Big(\frac{G'}{G}\Big) + a_0^2 \tag{20}$$

$$V^4(\xi) = a_1^4\Big(\frac{G'}{G}\Big)^4 + 4a_0a_1^3\Big(\frac{G'}{G}\Big)^3 + 6a_0^2a_1^2\Big(\frac{G'}{G}\Big)^2 + 4a_0^3a_1\Big(\frac{G'}{G}\Big) + a_0^4 \tag{21}$$

Substituting Eq. (17)-(21) into Eq. (16), collecting the coefficients of $(G'/G)^i, (i = 0, ..., 3)$ and setting them to be zero, we obtain the system

$$\begin{aligned}
(-4c-2)a_1^3 + (\gamma+1)a_1^2 + 2\xi_0 a_1^4 &= 0\\
(-4c-2)(\lambda a_1^3 + 2a_0a_1^2) + (\gamma+1)2\lambda a_1^2 + 8\xi_0 a_0 a_1^3 &= 0\\
(-4c-2)(\mu a_1^3 + 2\lambda a_0 a_1^2 + a_0^2 a_1) + (\gamma+1)(\lambda^2 a_1^2 + 2\mu a_1^2) + 12\xi_0 a_0^2 a_1^2 &= 0\\
(-4c-2)(2\mu a_0 a_1^2 + \lambda a_0^2 a_1) + (\gamma+1)2\lambda\mu a_1^2 + 8\xi_0 a_0^3 a_1 &= 0\\
(-4c-2)\mu a_0^2 a_1 + (\gamma+1)\mu^2 a_1^2 + 2\xi_0 a_0^4 &= 0
\end{aligned} \tag{22}$$

Solving this system by symbolic computation gives

$$a_0 = a_0, a_1 = \pm\frac{a_0}{\sqrt{\mu}}$$

$$c = \pm\frac{\sqrt{\mu}(\gamma+1)}{2a_0} - \frac{1}{2}, \xi_0 = \frac{\mu(\gamma+1)}{2a_0^2} \tag{23}$$

where λ and μ are arbitrary constants.
Applying (G'/G)-expansion method and using the results obtained above into Eq. (17), the following exact solutions to Eq. (11) is obtained when $\lambda^2 - 4\mu = 0$

$$V_{1,2}(\xi) = a_0 \pm \frac{a_0}{\sqrt{\mu}}\Big[\Big(\frac{C_2}{C_1 + C_2\xi} - \frac{\lambda}{2}\Big)\Big] \tag{24}$$

where $\xi = x + y - \Big(\pm\frac{\sqrt{\mu}(\gamma+1)}{2a_0} - \frac{1}{2}\Big)t$.
From $U(\xi) = V^{-1}(\xi)$ transformation, we obtain as follows:

$$U_{1,2}(\xi) = \Big(a_0 \pm \frac{a_0}{\sqrt{\mu}}\Big[\Big(\frac{C_2}{C_1 + C_2\xi} - \frac{\lambda}{2}\Big)\Big]\Big)^{-1} \tag{25}$$

where $\xi = x + y - \Big(\pm\frac{\sqrt{\mu}(\gamma+1)}{2a_0} - \frac{1}{2}\Big)t$.
If $\gamma = 0$, the exact solutions of the LRT equation for an unsteady transonic gas flow are found as

$$U_{1,2}(\xi) = \Big(a_0 \pm \frac{a_0}{\sqrt{\mu}}\Big[\Big(\frac{C_2}{C_1 + C_2\xi} - \frac{\lambda}{2}\Big)\Big]\Big)^{-1} \tag{26}$$

where $\xi = x + y - \Big(\pm\frac{\sqrt{\mu}}{2a_0} - \frac{1}{2}\Big)t$.

3.2 Kármán-Fal'kovich-Guderley (KFG) equation

Consider

$$u_t u_{tt} = \Delta u, R^N \times R^1 \tag{27}$$

which is called the Kármán-Fal'kovich-Guderley (KFG) equation of transonic gas flows. Using traveling wave variable

$$u(x,t) = U(\xi), \xi = x - ct \tag{28}$$

where c is nonzero arbitrary constant, when substituting Eq.(26) with Eq. (27) can be turned into an ODE

$$-c^3U'U'' - U'' = 0 \tag{29}$$

where $U' = \frac{dU}{d\xi}$.
Integrating Eq.(28) , we get

$$-c^3(U')^2 - 2U' + 2\xi_0 = 0 \tag{30}$$

and we let

$$U(\xi) = V^{-1}(\xi) \tag{31}$$

Substituting Eq. (30) into Eq. (29), we have

$$-c^3(V')^2 + 2V^2V' + 2\xi_0 V^4 = 0 \tag{32}$$

By using the ansatz, balancing V^2V' with V^4 in Eq.(32) gives $M = 1$. Assume that the solution of Eq.(32) can be found by a polynomial in (G'/G) as follows:

$$V(\xi) = a_0 + a_1\Big(\frac{G'}{G}\Big), a_1 \neq 0 \tag{33}$$

and we have

$$V'(\xi) = -a_1\Big(\frac{G'}{G}\Big)^2 - a_1\lambda\Big(\frac{G'}{G}\Big) - a_1\mu \tag{34}$$

Substituting Eq. (17)-(21) into Eq. (32), equating each coefficient of $(G'/G)^i, (i = 0, ..., 3)$ to zero, we can obtain the system

$$\begin{aligned}
-c^3a_1^2 - 2a_1^3 + 2\xi_0 a_1^4 &= 0\\
-c^3 2\lambda a_1^2 - 2(\lambda a_1^3 + 2a_0a_1^2) + 8\xi_0 a_0 a_1^3 &= 0\\
-c^3(2\mu a_1^2 + \lambda^2 a_1^2) - 2(\mu a_1^3 + 2\lambda a_0 a_1^2 + a_0^2 a_1) + 12\xi_0 a_0^2 a_1^2 &= 0\\
-c^3 2\lambda\mu a_1^2 - 2(2\mu a_0 a_1^2 + \lambda a_0^2 a_1) + 8\xi_0 a_0^3 a_1 &= 0\\
-c^3\mu^2 a_1^2 - 2\mu a_1 a_0^2 + 2\xi_0 a_0^4 &= 0
\end{aligned} \tag{35}$$

Solving the above system by symbolic computation, we have the set of coefficients as given below:

$$\begin{aligned}
a_0 = a_0, a_1 = \pm\frac{a_0}{\sqrt{\mu}}\\
c = \Big(\mp\frac{a_0}{\sqrt{\mu}}\Big)^{1/3}, \xi_0 = -\frac{\sqrt{\mu}}{2a_0}
\end{aligned} \tag{36}$$

where λ and μ are arbitrary constants.
Applying (G'/G)-expansion method and using the results obtained above into Eq. (32) , we can obtain the following exact solutions to Eq. (27) when $\lambda^2 - 4\mu = 0$

$$V_{1,2}(\xi) = a_0 \pm \frac{a_0}{\sqrt{\mu}}\Big[\Big(\frac{C_2}{C_1 + C_2\xi} - \frac{\lambda}{2}\Big)\Big] \tag{37}$$

where $\xi = x - \Big(\mp\frac{a_0}{\sqrt{\mu}}\Big)^{1/3}t$.
From $U(\xi) = V^{-1}(\xi)$ transformation, we get as follows:

$$U_{1,2}(\xi) = \Big(a_0 \pm \frac{a_0}{\sqrt{\mu}}\Big[\Big(\frac{C_2}{C_1 + C_2\xi} - \frac{\lambda}{2}\Big)\Big]\Big)^{-1} \tag{38}$$

where $\xi = x - \Big(\mp\frac{a_0}{\sqrt{\mu}}\Big)^{1/3}t$, which are the exact solutions of the KFG equation.

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