Degenerate neutrino mass model revisited

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Abstract

The estimation of absolute neutrino masses in degenerate neutrino mass models and the effects of Majorana CP phases on neutrino masses, are addressed in the present investigation. Degenerate Majorana neutrino mass model with CP-parity pattern $(+-+)$ (referred to as Type IA) in the mass eigenvalues, exhibits a strong variation of the absolute neutrino masses $(m_0 \sim m_i)$ with the solar mixing angle $tan^2\theta_{12}$. Numerical analysis shows that the range of solar angle $tan^2\theta_{12} = (0.50 - 0.45)$ corresponds to the range of the absolute neutrino mass scale $m_0 = (0.4 - 0.1)$. Since the lower values are acceptable, the model is far from discrimination from the bounds on absolute neutrino masses from $0\nu\beta\beta$ decay and cosmology. The other two models with CP parity patterns $(+++)$ (type IB) and $(++-)$ (type IC) respectively, do not show such variations of solar angle with absolute neutrino masses. It has been found that the absolute neutrino mass scale m_0 is almost fixed at about $0.4eV$ for a wide range of solar angle, $tan^2\theta_{12} = (0.50 - 0.45)$. These models are therefore disfavoured by the available data from absolute neutrino masses. The present investigation on degenerate models is based on a new parameterization of the degenerate neutrino mass matrix obeying $\mu - \tau$ symmetry which has the ability to lower the solar mixing angle below the tri-bimaximal value. The breaking of $\mu - \tau$ symmetry can be carried out with the inclusion of charged lepton corrections in the neutrino mixing angle.

Keywords: Degenerate neutrino mass model, absolute neutrino masses, solar mixing angle below tri-bimaximal mixings.

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1. Introduction

Discrimination of neutrino mass patterns among three possible cases, viz., degenerate, normal and inverted hierarchical models, has drawn renewed interest with the presently available precise observational data from neutrino oscillation experiments as well as bounds on absolute neutrino masses. The present neutrino oscillation data gives the following information for the parameters at 1σ level [1]:

$$\Delta m_{21}^2(10^{-5}eV^2) = (7.43 - 7.81)$$
$$\Delta m_{32}^2(10^{-3}eV^2) = (2.46 - 2.61)$$
$$tan^2\theta_{12} = (0.435 - 0.506)$$
$$tan^2\theta_{23} = (0.667 - 0.855)$$
$$sin^2\theta_{13} = (0.0218 - 0.0275)$$

At present there is no precise information about the absolute neutrino masses and one could expect only the upper bounds on the absolute neutrino masses from various experiments like the Tritium β decay, Neutrinoless double beta decay [2] and the cosmological observations [3]:

$$m_{\nu_e} = \left(\sum_i m_i^2 |U_{ei}|^2\right)^{1/2} < 2.3\ eV \quad \text{(Tritium β decay)}$$
$$m_{ee} = \left|\sum_i m_i U_{ei}^2\right| \leq 0.27\ eV \quad (0\nu\beta\beta \quad \text{decay})$$
$$m_{cosmo} = \sum_i |m_i| \leq 0.28\ eV \quad \text{(Cosmological bounds)}$$

Both normal and inverted hierarchical models are now well explained within the above observational bounds, and are therefore far from discrimination at the moment, but the degenerate models are considered almost disfavoured in the literature, following the present bounds on absolute neutrino masses, particularly from cosmological and neutrinoless double beta decay bounds. However a closer analysis in the present work will reveal that such assertion is specifically correct for larger absolute neutrino masses $m_0 \sim 0.4\ eV$. There are still possibilities for certain degenerate models which allow lower values of neutrino masses $m_0 \sim 0.1\ eV$ valid for lower values of solar mixings.

We classify for convenience the degenerate models following their CP-parity patterns in their mass eigenvalues $m_i = (m_1, m_2, m_3)$, viz., Type IA: $(+-+)$, Type IB: $(+++)$, Type IC: $(++-)$ respectively. It will be shown in the present work that both Type IB and IC are almost disfavoured by the presently available data on absolute neutrino masses. This is due to the fact that the predicted absolute neutrino masses are in the range $m_0 \sim 0.4\ eV$ in these two cases. On the other hand, Type IA has many interesting properties which are yet to be explored, particularly variation of m_0 with solar mixing $tan^2\theta_{12}$, and also a partial cancellation of even and odd CP parity in the first two mass eigenvalues appeared in the expressions of m_{ν_e} and m_{ee}. This makes Type IA degenerate model far from discrimination. It provides enough scope for future experiments to go the sensitivity down to $|m_{ee}| = 0.03\ eV$.

In the theoretical front there are several attempts to find out the most viable neutrino mass models, and among them the neutrino mass models obeying μ–τ symmetry [9, 10, 11, 12, 13], have drawn considerable attention. Neutrino mass matrix having $\mu - \tau$ symmetry, leads to maximal atmospheric mixings ($tan^2\theta_{23} = 1$) and zero reactor angle ($\theta_{13} = 0$), whereas the solar angle is purely arbitrary. This has to be fixed by the input values of the parameters in the mass matrix. The tri-bimaximal mixings (TBM) [4, 5] with

$tan^2\theta_{12}$=0.5 is a special case of this symmetry. There are four unknown elements present in a general μ−τ symmetric mass matrix and it is difficult to solve these four unknown elements from only three equations involving observational data on $tan^2\theta_{12}$, Δm_{21}^2 and Δm_{32}^2 . Thus

$$m_{LL} = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \quad (1)$$

where the eigenvalues and solar mixings are

$$\begin{gathered} m_1 = m_{11} - \sqrt{2}\tan\theta_{12}\, m_{12} \\ m_2 = m_{11} + \sqrt{2}\cot\theta_{12}\, m_{12} \\ m_3 = m_{22} - m_{23} \\ \tan 2\theta_{12} = \frac{2\sqrt{2}\, m_{12}}{m_{11} - m_{22} - m_{23}} \end{gathered}$$

In our earlier works we simply considered possible parameterization with lesser numbers of free parameters consistent with available data for practical solution of the mass matrix. We have already reported our method for parameterization with only two parameters η and ε and the ratio of the these two parameters [6], is responsible for lowering the solar angle below tri-bimaximal solar mixing. We have already parameterized both normal and inverted hierarchical neutrino mass matrices [7]. In TBM mixing we have the value of the solar mixing angle, ($tan^2\theta_{12}$=0.5). But, the recent global 3ν oscillation analysis has shown a mild deviation from the tri-bimaximal neutrino mixings i.e., $tan^2\theta_{12}$=0.45 and the present work has its own relevance here. Such parameterization thus reduces to two unknown free parameters η and ε in addition to an overall neutrino mass scale m_0, making the three equations solvable in practice.

In section 2 we first extend our earlier method of parameterization of μ−τ symmetric mass matrices, to a particular degenerate neutrino mass model Type IA $(m_1, -m_2, m_3)$ denoted by CP-parity pattern $(+-+)$, and also identify the flavour twister term to obtain tri-bimaximal mixings and then modify it for deviations from tri-bimaximal mixings. Similar analysis is also carried out for Type IB and IC degenerate models. Section 3 we apply the charged lepton correction to break the μ − τ symmetry. Section 4 concludes with a brief summary.

2. Parameterization of degenerate models

2.1 Degenerate Type IA $(m_1, -m_2, m_3)$

The zeroth order left-handed Majorana mass matrix [8] for Type IA degenerate model, is given by

$$m_{LL}^0 = \begin{pmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & -1/2 & 1/2 \end{pmatrix} m_0 \quad (2)$$

where m_0 is the overall scale factor and mass eigenvalues are Diag $(1, -1, 1)m_0$. A complete light neutrino mass matrix m_{LL} for the degenerate neutrino mass model (Type IA) with $\mu - \tau$ symmetry in eq.(2) is now modified as

$$m_{LL} = \begin{pmatrix} \varepsilon - 2\eta & -c\,\varepsilon & -c\,\varepsilon \\ -c\,\varepsilon & 1/2 - d\,\eta & -1/2 - \eta \\ -c\,\varepsilon & -1/2 - \eta & 1/2 - d\,\eta \end{pmatrix} m_0 \tag{3}$$

Where c and d are just real constants, and η and ε are the unknown parameters. The mass matrix still maintains the $\mu - \tau$ symmetry but the arbitrary solar angle can be fixed at the particular value by choosing a set of values for c, d, η and ε .

The mass matrix in eq.(3) predicts an expression for solar mixing angle,

$$\tan 2\theta_{12} = -\frac{2\sqrt{2}\ c}{1+(d-1)\eta/\ \varepsilon} \tag{4}$$

In general, any mass matrix obeying $\mu - \tau$ symmetry, can be diagonalized by the following unitary matrix,

$$U_\nu = \begin{pmatrix} \cos\theta_{12} & -\sin\theta_{12} & 0 \\ \sin\theta_{12}/\sqrt{2} & \cos\theta_{12}/2 & -1/2 \\ \sin\theta_{12}/\sqrt{2} & \cos\theta_{12}/2 & 1/2 \end{pmatrix} \tag{5}$$

Diagonalizing the mass matrix in eq.(3) we get the three mass eigenvalues as,

$$m_1 = \frac{1}{2}(\varepsilon - 3\eta - d\,\eta + \chi)m_0 \tag{6}$$
$$m_2 = \frac{1}{2}(\varepsilon - 3\eta - d\,\eta - \chi)m_0 \tag{7}$$
$$m_3 = (1 + \eta - d\,\eta)m_0 \tag{8}$$
$$\chi^2 = \varepsilon^2 + 8c^2\varepsilon^2 - 2\eta\,\varepsilon + 2d\,\eta\,\varepsilon + \eta^2 - 2d\,\eta^2 + d^2\eta^2 \tag{9}$$

The choice of $c = d = 1$ in eq.(4) leads to tri-bimaximal mixings (TBM), $tan^2\theta_{12} = 0.5$ ($\tan 2\theta_{12} = 2\sqrt{2}$) . With the input value $m_0 = 0.4\ eV$, the values of the free parameters are extracted from the data of neutrino oscillation mass parameters, as $\varepsilon = 0.661145$ and $\eta = 0.165348$ respectively. As seen in eq.(4) there is no flavour twister term in this case.

In the next step the flavour twister $\eta/\ \varepsilon$ responsible for lowering solar angle, is then introduced by taking $c < 1$ and $d > 1$. Using the known values of η and ε already fixed in TBM, along with those of c and d for a particular choice of solar angle, the value of m_0 is extracted. Table 1 gives the numerical results for a few selected cases. A scattered plot in Fig.1 using the points generated within the allowed ranges of neutrino oscillation mass parameters, depicts the dependence of $tan^2\theta_{12}$ on m_0 . This leads to m_{cosmo} within the upper bound from cosmology. For $tan^2\theta_{12} = 0.45$, we get $\sum|m_i| = 0.275\ eV$ and $|m_{ee}| = 0.033\ eV$. It can be emphasised that the predictions on other oscillation parameters are consistent with latest data.

2.2 Degenerate Types 1B and 1C

We now briefly perform similar analysis for other two degenerate models - Type IB with CP-Parity $(+ + +)$ and Type IC with $(+ + -)$. The structures of their mass matrices are given below:

Degenerate Type IB (m_1, m_2, m_3)

$$m_{LL} = \begin{pmatrix} 1-\varepsilon-2\eta & c\,\varepsilon & c\,\varepsilon \\ c\,\varepsilon & 1-d\,\eta & -\eta \\ c\,\varepsilon & -\eta & 1-d\,\eta \end{pmatrix} m_0 \qquad (11)$$

where the zeroth-order mass matrix with mass eigenvalues Diag. $(1,1,1)m_0$, is given by

$$m_{LL}^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_0 \qquad (12)$$

Degenerate Type IC $(m_1, m_2, -m_3)$

$$m_{LL} = \begin{pmatrix} 1-\varepsilon-2\eta & c\,\varepsilon & c\,\varepsilon \\ c\,\varepsilon & -\eta & 1-d\,\eta \\ c\,\varepsilon & 1-d\,\eta & -\eta \end{pmatrix} m_0 \qquad (13)$$

where the zeroth-order mass matrix with mass eigenvalues Diag. $(1,1,-1)m_0$, is given by

$$m_{LL}^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m_0 \qquad (14)$$

The above two models (Type IB and IC) are similar except the interchange of (22) and (23) elements, which imparts a negative sign before the third mass eigenvalue m_3 . Both give the same expression for the solar mixing angle as

$$\tan 2\theta_{12} = \frac{2\sqrt{2}\ c}{1+(d-1)\eta/\ \varepsilon} \qquad (15)$$

Using the condition c = d = 1 for tri-bimaximal mixings and input value $m_0 = 0.4\ eV$, the values of ε and η are solved as $\varepsilon = 8.3138 \times 10^{-5}$ and η = 0.00395. These two values are again used for lowering the solar angle corresponding to the condition c < 1 and d < 1. However the neutrino mass scale is always obtained at $m_0 = 0.4\ eV$ leading to $\sum|m_i| = 1.194\ eV$ and $|m_{ee}| = 0.033\ eV$ respectively. Table 2 gives some representative numerical examples. There is no variation of solar angle with neutrino mass as shown in Fig.2. Such degenerate models are generally disfavoured by the bounds with absolute neutrino masses.

4. Charged lepton corrections for breaking μ − τ symmetry

A very common practice to break the μ − τ symmetry in the mass matrices discussed in the previous sections, is the introduction of charged lepton corrections [14] to the neutrino mixing matrix $U_{PMNS} = U_{eL}^{\dagger} U_{\nu}$, where U_{ν} is given by eq.(5) for μ − τ symmetric neutrino mass matrices and the contribution from charged lepton diagonalizing matrix U_{eL} can be taken of the following form,

$$U_{eL} = \begin{pmatrix} 1-\frac{\lambda^2}{4} & -\frac{\lambda}{2} & \frac{\lambda}{2} \\ -\frac{\lambda}{2} & 1-\frac{\lambda^2}{8} & -\lambda^2 \\ \frac{\lambda}{2} & \lambda^2 & 1-\frac{\lambda^2}{8} \end{pmatrix} \tag{16}$$

Then $U_{PMNS} = U_{eL}^{\dagger} U_{\nu}$ is given by

$$\begin{pmatrix} c_{12}\left(1-\frac{\lambda^2}{4}\right) & -s_{12}\left(1-\frac{\lambda^2}{4}\right) & \frac{\lambda}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} - c_{12}\frac{\lambda}{2} + \frac{7}{8\sqrt{2}} s_{12}\lambda^2 & \frac{c_{12}}{\sqrt{2}} - s_{12}\frac{\lambda}{2} + \frac{7}{8\sqrt{2}} c_{12}\lambda^2 & -\frac{1}{\sqrt{2}}\left(1-\frac{9}{8}\lambda^2\right) \\ \frac{s_{12}}{\sqrt{2}} + c_{12}\frac{\lambda}{2} - \frac{9}{8\sqrt{2}} s_{12}\lambda^2 & \frac{c_{12}}{\sqrt{2}} - s_{12}\frac{\lambda}{2} - \frac{9}{8\sqrt{2}} c_{12}\lambda^2 & \frac{1}{\sqrt{2}}\left(1+\frac{7}{8}\lambda^2\right) \end{pmatrix} \tag{17}$$

where $c_{12} = \cos\theta_{12}$ and $s_{12} = \sin\theta_{12}$. The predictions give the same arbitrary solar angle without any affect from charged lepton corrections, but the atmospheric and Chooz angles are deviated from tri-bimaximal values viz., $tan^2\theta_{23} = \left(1-\frac{9}{8}\lambda^2\right)^2 / \left(1+\frac{7}{8}\lambda^2\right)^2 = 0.816$ and $sin^2\theta_{13} = 0.0253$ for the value of Wolfestein parameter $\lambda = 0.225$. These results are consistent with data [1]. Such charged lepton correction is valid for all three models discussed above for arbitrary value of solar mixing angle, including the best value $tan^2\theta_{12} = 0.45$.

Eq.(17) reduces to Tri-bimaximal case $U_{\nu} = U_{TBM}$ when $c_{12} = \sqrt{2/3}$, $s_{12} = \sqrt{1/3}$. Then $U_{PMNS} = U_{eL}^{\dagger} U_{TBM}$ is given by

$$\begin{pmatrix} \sqrt{\frac{2}{3}}\left(1-\frac{\lambda^2}{4}\right) & -\sqrt{\frac{1}{3}}\left(1-\frac{\lambda^2}{4}\right) & \frac{\lambda}{\sqrt{2}} \\ \sqrt{\frac{1}{6}}\left(1-\lambda+\frac{7}{8}\lambda^2\right) & \sqrt{\frac{1}{3}}\left(1+\frac{\lambda}{2}+\frac{7}{8}\lambda^2\right) & -\frac{1}{\sqrt{2}}\left(1-\frac{9}{8}\lambda^2\right) \\ \sqrt{\frac{1}{6}}\left(1+\lambda-\frac{9}{8}\lambda^2\right) & \sqrt{\frac{1}{3}}\left(1-\frac{\lambda}{2}-\frac{9}{8}\lambda^2\right) & \frac{1}{\sqrt{2}}\left(1+\frac{7}{8}\lambda^2\right) \end{pmatrix} \tag{18}$$

which differs from that of King [15] in the prediction of atmospheric mixing angle.

3. Summary and conclusion

To summarise, we have studied the effects of Majorana CP phases on the prediction of absolute neutrino masses and the variation of the solar mixing angle with the overall ab- solute neutrino mass scale in degenerate neutrino mass models. This was possible through a parameterization of the degenerate neutrino mass models obeying $\mu - \tau$ symmetry, which enables us to lower the solar mixing angle below the tri-bimaximal value $tan^2\theta_{12} = 0.45$, whereas the maximum atmospheric mixing angle and zero reactor angle are then modified by the application of charged lepton corrections in neutrino mixing matrix. The predictions from these mass models are also consistent with the data on the mass-squared differences derived from various oscillation experiments, and also from the bounds on absolute neutrino masses from $0\nu\beta\beta$ decay and cosmology. Degenerate model with CP-parity pattern $(+-+)$ (Type IA) predicts a variation of absolute neutrino mass scale with the solar mixing angle, and the best-fit value $tan^2\theta_{12} = 0.45$

corresponds to the absolute neutrino mass scale 0.1eV consistent with the cosmological upper bound [3]. The model is not yet ruled out and are therefore far from discrimination. The other two having CP parity patterns $(+++)$ (type IB) and $(++-)$ (type IC) respectively, do not show such variations and the neutrino mass scale is almost fixed at about 0.4eV for a wide range of solar mixing angle. Such degenerate models are found to be almost disfavoured by the available data from absolute neutrino masses. Moreover, in all these cases the patterns of neutrino masses are always found to be mild normal hierarchical. This agrees with the earlier result that inverted hierarchical form of neutrino mass eigenvalues in degenerate model, is not allowed in radiative magnification [16]. Further it also agrees with observation [17] that for neutrino absolute mass $m_\nu \sim 0.1\ eV$ with $\tan\beta \sim 6$ are required. The correction from charged lepton mass matrix leads to desired values of atmospheric and Chooz mixing angles without affecting the solar mixing angle. The present result has profound implications on the sensitivity of the future experiments on the discrimination of neutrino mass models.

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Parameters	I	II	III
c	1.0	0.931	0.868
d	1.0	1.011	1.025
m_0 (eV)	0.4	0.141	0.100
m_1 (eV)	0.39664	0.13119	0.08749
m_2 (eV)	-0.39674	-0.13148	-0.08793
m_3 (eV)	0.40000	0.14074	0.09959
Δm_{21}^2 $(10^{-5}\ eV^2)$	7.838	7.651	7.685
Δm_{32}^2 $(10^{-3}\ eV^2)$	2.600	2.521	2.186
$tan^2\theta_{12}$	0.5	0.475	0.450
$\lvert m_{ee}\rvert$ (eV)	0.132	0.047	0.033
$\sum\lvert m_i\rvert$ (eV)	1.193	0.403	0.275

Table 1 : Numerical calculation for Type IA degenerate model. Different values of parameters c , d , η and ε along with the corresponding ranges of Δm_{21}^2 , Δm_{32}^2 , $tan^2\theta_{12}$, $|m_{ee}|$ and $\sum|m_i|$ for fixed value of $tan^2\theta_{12} = 1$ and $\sin\theta_{13} = 0$

Parameters	I	II	III
c	1.0	0.945	0.868
d	1.0	0.998	0.998
m_0 (eV)	0.4	0.400	0.400
m_1 (eV)	0.39677	0.39678	0.39678
m_2 (eV)	-0.39687	-0.39687	-0.39687
m_3 (eV)	0.40000	0.40000	0.40000
Δm_{21}^2 $(10^{-5}\ eV^2)$	7.936	7.619	7.143
Δm_{32}^2 $(10^{-3}\ eV^2)$	2.492	2.494	2.494
$tan^2\theta_{12}$	0.5	0.45	0.421
$\lvert m_{ee}\rvert$ (eV)	0.397	0.397	0.397
$\sum\lvert m_i\rvert$ (eV)	1.194	1.194	1.194

Table 2 : Numerical calculation for Type IB degenerate model. Different values of parameters c , d , η and ε along with the corresponding ranges of Δm_{21}^2 , Δm_{32}^2 , $tan^2\theta_{12}$, $|m_{ee}|$ and $\sum|m_i|$ for fixed value of $tan^2\theta_{12} = 1$ and $\sin\theta_{13} = 0$

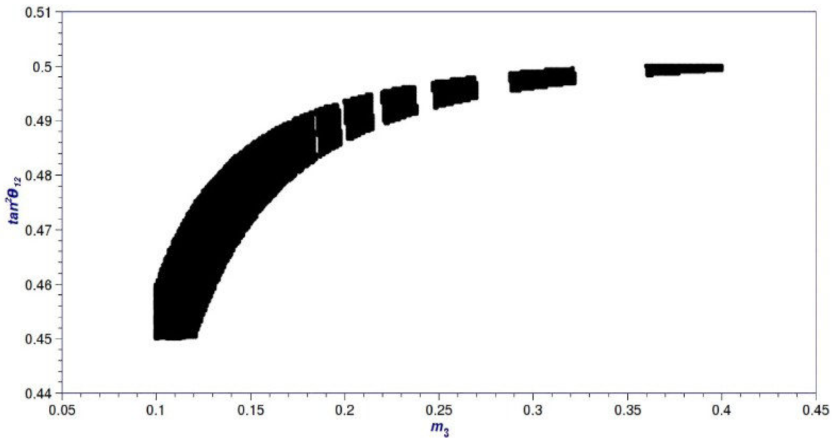

Figure 1: Scatter plot showing the variation of solar mixing $tan^2\theta_{12}$ with absolute neutrino mass m_3 (eV) in Type IA degenerate model with CP parity pattern $(+-+)$.

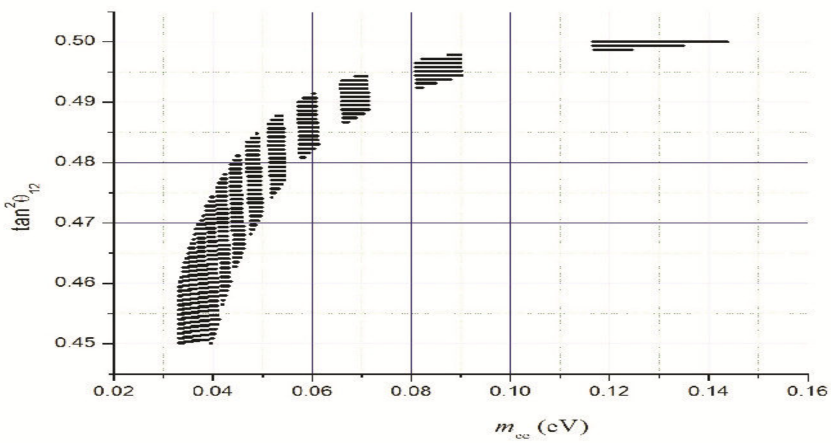

Figure 2: Scatter plot showing the variation of solar mixing $tan^2\theta_{12}$ with 0νββ absolute neutrino mass m_{ee} (eV) in Type IA degenerate model with CP parity pattern $(+-+)$.

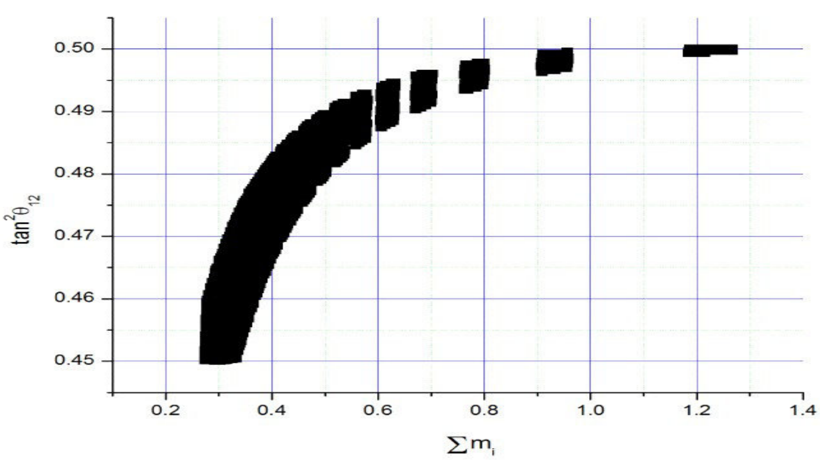

Figure 3: Scatter plot showing the variation of solar mixing $tan^2\theta_{12}$ with sum of the three absolute neutrino masses $\sum|m_i|$ (eV) in Type IA degenerate model with CP parity pattern $(+-+)$.

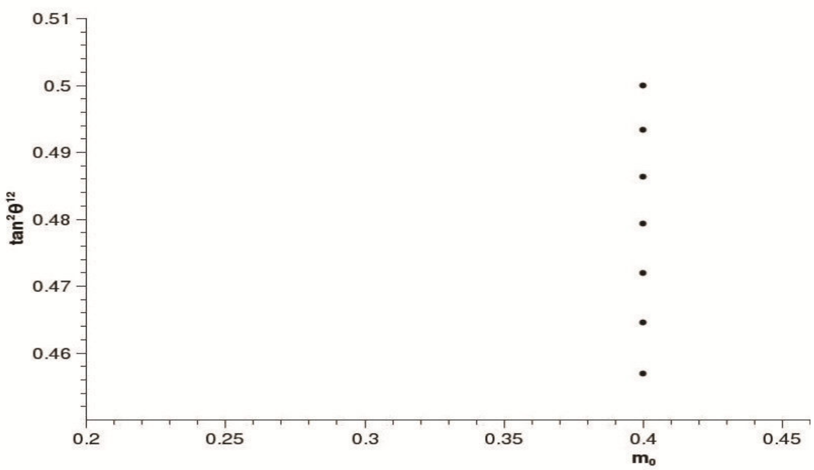

Figure 4: Scatter plot showing a relation of solar mixing $tan^2\theta_{12}$ with the scale of absolute neutrino masses m_0 (eV) in Type IB degenerate model with CP parity pattern (+++) .