MATHEMATICAL APPLICATION OF SIMPLEX NUMERICAL METHOD IN THE ALLOCATION OF WATER IN ABOINE RIVER BASIN OF NIGERIA

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ABSTRACT

The research was based on the mathematical application of Simplex Numerical method on the allocation of one of the finite natural resources with limited availability called water. The research focused on how the mathematical method can be utilized to allocate water to the catchment basin of Aboine river. The study necessitated the collection of data from the communities that constituted the catchment basin and the communities included Obollo Afor, Ikem, Eha Amufu and Nkalaha. The data collected were the population, industrial gross domestic product, ecological and power generation characteristics of the rural communities within the basin. A design provision was made for the mitigation of water losses and leakages and this was fully integrated in the allocation process. With the collected data linear programming (LP) model equation were formulated. Water was allocated to the prioritized areas of power generation, domestic use, ecology, industrialization and they were reflected in the developed dynamic model equations. These equations were solved using the Simplex method and the solutions resulted to ten iterations with the optimal values of 1.004, 1.010, i.079 and 0.776 for Obollo Afor, Ikem, Eha-Amufu and Nkalaha communities respectively. The optimal values were applied to allocate water for various uses in the respective areas.

Keywords: simplex, allocation, model, water resources,

1.0 INTRODUCTION

As a matter of necessity water resource allocators or operators should evolve realistic policies and institutional arrangements that will possess the potentiality of controlling demand through the apportionment of available quantities along economic efficient lines to ensure that water is used more efficiently in various demand sectors. It has been established that the provision of water to households in developing countries would contribute to the eradication of poverty. However, providing water in an efficient, sustainable and equitable manner is a difficult task, because of the existence of equilibrium trap in the water sector in developing countries (Singh et al 1993). Provision of water to people needs a flexible approach based on an in depth analysis of the given situation, the players involved and their potentials. Good and accountable governance and a sound all-round management of water are keys for sustainable water services in economic, social and ecological relationships and dimensions.

The effective management of available water in arid and semi-arid regions in general and Nigeria in particular has increased in importance in recent times due to limited water availability. In Nigeria, water supply for various uses is highly unsustainable because of the neglect of water resources allocation concept. The attendant consequences have been lack of potable water in the midst of many water resources and the absence of considerations for apportioning available water resources for the ever increasing demands for ecological, industrial, hydropower, and recreational purposes. It is obvious that the world's craze for industrial, technological and recreational increasing demand and even the domestic and crop production needs make proper water distribution or allocations for various entitlements inevitable. The issue of thorough understanding of available water and the competing needs remains a challenge to engineers and others who must guarantee that future generations will be served with adequate water supply and equally ensure high level of environmental sanitation.

The total water cycle is global in nature. World water problem deserves conscious analytical studies on regional and national scales. The authenticity of the fact that the total supply of fresh water to the earth is limited and very small when compared with the salt water content of the oceans, has received very little attention. According to Raymond L. Nace of the U.S. Gelological Survey, water resources are a global problem with local roots. In view of the problems facing water resources, a veritable approach for water resources managers will be to create a balance between supply and demand of water. Increased population growth, rapid economic growth and environmental degradation have driven an increasing demand on water.

The United Nations conference on sustainable development in Rio de Janeiro in June 2012 (Rio+ 20 Summit) stated that water is the blood stream of green economy. Water, energy and food are interlinked and interdependent, securing them is central to alleviating poverty and to creating a climate of resilient and robust green economy. Population growth, expanding cities and accelerating economic activity increase the demand for energy and food and create unsustainable pressure on our water and land resource.

Water, said the eminent Greeks philosopher, Pindar "is the best of all things". As the 21st century dawns, the importance of this statement is becoming evident in many parts of the world, even though Pindar lived some two-and-a- half millennia ago. Increasing population and higher levels of human activities, including effluent disposals to surface and groundwater source, have made sustainable management of water resources a very complex task throughout the world. In addition, per capita demand for water in most countries is steadily increasing as more and more people achieved higher standards of living and as lifestyles are changing progressively. For example even in an industrialized nation like Japan, per capita water demand exactly doubled during the period 1985 to 1991. (Biswas, 1997). The situation is even more critical for developing countries, where the rates of population growth are the highest and also, per capita demand for water for steadily increasing numbers of affluent people are putting a stress on the management of water resources. Accordingly, sustainable planning and management of water resources has become a priority consideration for the future welfare of mankind. Water is a resource which has limited availability but always employed in satisfying humans unlimited demand. Water in its limited availability has a lot of purposes to serve and this necessitated the optimal allocation of this resource to many users. Water is one of the basic natural resources that occupy a unique position for the sustenance of life on planet earth. This idea goes a long way to portray the indispensability of water in the existence of life. In view of this, countries, cities and communities do everything within their powers to preserve their sources of water. The planet earth has various oceans, seas, rivers, lakes, streams, springs etc and the quantity they contained was approximately put at 1.4×10^9 cubic kilometers. Surprising, it is only 3% of the total quantity of water on earth that is available in rivers, lakes and ground water. The unimaginable growth in urbanization and population has made the available fresh water grossly inadequate.

The Stockholm statement to the 2012 United Nations Conference on Sustainable Development in Rio De Janeiro (Rio + 20 Summit) stated that by 2030, in a business as usual scenario, humanity's demand for water could outstrip supply by as much as 40 per cent. This would place water, energy and food security at risk, increase public health costs, constrain economic development, lead to social and geopolitical tensions and cause lasting environmental damage.

Countries lack plans and facilities to deal with the calamity that will result from the next widespread regional drought, and the affected public does not even conceive of the impending disaster. Water resources in Nigeria (like in many other parts of the world) are poorly managed, under-utilized and utterly neglected and the trend has locked up the capacity for the appropriation of the potentialities of these natural resources. The neglect of the water resources are partly attributable to the under-development of most communities, local government areas, states and the nation at large. The prevailing deterioration and eventual extinction of our water resources are the precipitations of the unco-ordinated activities allowed in the water resources. The unco-ordinated activities have exacerbated the problem of aquatic life destruction, watershed degradation, over-cultivation of the river banks, near bank excavation, distortion of allocation equilibrium, etc. The attendant consequences include under-utilization, extinction, deterioration, under-development and conflict and these have ultimately created an imbroglio in the scenario of water resources planning and management.

When a country lacks the capacity to manage water resources efficiently her citizens or populace will inevitably bear a growing cost burden as her water deteriorates quantitatively and qualitatively. As a result of inappropriate management; inadequate specifications, poor standards and ineffectiveness on the part of the institutions that are supposed to provide such oversight functions arises. The research is necessary because of the spatial, temporal and purposeful imbalance in the allocation of water resources in the Aboine River Basin. The imbalance had impacted negatively on the economic, health and environmental integrity of the sub-areas and localities in the basin.

According to UN conference on sustainable development in 2012, the foundation for a resources efficient green economy must be built upon water, energy and food security and these issues must be addressed in an integrated, holistic manner that values the natural environment and recognizes the carrying capacity of the planet. Action is critical at all levels to address inequalities, especially for the "Bottom billion" who live in slums and impoverished rural areas and survive without access to safe drinking water, adequate sanitation, sufficient food and energy services.

Throughout history water has been considered a natural resource critical to human survival. From the earliest evaluation of hominid species around the lake shores of northern Kenya to the development of the main civilization on the banks of certain major rivers, human history can generally be considered to be water-centered. The early important civilization developed and flourished on the banks of major rivers such as the Nile, Euphrates, Tigris and Indus. Human history can, in fact, be written in terms of interactions and interrelations between humans and water (Biswas, 1997).

It is not difficult to realize why civilization and habitat often developed along the banks of several strategically important rivers. Easy availability of water for drinking, farming and transportation was important requirement for survival. Human survival and welfare generally depended on regular availability of and control of water. Floods and droughts inflicted major pains, often contributing to deaths of human beings and livestock. Because water played a very important role, when Rishi Narada of India, probably the earliest leading authority on politics who lived many centuries before the Christian era, met the great Pandava kings, Yuddhistira, his greeting was water centered because of its importance; "I hope your realm has reservoirs that are large and full of water, located in different parts of the land, so that agriculture does not depend on the caprice of the Rain God" Proper management and control of water means that the ravages due to drought and subsequent famines could be significantly reduced.

It is unarguably certain that water is one of the most important resources of a country and the entire communal society. After air, water is the next essential need for the sustenance of life on planet earth. This is true because no life is possible without water. Water occurs in the three basic states of liquid, solid and gas. Chemically, its creation comes from the bond of two atoms of hydrogen and one atom of oxygen (H_2O).

Water can be harnessed to satisfy various purpose and needs. Water can therefore be put into such uses (water allocation priorities) as

- Running of hydroelectric turbines
- Navigation of ships, recreation etc
- Domestic use/Drinking Water
- Irrigation
- Agro-industries and non-agricultural industries
- Ecology

Water is the basis of life, an ecological resource for the flora and fauna of the earth and a fundamental necessity for human life. Without an adequate supply of water, there is no hope of improving the health of the people in any given community. World Health Organization estimates that 80% of all diseases is in one way or the other connected with contaminated water usage. Without a good functional system, human water system, human productivity will tend to zero, be it agriculture, industry or trade. Despite, the estimable importance of water, almost half of the population of developing countries have neither the quantity nor the quality of water they need and even fewer people have access to sustainable disposal facilities for sewage. Sustainable development means overcoming conflicts between environmental protection and economic growth and implementing concepts that are socially balanced, economically efficient and environmentally sound.

1.2 NECESSITY OF WATER RESOURCES ALLOCATION

The obvious nature of most basins where some sub – areas are under acute water shortages also necessitate the spatial water allocation in such a basin. Fairness and efficiency of distribution of water are overriding principles. Each local area requires adequate water for community and ecological purposes as well as a supply sufficient to maximize economic growth. Within the basin, there are conflicts between the water uses for the various purposes and this can erupt into violent confrontation between the user groups in future.

There has been great imbalance between the water supply and demand and between the demand capacities of the user groups. The basic principles for the allocation of water resources are efficiency, equity and sustainability, with the aim of pursuing maximum benefit for the society, maintaining fair allocation among the various areas and people. It is therefore very imperative to develop a model for the distribution of scarce water resources for community, ecological and economic uses among different sub-areas in the basin using the principles of efficiency and equity through a mathematical modeling.

1.3 RELATIONSHIP BETWEEN MATHEMATICS AND ENGINEERING

Mathematics is a very strong tool employed by engineers to solve the problem of humanity and the possession of high computational dexterity and skill is an important prerequisite for a successful career in the engineering profession. Quantification of parameters is a common feature in the engineers' day to day operations and transactions. Engineering is a discipline of precision engaged in the business of analytical appraisal of concept values both in quantities and directions.

1.3.1 The Simplex method

Most LP problems when formulated has more than two variables and they are difficult to be interpreted with the help of the graphical solution method. Following this difficulty, the Simplex method was developed by G.B. Dantzing. During the World War II, George B. Dantzing who was then working with the U.S. Air Force developed a Linear Programming model in 1947 (Sharma, 2009). The LP model was specifically developed primarily to military logistics problems. Its application has now been extended functional areas of management, engineering, airlines, agriculture, military operations, oil refining, education, energy planning,

The simplex method is one of the most general and powerful methods of solving linear programming problems. The simplex method rests on two concepts:

- Feasibility and
- Optimality

The search for the optimal solution starts a basic feasible solution or program. We look out for only the extreme-point solutions. The solution is tested for optimality and if it is optimal the search is stopped. If the test of optimality shows that the current solution is not optimal, a new and better feasible solution is designed. The feasibility of the new solution is guaranteed by the mechanics of the simplex method as is the fact that each successive solution is achieved only if it is better than each of the previous solutions. This iterative process is continued until an optimal solution has been attained.

1.3.2 Rationale for the Simplex Method

The Simplex Method is based on the property that the optimal solution to a linear programming problem, can always be found in one of the basic feasible solutions. Thus, in the simplex method, the first step is to obtain a basic feasible solution. This solution is then tested for optimality by examining the net effect on the linear objective function of introducing one of the non basic variables to replace at least one of the current basic variables.

If any improvement potential is noted, the replacement is made, always by introducing only the non-basic variables at a time. The replacement process is such that the new solution is always feasible. The method is quite mechanical in nature. The iterative steps of the simplex methods are repeated until a finite optimal solution is determined.

1.3.3 DESIGNING AN INITIAL PROGRAMME

In the simplex method, each programme is given in the form of a matrix or tableau. The first programme is that which involves the slack variables.

(1) In the column labeled “programme” are listed the variables that are included in the solution. These are the basic variables.

(2) The next column are listed the profit, output or cost coefficients of the basic variables that are included in the specific programme.

(3) In the column labeled “Quantity” are listed the value of basic variables included in the solution. Any variables that are not listed under the “programme” column are the non-basic variables and their values are by definition, zero.

(4) The total profit, output or cost contribution resulting from a specific programme can be calculated by multiplying corresponding entries in the “profit or cost per unit” column and the “Quantity” column and adding the products.

(5) Number in the main body can be interpreted to mean physical ratios of substitution if the programme consists only of the slack variables. The entries in the identity column represent the ratios exchange.

(6) The C_j number at the top of the column of all the variables, represent the coefficients of the respective variables in the objective function.

(7) The number in the Ƶj row, under each variable, give the total gross amount until of outgoing profit, output or cost, when we consider the exchange between one unit of column variable and the basic variables.

(8) The number in the net evaluation row, $C_j - Z_j$, give the net effect of exchange between one unit of each variable and basic variable. They are always zero under the basic variables. Under the non-basic variables, they can be positive, negative or zero.

(9) Test of optimality can be obtained in terms of the signs of the entries ($C_j - Z_j$) in the net evaluation row. In maximization problems, the programme is optimal if each $C_j - Z_j$ is either zero or negative. In the minimization problems, the program is optimal if each $Z_j - C_j$ is zero or negative.

1.3.4 REVISE THE CURRENT PROGRAMME

Once a programme is not optimal, a new and better programme is designed. This is achieved by:

(1) Identify, from the set of non basic variables, the incoming variable (or the key column) or the pivot column. The rule for determining the key column is: the key column is the column under which the largest negative $C_j - Z_j$ appears.

(2) Identify the outgoing variables (or the key row or the pivot row). This is accomplished by, dividing the entries under the "Quantity" column by the corresponding positive entries of the key column and compare these ratios. The row in which the smallest ratio falls is key row. It is convenient to place these replacement quantity calculations on the extreme right-hand side of a given tableau. The limiting replacement quantity of the incoming variable is then identified by the lowest non-negative value.

(3) Identify the key number (or the pivot element).
Once the key row and the key column have been determined the identification of the number is next. The number that lies at the intersection of the key row and the key column of a given tableau is given key number.

The simplex tableau are so constructed that the number of rows in each tableau is the same even though in some cases the value of one or more basic variables appear as zero under the "Quantity" column. Any given tableau, during solution stages, has two types of rows, the key row and the non key rows. Therefore, to drive a new tableau from an old tableau, the rules of transformation for these two types of rows will have to be established. These rules are to be applied to the entire set of entries of each row, starting with and to the right of the "Quantity" column

1.3.5 Transformation of the key Row and Non-key Rows

The rule for transforming the key row is:
The rule for transforming the key row is to divide all the numbers in the key row by the key number. The resulting numbers from the corresponding row in the next tableau, to be placed in exactly the same position. The rule for transforming a non-key row is, subtract from the old row number, in each column, the product of the n key-row number and the corresponding fixed ratio (formed by dividing the row number in key column by the key). The result will give rule corresponding new row number to be placed in exactly the some position. This rule can be placed in the following equation form:

New Row Number = Old Row Number – corresponding number
In key Row x (corresponding fixed Ratio)

Where fixed Ratio = $\dfrac{\text{Old Row number in key Column}}{\text{Key number}}$

Minimization problems are solved exactly as maximization problems apart from little modifications such as stated below (Agunwamba, 2001):

(1) Transformation of all ≥ inequality constraints to equations by subtracting a surplus variable and adding an artificial variable.
(2) Assigning a $C_j - Z$ row, to $Z_j - C_j$. This represents the objective function.
(3) Changing the $C_j - Z_j$ row, to $Z_j - C_j$. This represents the net per unit decrease in costs or output.
(4) Surplus variable are subtracted instead of adding slack variables. While a slack variable reflects an unused resource, a surplus variable in the expression reflects the excess above a minimum resources requirement level.

Flow chart showing the sequential order of solving LP problems whether they are the minimization or maximization type is shown in the figure 2. 1 below;

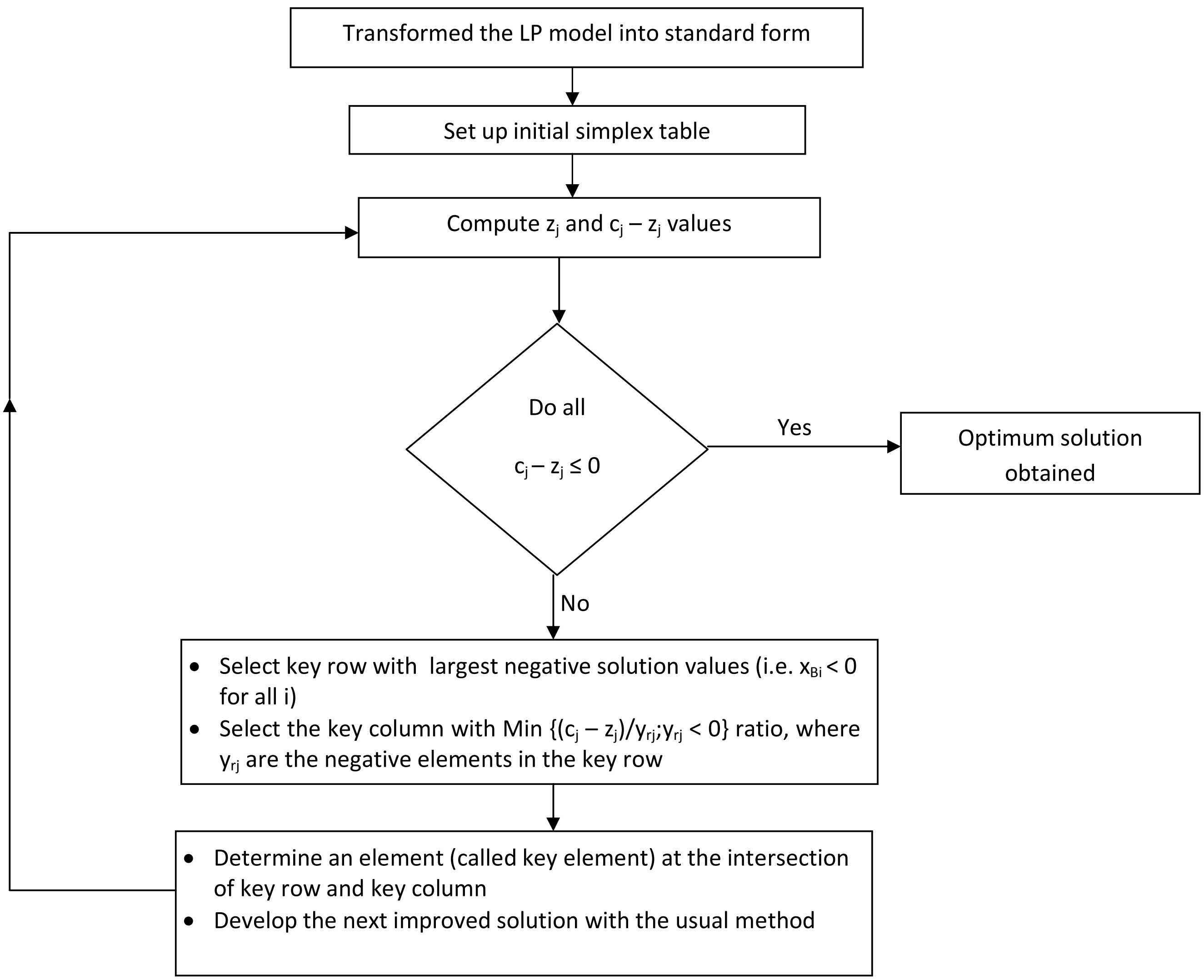

Fig 1.1: Flow chart for Maximization and Minimization

1.3 LINEAR PROGRAMMING MATHEMATICAL MODEL

The optimization problem is one requiring the determination of the optimal (minimum) value of a given function called the objective function, subject to a set of stated restrictions or constraints placed on the variables concerned. Optimization therefore is the process of seeking the best value, condition or solution to a problem. In optimization, we first describe a given system in terms of a mathematical model of the form:
The objective function:

$$Min\ Z = \sum_{j=1}^{n} C_i X_j ----------------3.0$$

Initial Model Equations

$$\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij} X_{ij}$$

Subject to the constraints:

$$\sum a_{ij} X_{ij} \leq b ----------------3.1$$

Where C_{ij} and a_{ij} are coefficients
X_i represents that quantity of variable i that produces the optimum value for the criterion
b = limitation
i = the sectoral allocations while
j = the sub – areas.

1.4 THE ABOINE RIVER BASIN

The Aboine river basin is also in the Anambra-Imo River Basin development Authority area of jurisdiction. It is located in the border between Benue State in the North and Ebonyi State in the South with Enugu in the center. The river flows along the Northern East boundaries of Obollo-Afor traversing Ikem town before emerging at Eha-amufu town and emerging itself into the Nkalaha River in Nkalaha , Ebonyi State. The river is perennial and suitable as a source for the community water use. The river bed sediment consists of medium to coarse sand and as such, excavation of aggregates takes place along the river bed.

Little information found on the flow level indicated that the river has a dry season flow that can stand a major water scheme. It is therefore considered acceptable for a rural water system. Due to climate changes and excessive abstraction by means of river banks or riverbed filtration, there have been reductions in river flow especially to town and communities at the lower reach of the river. The river also serves as a recreation spot for swimmers and divers as the case may be. Along the coastlines are crude furrow for broad irrigate farmlands. During raining season, the river level is very high, those interviewed attested to the presence of high number catfish in some sections of the river channel.

Aboine river basin comprises four communities of Obollo-afor, Ikem, Eha-amufu and Nkalaha towns. Due to demand for water rights, conflicts usually arise and if unchecked could deteriorate and endanger economic activities and development in the affected areas.

2.0 METHODOLOGY

2.1 THE STUDY AREA

The study area consists of communities that are within the catchment basin of the Aboine river and the communities included Obollo Afor, Ikem, Eha-Amufu and Nkalaha and this river basin is within the larger Anambra –Imo River.

2.2 DATA COLLECTION

In the research, data were collected from the catchment areas consisting of communities on the critical parameters of population, industrial gross domestic product, hydropower usage capacity and ecological water usage.

2.3 THE MATHEMATICAL MODEL

The data collected were utilized in the formulation of a linear programming mathematical model as shown below:

Objective function:

$$Min\ Z = \sum_{j-i}^{4} C_i X_j --------------3.14$$

Initial Model Equation

$$Min\ Z = \sum_{i=1}^{5}\sum_{j=1}^{4} C_{ij} X_{ij} ----------- 3.15$$

Subject to the constraints

$$\sum_{j=1}^{4} X_{1j} \leq 189511$$

$$\sum_{j=1}^{4} X_{2j} \leq 104$$

$$\sum_{j=1}^{4} X_{3j} < 140$$

$$\sum_{j=1}^{4} X_{4j} \leq 65$$

$$S\sum_{j=1}^{4} X_{5j} \leq 31 --------------- 3.16$$

Under the Non-negativity conditions

$X_{11}, X_{12}, X_{13}, X_{14}, X_{21}, X_{22}, X_{23}, X_{24}, X_{41}, X_{42}, X_{43}, X_{44}, X_{51}, X_{52}, X_{53}, X_{54}, \geq 0$

These formulated equations can be written in full blown format as presented below:

Objective function

$Min\ Z = 57X_1 + 64X_2 + 54X_3 + 65X_4$

INITIAL MODEL EQUATIONS

$Min\ Z = 52422X_{11} + 24568X_{12} + 92787X_{13} + 19734X_{14} + 27X_{21} + 24X_{22} + 30X_{23} + 15X_{24} + 75X_{31} + 20X_{32} + 30X_{33} + 15X_{34} + 20X_{41} + 10X_{42} + 25X_{43} + 10X_{44} + 7.5X_{51} + 8.4X_{52} + 7.2X_{53} + 8.5X_{54}$

Subject to the constraints

$X_{11} + X_{12} + X_{13} + X_{14} \leq 189{,}511$

$X_{21} + X_{22} + X_{23} + X_{24} \leq 104$

$X_{31} + X_{32} + X_{33} + X_{34} \leq 140$

$X_{41} + X_{42} + X_{43} + X_{44} \leq 65$

$X_{51} + X_{52} + X_{53} + X_{54} \leq 31$

Under the non-negativity conditions;

$X_{11}, X_{12}, X_{13}, X_{14}, X_{21}, X_{22}, X_{23}, X_{24}, X_{31},$

$X_{32}, X_{33}, X_{34}, X_{41}, X_{42}, X_{43}, X_{44}, X_{51},$

$X_{52}, X_{53}, X_{54} \geq 0$

2.4 APPLICATION OF SIMPLEX NUMERICAL METHOD

The developed model equations were solved through the application of the Simplex Numerical method. The objective function and the model equation constraints were arranged in a tableau with the constraint inequalities transformed into equalities. The principles of the Simplex method were applied until the optimal solutions were attained. Based on the obtained optimal solutios, the available water was allocated to the various communities within the catchment basin for purposes domestic use, ecology, hydropower generation, industrial use and leakages/losses mitigation.

3.0 PRESENTATION AND ANALYSIS OF RESULTS

3.1 PRESENTATION OF RESULTS

The original data collected from the communities were presented on Table 3.1

Table 3.1: Aboine River Basin Linear Program – Original Data

Basic	X_1	X_2	X_3	X_4	b
Minimize	57.61	64.18	54.84	65.38	
Subject to					
1	52422	24568	92787	19734	= 189511
2	27.40	24.50	30.35	21.40	= 104
3	75.00	20.00	30.00	15.00	= 140
4	20.00	10.00	25.00	10.00	= 65
5	7.50	8.40	7.20	8.50	= 31
Lower Bound	0.00	0.00	0.00	0.00	
Upper Bound	Infinity	Infinity	Infinity	Infinity	
Unrest(y/n)?	n	n	n	n	

The solution of the equations were furthered through the Simplex method and the results gave rise to tableau as presented in Tables

Table 3.2: Initial Simplex Iteration Tableau

C_j			57	64	54	65	0	0	0	0	0	M	M	M	M	M
	BV	Q	X_1	X_2	X_3	X_4	S_1	S_2	S_3	S_4	S_5	A_1	A_2	A_3	A_4	A_5
M	A_1	189511	52422	24568	92787	19734	0	0	0	0	0	1	0	0	0	0
M	A_2	96	27	24	30	15	0	1	0	0	0	0	1	0	0	0
M	A_3	140	75	20	30	15	0	0	1	0	0	0	0	1	0	0
M	A_4	65	20	10	25	10	0	0	0	1	0	0	0	0	1	0
M	A_5	31	7.5	8.4	7.2	8.5	0	0	0	0	1	0	0	0	0	1
Zj			52551.5m	24630.4m	92879.2m	19782.5m	M	M	M	M	M	M	M	M	M	M
Zj - Cj			52551.5m - 57	24630.4m - 64	92879.2m - 54	19782.5m - 65	M	M	M	M	M	O	O	O	O	O

Table 3.3: 2[nd] Simplex Iteration Tableau

C_j			57	64	54	65	0	0	0	0	0	M	M	M	M	M
	BV	Q	X_1	X_2	X_3	X_4	S_1	S_2	S_3	S_4	S_5	A_1	A_2	A_3	A_4	A_5
M	A_1	2.04	0.56	0.26	1.00	0.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
54	A_2	34.8	10.2	16.2	0.0	8.7	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0
M	A_3	78.8	58.2	12.2	0.00	8.7	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.0	0.00	0.00
M	A_4	14.0	6.0	3.5	0.00	4.75	0.00	0.00	0.00	1.0	0.00	0.00	0.00	0.00	1.0	0.00
M	A_5	16.31	3.47	6.53	0.00	6.99	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
Zj			68.23m + 550.8	22.49m + 874.8	M	20.65m + 469.8	0.00	54	M	M	M	0.00	54	M	M	M
Zj - Cj			68.23m + 493.8	22.49m + 810.8	M - 54	20.65m + 404.8	0.00	54	M	M	M	- M	54-M	0.00	0.00	0.00

Table 3.4: 3rd Simlex Iteration Tableau

C_j			57	64	54	65	0	0	0	0	0	M	M	M	M	M
	BV	Q	X_1	X_2	X_3	X_4	S_1	S_2	S_3	S_4	S_5	A_1	A_2	A_3	A_4	A_5
M	X_1	1.28	0.00	0.14	1.00	0.13	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	-0.01	0.00	0.00
54	X_3	21.03	0.00	14.06	0.00	7.17	0.00	1.00	-0.20	0.00	0.00	0.00	1.00	-0.20	0.00	0.00
M	A_3	1.35	1.00	0.21	0.00	0.15	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.02	0.00	0.00
M	A_4	5.9	0.00	2.24	0.00	3.85	0.00	0.00	-0.12	1.00	0.00	0.00	0.00	-0.12	1.00	0.00
M	A_5	11.63	0.00	5.80	0.00	6.47	0.00	0.00	-0.07	0.00	1.00	0.00	0.00	-0.07	0.00	0.00
Zj			M	8.25M + 67.22	57	10.47m + 394.59	0.00	54	-.17m -11.37	M	M	0.00	54	- 0.17m -11.37	M	0.00
Zj – Cj			M - 57	8.25M + 703.22	3.00	10.47m + 329.59	0.00	54	-.17m -11.37	M	M	- M	54-M	-1.17 -11.37	0.00	-m

Table 3.5: 4th Simplex Iteration Tableau

C_j			57	64	54	65	0	0	0	0	0	M	M	M	M	M
	BV	Q	X_1	X_2	X_3	X_4	S_1	S_2	S_3	S_4	S_5	A_1	A_2	A_3	A_4	A_5
M	X_1	1.08	0.00	0.06	1.00	0.00	0.00	0.00	-0.01	-0.03	0.00	0.00	0.00	-0.01	-0.03	0.00
54	X_3	10.06	0.00	9.90	0.00	0.00	0.00	1.00	0. 02	-1.86	0.00	0.00	1.00	0. 02	-1.86	0.00
M	A_3	1.12	1.00	0.12	0.00	0.00	0.00	0.00	0.024	-0.039	0.00	0.00	0.00	0.024	-0.039	0.00
M	A_4	1.53	0.00	0.58	0.00	1.00	0.00	0.00	0.03	0.26	0.00	0.00	0.00	-0.03	0.26	0.00
M	A_5	1.73	0.00	2.05	0.00	0.00	0.00	0.00	0.124	1.68	1.00	0.00	0.00	0.124	-1.68	0.00
Z_j			M	2.17M + 575.72	57	65	0.00	54	0.148M – 1.44	-1.719M – 104.10	M	0.00	54	0.148M – 1.44	-1.719M -85.25	0.00
Z_j - C_j			M - 57	2.17M + 511.72	3	0.00	0.00	54	0.148M – 1.44	-1.719M – 104.10	M	0.00	54-M	-0.852M – 1.44	-0.719M -85.25	-M

Table 3.6: 5th Simplex Iteration Tableau

C_j			57	64	54	65	0	0	0	0	0	M	M	M	M	M
	BV	Q	X_1	X_2	X_3	X_4	S_1	S_2	S_3	S_4	S_5	A_1	A_2	A_3	A_4	A_5
57	X_1	1.029	0.00	0.00	1.00	0.00	0.00	0.00	-0.014	0.019	-0.03	0.00	0.00	-0.014	0.0192	0.00
54	X_3	2.344	0.00	0.00	0.00	0.00	0.00	0.00	-0.574	6.258	8.138	0.00	1.00	-0.574	6.258	0.00
M	A_3	1.019	0.88	0.00	0.00	0.00	0.00	0.00	0.017	0.059	-0.059	0.00	0.00	0.017	0.059	0.00
65	X_4	1.040	0.00	0.00	0.00	1.00	0.00	0.00	-0.038	0.736	-0.283	0.00	0.00	-0.065	0.736	0.00
64	X_2	0.844	0.00	1.00	0.00	0.00	0.00	0.00	0.060	-0.820	0.488	0.00	0.00	0.060	-0.820	0.00
Zj			0.88M	64	57	65	0.00	0.00	0.017M -30.424	0.059M +334.375	-0.059M +388.115	0.00	54	-0.017M -32.179	0.059M +334.386	0.00
Zj - Cj			0.88M -57	0.00	3	0.00	0.00	0.00	0.017M -30.424	0.059M +334.375	-0.059M +388.115	- M	54-M	-0.017M -32.179	1.059M +334.386	-M

Table 3.7: 6th Simplex Iteration Tableau

C_j			57	64	54	65	0	0	0	0	0	M	M	M	M	M
	BV	Q	X_1	X_2	X_3	X_4	S_1	S_2	S_3	S_4	S_5	A_1	A_2	A_3	A_4	A_5
57	X_1	1.022	0.00	0.00	1.00	0.00	0.00	0.00	-0.001	0.00	-0.055	0.00	-0.003	-0.012	0.00	0.00
54	X_3	0.359	0.00	0.00	0.00	0.00	0.00	0.00	-0.092	1.00	1.300	0.00	0.160	-0.092	1.00	0.00
M	A_4	0.998	0.88	0.00	0.00	0.00	0.00	0.00	0.012	0.00	-0.136	0.00	-0.009	0.022	0.00	0.00
65	X_4	0.776	0.00	0.00	0.00	1.00	0.00	0.00	0.030	0.00	-1.240	0.00	-0.118	0.003	0.00	0.00
64	X_2	1.138	0.00	1.00	0.00	0.00	0.00	0.00	-0.015	0.00	1.554	0.00	0.131	-0.015	0.00	0.00
Zj			0.88 M	64	57	65	0.00	0.00	0.012M - 4.035	54	-0.136M +85.921	0.00	-0.009M +9.183	0.022M -5.653	54	0.00
Zj - Cj			0.88 M - 57	0.00	3	0.00	0.00	0.00	0.012M - 4.035	54	-0.136M +85.921	- M	-0.009M +9.183	-0.978M - 5.653	54 – M	-M

Table 3.8: 7th Simplex Iteration Tableau

C_j			57	64	54	65	0	0	0	0	0	M	M	M	M	M
	BV	Q	X_1	X_2	X_3	X_4	S_1	S_2	S_3	S_4	S_5	A_1	A_2	A_3	A_4	A_5
57	X_1	1.022	0.00	0.00	1.00	0.00	0.00	0.00	-0.001	0.00	-0.055	0.00	-0.003	-0.012	0.00	0.00
54	X_3	0.359	0.00	0.00	0.00	0.00	0.00	0.00	-0.092	1.00	1.300	0.00	0.160	-0.092	1.00	0.00
M	S_1	1.134	1.00	0.00	0.00	0.00	0.00	0.00	0.014	0.00	-0.155	0.00	-0.010	0.025	0.00	0.00
65	X_4	0.776	0.00	0.00	1.00	1.00	0.00	0.030	0.00	- 1.240	0.00	-0.118	0.003	0.003	0.00	0.00
64	X_2	1.138	0.00	1.00	0.00	0.00	0.00	0.00	-0.015	0.00	1.554	0.00	0.131	-0.015	0.00	0.00
Zj			0.00	64	122	-65	0.00	1.95	-5.985	-26.6	166.521	-7.67	17.048	-6.417	54	0.00
Zj - Cj			-57	0.00	68	0.00	0.00	1.95	-5.985	-26.6	166.521	-7.67 - M	-17.048M	-6.417 -M	54 –M	-M

Table 3.9: 8th Simplex Iteration Tableau

Cj			57	64	54	65	0	0	0	0	0	M	M	M	M	M
	BV	Q	X_1	X_2	X_3	X_4	S_1	S_2	S_3	S_4	S_5	A_1	A_2	A_3	A_4	A_5
57	X_1	1.087	0.00	0.00	1.00	0.00	0.00	0.00	-0.005	0.042	0.00	0.00	0.004	-0.016	0.042	0.00
54	X_3	1.076	0.00	0.00	0.00	0.00	0.00	0.00	-0.071	0.769	1.00	0.00	0.123	-0.071	0.769	0.00
64	X_2	1.077	1.00	0.00	0.00	0.00	0.00	0.00	0.003	0.119	0.00	0.00	0.009	0.014	0.119	0.00
65	X_4	0.776	0.00	0.00	0.00	0.00	0.00	0.030	0.00	-1.240	0.00	-0.118	0.003	0.003	0.00	0.00
O	S_2	0.709	0.00	1.00	0.00	0.00	0.00	0.00	1.248	-1.195	0.00	0.00	-0.060	0.095	-1.195	0.00
Zj			64	64	54	0.00	0.00	1.95	75.753	- 112.62	54	-7.65	1.529	1.529	-32.56	0.00
Zj-Cj			-3	0.00	-3	-65	0.00	1.95	75.753	- 112.62	54	- 7.67M	1.529 -M	1.529 -M	-32.56 -M	-M

Table 3.10: 9th Simplex Iteration Tableau

Cj			57	64	54	65	0	0	0	0	0	M	M	M	M	M
	BV	Q	X_1	X_2	X_3	X_4	S_1	S_2	S_3	S_4	S_5	A_1	A_2	A_3	A_4	A_5
57	X_1	1.040	0.00	0.004	1.00	0.00	0.00	0.00	0.00	0.037	0.00	0.00	0.004	-0.016	0.037	0.00
54	X_3	1.015	0.00	-0.057	0.00	0.00	0.00	0.00	0.00	0.701	1.00	0.00	0.120	0.066	0.641	0.769
64	X_2	1.175	1.00	-0.007	0.00	0.00	0.00	0.00	0.00	0.122	0.00	0.00	0.009	0.014	0.122	0.00
65	X_4	0.776	0.00	0.00	0.00	0.00	0.00	0.030	0.00	-1.240	0.00	-0.118	0.003	0.003	0.00	0.00
O	X_2	0.568	0.00	0.801	0.00	0.00	0.00	0.00	1.00	-0.957	0.00	0.00	-0.048	0.076	-0.958	0.00
Zj			0.00	57.57	1.14	0.00	0.00	1.95	64	- 102.117	54	-7.67	-3.831	8.431	-24.591	41.526
Zj-Cj			-57	-6.43	60	-65	0.00	1.95	64	- 102.117	54	7.67 - M	3.831 -M	8.431 -M	-24.591 -M	41.526 -M

Table 3.11: 10th Simplex Iteration Tableau

Cj			57	64	54	65	0	0	0	0	0	M	M	M	M	M
	BV	Q	X_1	X_2	X_3	X_4	S_1	S_2	S_3	S_4	S_5	A_1	A_2	A_3	A_4	A_5
57	X_1	1.004	0.00	0.004	1.00	0.00	0.00	0.00	0.00	0.019	0.00	0.00	+0.004	-0.008	0.019	0.00
54	X_3	1.010	0.00	-0.057	0.00	0.00	0.00	0.00	0.00	0.701	0.00	0.00	-0.20	-0.066	-0.641	0.769
64	X_2	1.079	-1.00	-0.002	-0.080	0.00	0.00	0.00	0.00	0.122	0.00	0.00	-0.009	-0.014	0.122	0.00
65	X_4	0.776	0.00	0.00	0.00	0.00	0.00	0.030	0.00	-1.240	0.00	-0.118	-0.063	-0.003	0.00	0.00
0	S_2	0.568	0.00	0.801	0.00	0.00	0.00	0.00	1.00	-0.957	0.00	0.00	-0.048	0.076	-0.958	0.00
Zj			-64	-2.978	51.880	0.00	0.00	0.00	0.00	-33.86	-54.00	-7.670	-15.243	-5.111	-25.723	41.53
Zj-Cj			-7.00	-66.978	-2.120	-65.00	0.00	0.00	0.00	-33.86	-54.00	-7.670 -M	-15.24 -M	-5.111 -M	-25.72 -M	-41.53 -M

Following the optimal values obtained, the available water in the river was allocated to the communities as shown on Table 3.12

Table 3.12: Computed Values For Subarea Sectoral Allocation In Aboine River Basin

S/N	USAGE AREAS		ALLOCATION LIMITS FOR SUBAREAS			
		Obollo afor (x_1)	**Ikem X_2**	**Eha-Amufu X_3**	**Nkalaha X_4**	**Total amount (m^3x10^2)**
	Consumption & Domestic Use	2312265	1169802	4100761	716622	82994.50
	Ecological use (m^3/yr)	331102	320448	368083	212887	12325.20
	Industrial use (m^3/yr)	919	595	665	454	26.33
	Energy use (m^3/yr)	342	185	429	141	10.57
	Leakages mitigation (m^3/yr)	396694	223524	670488	139516	24302.22
	Total	**3041322**	**2214554**	**5140426**	**1069620**	**114659.22**

3.2 ANALYSIS OF RESULTS

The model equations were solved using Simplex method and it resulted to ten iterations. It was at the tenth iteration that optimal results were obtained because it was at this point that the Cj-Zj values were zero and negative. From the allocations done, it was observed that the domestic and consumption use had the highest amount of allocated water while the energy use was given the least allocation. The allocations were further analyzed to view how made for the various uses vary with each other. This was done as shown in Table 3.1:

Table 3.13: Degree of Allocations for Various Uses in Aboine River Basin

S/NO	Sectors for Water Use	Amount (%)
1	Consumption &Domestic Use	69.36
2	Ecological Use	10.3
3	Industrial Use	0.02
4	Energy Use	0.01
5	Leakages/Losses Mitigation	20.3

With the optimal allocations, it can be observed that all the purposes of domestic use, industrial, energy and ecology were satisfied. Water was also optimally apportioned for the possible mitigation of water losses which will accrue from leakages and other means of water wastages. This optimal allocation will make for adequate water provision for the communities within the catchment basin.

3.0 CONCLUSION AND RECOMMENDATIONS:

It can be concluded from this research that the Simple Numerical method is a valuable mathematical tool for the optimal allocation of water in any given river. The application of this mathematical method entrenches objectivity, fairness and efficiency in the business of water resources allocation.

From the results this research, the following recommendations are made

1. The Simple numerical method has a high potency of allocating water based on the principles of fairness, efficiency and optimality.
2. Water allocations for communities in any river basin should be subjected to an optimization process to ensure that the available water is fully utilized and appropriated.
3. The application of this method will likely stimulate economic activities and accelerate industrial and infrastructural development of areas where it is applied.

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