Comprehensive formulations forthe total normal-incidence optical reflectance and transmittance of thin films laid on thick substrates

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Abstract

The general formulae for the total spectral transmittance $T(\lambda)$and specular reflectance $R(\lambda)$of an "ideal"four-layered optical system of the {air/film/thickdielectric substrate/air}-pilingwere fully derived for thin and thick films using the $E^{+}- E^{-}$ transfer matrix method, combined with incoherent treatment of light plane waves normally incident upon the system. The attained$T(\lambda)$- and$R(\lambda)$- formulae at the wavelength $\lambda$are explicitly expressed in terms of n and n_s, the indices of refraction of the film and substrate,as well as of $d$and κ, the thickness and extinction coefficient of the film. These $T(\lambda)$- and$R(\lambda)$-formulae are akin to those quoted in the literature for analyzing experimental transmission and reflection data of "ideal" four-layered structures in their transparent and absorbing spectral regions. The derived $R(\lambda)$ and $T(\lambda)$formulaecan readily be extended to the oblique-incidence case and to include the effect of optical absorption of the substrate.

1. Introduction

Knowledge of accurate values of the *complex*index of refraction$\hat{n}(\lambda)$of thin solid films as a function ofthe spectral wavelength $\lambda$of the light incident upon them is essential, both from fundamental and technological viewpoints. This yields good information ontheirband-energy structures and on the wavelength dependence (i.e., dispersion) of their optical constants[1-24].In the present article, Ishall adopt the nomenclature $\hat{n}(\lambda) \equiv n(\lambda) - j\kappa(\lambda)$,with $j \equiv \sqrt{-1}$, where $n(\lambda)$ and $\kappa(\lambda)$are its real and imaginary parts, and use*thecircumflexon the algebraic symbols to designate complex quantities*.Often, $n(\lambda)$is called the ordinary index of refraction of the material, which largely determines how reflective and refractive a film made from this material will be,and $\kappa(\lambda)$ is itsextinction coefficient, which governs the optical absorptionof electromagnetic waves propagating throughoutit. The absorption phenomena in a substance can also be specified by the absorption coefficient $\alpha(\lambda)$, which is related to $\kappa(\lambda)$through the relation:$\alpha(\lambda) \equiv 4\pi\kappa(\lambda)/\lambda$.

The experimental optical findings usually exemplify the quality of the investigatedlayers(films), existence of native impurities and structural defects in them as well as the nature of dispersion and optical transition phenomena taking place in thesefilms. This is important for assessing the performance of electronic and optical devices incorporating them. Also, the index of refraction of a material is necessary for the design and modeling of a variety of optical systemsintegratingthin films made from this material.If the model of a linear, isotropic, homogeneous, and nonmagneticplane-parallel thin film (hereafter called an *"ideal" film*) is applicable, the real $n(\lambda)$and imaginary $\kappa(\lambda)$parts of its complex index of refraction completely determine its optical properties, in addition to otherrelated physical features[16-58]. When theseideal requirements are *not met* as in case of inhomogeneous and rough films, it is still possible to model the optical properties of these "non-ideal" thin films, albeit with much more complexity [59-73].

Independent measurements of two macroscopic optical quantities are in principle necessary at each spectral wavelengthλ in order to solve their theoretical expressions for the two unknowns $n(\lambda)$ and $\kappa(\lambda)$ of the layer under study [17, 24]. One type of optical measurement may be sufficient if the data is scanned over a wide range of wavelengths of the light incident onto the layer, but the two independent macroscopic optical measurements are sometimes necessary.Further, the knowledge of $n(\lambda)$ and $\kappa(\lambda)$ of a layer enables one to determine $\alpha(\lambda)$ and the band-gap energyE_g of its material. A variety of optical characterization techniquesare normally employed to getthe experimental data required for determining optical parameters; with each techniquehaveits own advantages and disadvantages [1-25, 70].

A powerfulmethodthat has been commonly implementedfor the evaluationof $n(\lambda)$ and $\kappa(\lambda)$of afilm is that based on non-destructive measurements of its normal-incidence transmittance $T(\lambda)$and/or specular reflectance $R(\lambda)$ over the ultraviolet-visible-infrared (UV-Vis-IR) regions of the electromagnetic spectrum. In this situation, most of the literature experimental work on optical properties of solid layers has been often realized on *three-layeredstructures* wherein a layerjust stands freely in air (an *air-supported layer*) or on simple *four-layeredstructures*, in which a filmis deposited onto a thick substrate, with the whole optical unit beingimmersed in air.

A rigorous analysis of the measured spectral transmittance and reflectance of three- or four-layered structuresto determine the optical constants of their stacked layers, however,requires propertheoretical formulae that adequatelydescribe theirexperimental $T(\lambda)$ and $R(\lambda)$spectra. In many of thereported journal articles[25-57], the theoretical formulae of $R(\lambda)$ and $T(\lambda)$describing "ideal"*four-layered* structures are just cited from references thatare occasionallyaccessibleto many readers, orare just quoted without reference or were presented in concisesymbolical approaches, in most of which model approximations were not explicitly emphasized. One may also encounterdiverseintricate$R(\lambda)$- and $T(\lambda)$-formulas forthe *idealfour-layered* structures in some books of optics thatwere derived by the use of different modeling approaches [9-17, 20-33].Further,Šantić and Scholz [58] were unable to re-derive the frequently adopted $T(\lambda)$-formula [35] that describesthe normal-incidence transmittance of an "ideal" four-layered structure made from for an absorbing film laid on a dielectric substrate, both of which are bounded by air. These authors questioned the correctness of such a $T(\lambda)$-formula and instead, based on certain assumptions and restrictive conditions, have derived a different normal-incidence$T(\lambda)$-formula for the optical system of concern. The authenticity, adequacy and correctness of the $T(\lambda)$-formula commonly reported in the literature for "ideal"*four-layered* structures would thus become questionable and illusive for the scientists working in the field.

Therefore, I have been driven to derive from scratch the spectral normal-incidence transmittance$T(\lambda)$-formula as well as the respective formula of the specular reflectance $R(\lambda)$forthe "ideal" four-layered structurehaving the {*air/film/thick dielectric substrate/air*} piling, whether the film is thin or thick as well as conducting or dielectric (transparent). The obtained $R(\lambda)$- and $T(\lambda)$- formulae for such a four-layered structure, which were found to match the universally accepted $R(\lambda)$- and $T(\lambda)$- formulations,encouraged me to claim that the presentarticle can serve as a self-contained, autonomous and thorough reference for those engaged in the physical interpretation and numerical analysis of measured optical data of both three- and four-layered structures. Further, the approach used here to accomplish the required$R(\lambda)$ and $T(\lambda)$formulations can, in principle,be generalized to the case of monochromatic light plane waves that are incident either normally or obliquely upon multi-layered structures whose individual layers made from a combination of dielectric and/orsemiconducting substances of diverse physical properties.

2. General theoretical aspects

The simplest solution of the coupled Maxwell's first-order linear partial differential equations fora beam of monoenergetic (monochromatic)electromagnetic (EM) radiation propagating intofree-space or in an extended materialregion isa plane waverepresented by spatially-dependent and time-harmonic complex electric$\hat{\mathbf{E}}(\mathbf{r},t)$and magnetic$\hat{\mathbf{B}}(\mathbf{r},t)$field vectors, whose real or imaginary parts have only physical meaning. For a monochromatic EMplane wave propagating through a linear, isotropic, homogenous, and conducting (or semiconducting) medium,$\hat{\mathbf{E}}(\mathbf{r},t)$- and $\hat{\mathbf{B}}(\mathbf{r},t)$- vector fields, which must satisfy allMaxwell's equations,are usuallyexpressed, for mathematical convenience, in complex formatsas below [9, 14-17, 20-24, 29-30, 74-76]

$$\begin{Bmatrix}\hat{\mathbf{E}}(\mathbf{r},t)\\ \hat{\mathbf{B}}(\mathbf{r},t)\end{Bmatrix} = \begin{Bmatrix}\hat{\mathbf{E}}_0\\ \hat{\mathbf{B}}_0\end{Bmatrix}\exp\left[-j(\hat{\mathbf{k}}\,.\,\mathbf{r}-\omega t\right] = \begin{Bmatrix}\hat{\mathbf{E}}_0\exp(-\mathbf{k}_{\mathrm{i}}.\,\mathbf{r})\\ \hat{\mathbf{B}}_0\exp(-\mathbf{k}_{\mathrm{i}}.\,\mathbf{r})\end{Bmatrix}\exp\left[-j(\mathbf{k}_{\mathrm{r}}\,.\,\mathbf{r}-\omega t\right] \tag{1}$$

The symbols $\mathbf{r}$and t represent, respectively, the spatial position vector and time, while ωisthereal angular frequency of photons comprising the incoming monochromatic EM planewave whose wavelength $\lambda_0 \equiv (2\pi c/\omega)$, where c is light speed in vacuum. Generally, $\hat{\mathbf{k}}$ is acomplex wave propagation vectordefined here, to be consistent with the definition of other complex physical quantities, as$\hat{\mathbf{k}} \equiv \mathbf{k}_{\mathrm{r}} - j\mathbf{k}_{\mathrm{i}}$, with $j \equiv \sqrt{-1}$, where $\mathbf{k}_{\mathrm{r}}$ and $\mathbf{k}_{\mathrm{i}}$ are its real and imaginary vector parts, respectively. The time- and spatially-independent pre-exponential quantities $\hat{\mathbf{E}}_0$ and $\hat{\mathbf{B}}_0$ are, respectively,the complex-vector amplitudes of the electric- and magnetic-vector fields of the original EMradiation striking the medium. In fact,$\hat{\mathbf{E}}_0$ and $\hat{\mathbf{B}}_0$determine the real energy carried by the respective plane-wave field,which is proportional to the timeaverage (over at least one period $T \equiv 2\pi/\omega$) of the square modulus of its associated fields- that is $\langle|\hat{\mathbf{E}}(\mathbf{r},t)|^2\rangle_t$and $\langle|\hat{\mathbf{B}}(\mathbf{r},t)|^2\rangle_t$.

An energy quantity which is usefulin the discussion of the behavior of EM waves crossing a discontinuous boundary of two dissimilaradjacent media is the Poynting vector$\hat{\mathbf{S}}(\mathbf{r},t)$, defined by$\hat{\mathbf{S}}(\mathbf{r},t) \equiv \hat{\mathbf{E}}(\mathbf{r},t) \times \hat{\mathbf{B}}(\mathbf{r},t)$. For a harmonically time-dependent plane EMwave, the *time-averaged*$\langle\hat{\mathbf{S}}(\mathbf{r},t)\rangle_t$is a *real quantity*that determines the amount of electromagnetic energy per unit area per unit time (energy flux per unit area or intensity) and the direction of wave propagation. As complex-vector fields of a plane EM wave are time-harmonic with the same frequency, but not necessarily of the same phase,$\langle\hat{\mathbf{S}}(\mathbf{r},t)\rangle_t$can be described by the formula[9, 22-23, 29-31, 74-76]

$$\langle\hat{\mathbf{S}}(\mathbf{r},t)\rangle_t \equiv \langle\hat{\mathbf{E}}(\mathbf{r},t) \times \hat{\mathbf{B}}(\mathbf{r},t)\rangle_t = \frac{1}{2}\,\mathrm{Re}\left[\hat{\mathbf{E}}(\mathbf{r},t) \times \hat{\mathbf{B}}^*(\mathbf{r},t)\right] \tag{2}$$

The quantity$\hat{\mathbf{B}}^*(\mathbf{r},t)$ is the complex vector conjugate of the magnetic-field vector$\hat{\mathbf{B}}(\mathbf{r},t)$.

Another important phenomenon in optics which governs the behavior of plane EM waves crossing aphysical interface of two dissimilar optical media is the so-called state of polarization of the wave. In accordance with traditional terminology, the direction of the time-harmonic vibrating electric field vector $\hat{\mathbf{E}}(\mathbf{r},t)$of an electromagnetic wave at apoint $\mathbf{r}$in space is taken to represent the state of polarization of the wave[9, 22-23, 74-76].In brief, a planeEM (light) wave is referred to as *unpolarized* if Re $\{\hat{\mathbf{E}}(\mathbf{r},t)\}$, the real partof its complex electric-field vector $\hat{\mathbf{E}}(\mathbf{r},t)$, is randomly vibrating in space and time in all possible directions, while alightwave is said to be *linearlypolarized*when its Re $\{\hat{\mathbf{E}}(\mathbf{r},t)\}$ is time-harmonically vibrating along a particulardirection in space. More generally, the complex vector amplitude $\hat{\mathbf{E}}_0$ of the electric-field vector $\hat{\mathbf{E}}(0,t)$of a plane EMwave can be resolved at a certain point, say $\mathbf{r}=0$, into two perpendicular complex vector components as$\hat{\mathbf{E}}_0=\mathbf{p}\hat{E}_{0\mathrm{p}}+\mathbf{s}\hat{E}_{0\mathrm{s}}=\mathbf{p}E_{\mathrm{p}}\exp(j\emptyset)+\mathbf{s}E_{\mathrm{s}}$,where $\mathbf{p}$ and $\mathbf{s}$ are two real orthogonalunit vectors. The angle $\emptyset$ is a phase difference between the vector components $\hat{\mathbf{E}}_{0\mathrm{p}}$ and $\hat{\mathbf{E}}_{0\mathrm{s}}$which determines the state of polarization of the plane wave[9, 22-23, 74-76]. For a linearly-polarized plane EM wave $\emptyset$ takes the values 0 or π, depending on the orientation of itsreal electric-field vector $\mathrm{Re}\left\{\hat{\mathbf{E}}(0,t)\right\}$, while a circularly-polarized plane EM wave corresponds to$\emptyset=\pm\pi/2$ with equal vector amplitudes of its $\mathrm{Re}\left\{\hat{\mathbf{E}}(0,t)\right\}$-components ($E_{\mathrm{p}}=E_{\mathrm{s}}$). The more general state of polarization of a plane EM wave is the elliptical polarization when $\emptyset=\pm\pi/2$ and $E_{\mathrm{p}}\neq E_{\mathrm{s}}$ or for other values of$\emptyset$ (even if $E_{\mathrm{p}}=E_{\mathrm{s}}$), so that the minor and major axes of the corresponding ellipse are at an angle to the **p**- and **s**- axes [9, 22-23, 74-76].

Consider alinear, homogeneous, isotropic, conducting, andnonmagnetic medium of a non-vanishing electrical conductivityσ_{c}, an electric permittivity $\epsilon=\epsilon_0\epsilon_{\mathrm{r}}$ and a magnetic permeability $\mu=\mu_0$, whereϵ_0 and μ_0 are the free-space permittivity and permeability, respectively, andϵ_{r} is the medium'srelative electric permittivity (dielectric constant). The solutions of Maxwell's equations given in Equation (1) for a monochromatic plane EM wave travelling through such a medium are only satisfied for a *complex*wave propagation vector $\hat{\mathbf{k}}$, of magnitude $\hat{\mathrm{k}}\equiv\sqrt{\hat{\mathbf{k}}\,.\,\hat{\mathbf{k}}}$, connected to the real angular frequency ω of the incoming EMradiation by a dispersion relation of the form

$$\hat{\mathrm{k}}=\sqrt{\mathrm{k}_{\mathrm{r}}^2-\mathrm{k}_{\mathrm{i}}^2-j(2\,\mathbf{k}_{\mathrm{r}}.\mathbf{k}_{\mathrm{i}})}=\frac{\omega}{c}\left(\epsilon_{\mathrm{r}}-j\frac{\sigma_{\mathrm{c}}}{\epsilon_0\omega}\right)^{\frac{1}{2}}=\frac{\omega}{c}\sqrt{\hat{\epsilon}_{\mathrm{r}}}\equiv\frac{\omega}{c}\hat{n}\equiv\frac{\omega}{c}(n-j\kappa)\qquad(3)$$

Equation (3) tells us that if n andκ of asubstance are given and real, the quantities ϵ_{r} and $(\sigma_{\mathrm{c}}/\epsilon_0\omega)$, the realand imaginary partsof its complex dielectric constant $\hat{\epsilon}_{\mathrm{r}}=\epsilon_{\mathrm{r}}-j(\sigma_{\mathrm{c}}/\epsilon_0\omega)$, can then be found(and vice versa)from the relationships

$$\epsilon_{\mathrm{r}}=n^2-\kappa^2\,\frac{\sigma_{\mathrm{c}}}{\epsilon_0\omega}=2n\kappa\qquad(4a)$$

However, the electric conductivity σ_{c}, just like the dielectric constantϵ_{r}, is not a true constant of the medium, but is generallya complex function of the angular frequencyω. Thus, to ensure the positivity of the real and imaginary parts of the complex optical parameters, a complex conductivity$\hat{\sigma}_{\mathrm{c}}(\omega)$that varies with frequencyis traditionally defined as$\hat{\sigma}_{\mathrm{c}}(\omega)\equiv\sigma'(\omega)+j\sigma''(\omega)$, where $\sigma'(\omega)$ and $\sigma''(\omega)$ are its real and imaginary parts, which isrelated to a complex dielectric constant $\hat{\epsilon}_{\mathrm{r}}(\omega)\equiv{\epsilon_{\mathrm{r}}}'(\omega)-j{\epsilon_{\mathrm{r}}}''(\omega)as\hat{\sigma}_{\mathrm{c}}(\omega)\equiv j\omega\epsilon_0\hat{\epsilon}_{\mathrm{r}}(\omega)$ [3, 9-12, 16-24, 31,74-77]. The real part of $\hat{\epsilon}_{\mathrm{r}}(\omega)$, i.e., ${\epsilon_{\mathrm{r}}}'(\omega)=\sigma''(\omega)/\omega\epsilon_0$, is hence a combination of aconstant dielectric constant ϵ_{r} and a frequency-dependent part arising from dispersiveprocesses, whereas $\sigma'(\omega)$could be due to a constant dc electrical conductivity$\sigma_{\mathrm{dc}}$and$\omega\epsilon_0{\epsilon_{\mathrm{r}}}''(\omega)$arising from some sort of a frequency-dependent (dispersive)process[3, 9-12, 16-24, 31, 74-77].The complex index of refraction $\hat{n}(\omega)$ of a linear, isotropic, homogeneous, dispersive and nonmagnetic medium is also related to its complex dielectric constant $\hat{\epsilon}_{\mathrm{r}}(\omega)$ via a Maxwell-like formula, namely $\hat{n}(\omega)=n(\omega)-j\kappa(\omega)\equiv\sqrt{\hat{\epsilon}_{\mathrm{r}}(\omega)}$[3, 9-12, 16-24, 31, 74-77]. Accordingly, Equation (4a) still holds through replacing the constant parametersϵ_{r} and $\sigma_{\mathrm{c}}/\epsilon_0\omega$ bythe frequency-dependent real and imaginary components of the complex dielectric constant$\hat{\epsilon}_{\mathrm{r}}(\omega)$- that is,

$\epsilon_r{}'(\omega) = [n(\omega)]^2 - [\kappa(\omega)]^2 \epsilon_r{}''(\omega) = 2n(\omega)\kappa(\omega)$ (4b)

Thus, the empirical or theoretical models of the frequency-dependent dielectric constant and electric conductivity, through which the physical properties of a substance are originally expressed, can be employed to validate the dispersive behavior of its optically measured index of refraction $n(\omega)$ and extinction coefficient$\kappa(\omega)$.

At this stage, it is worth to discusssome physical realities regarding the propagation of a monochromatic plane electromagnetic (light)wave of a real angular frequency ω(corresponds to a free-space wavelength $\lambda_0 = 2\pi c/\omega = 2\pi/k_0$, where$c$ is the speed of light in vacuum and $k_0$is the magnitude of EM plane wave propagation vector$\mathbf{k_0}$) in both conducting and non-conducting or insulating (dielectric)media.

In a linear, isotropic, homogeneous and nonmagnetic dielectric medium of a real index of refraction n (κ = 0), such a light plane wave will travel along the direction of a real propagation vector $\mathbf{k} = k\,\mathbf{u}$, with$\mathbf{u}$being a real unit vector and $k = 2\pi/\lambda$, where λ is the wavelength of the light plane wave travelling inside such a medium. The associatedEMcomplex field vectors will thus be expressed as$\hat{\mathbf{E}}(\mathbf{r},t) = \hat{\mathbf{E}}\exp[-j(\mathbf{k}.\mathbf{r} - \omega t]$and $\hat{\mathbf{B}}(\mathbf{r},t) = \hat{\mathbf{B}}\exp[-j(\mathbf{k}.\mathbf{r} - \omega t]$, where $\hat{\mathbf{E}}$ and $\hat{\mathbf{B}}$ are their complex constant vector amplitudes that are interlinkedas$\hat{\mathbf{B}} = (k/\omega)\big[\mathbf{u} \times \hat{\mathbf{E}}\big]$and are perpendicular to each other (i. e., $\hat{\mathbf{E}}.\hat{\mathbf{B}} = 0$).If $\epsilon_r \neq 0$,both $\hat{\mathbf{E}}$ and $\hat{\mathbf{B}}$ vectors are perpendicular to $\mathbf{k}$(or $\mathbf{u}$)- that is,$\mathbf{u}.\hat{\mathbf{E}} = \mathbf{u}.\hat{\mathbf{B}} = 0$ always and the light wave isthus called*transverse*. A non-vanishing *polarization* charge density in a linear, isotropic, homogeneous and stationary medium free of both external charges and conduction-current sources can only produce a *longitudinal* electric field when $\epsilon_r = 0$, with $\hat{\mathbf{H}}(\mathbf{r},t) = 0$ and $\mathbf{k} \mathbf{x}\, \hat{\mathbf{E}}(\mathbf{r},t) = 0$, since $\boldsymbol{\nabla}.\hat{\mathbf{E}}(\mathbf{r},t) \neq 0$ even though $\boldsymbol{\nabla}.\hat{\mathbf{D}}(\mathbf{r},t) = 0$[74-77]. The quantities $\hat{\mathbf{H}}(\mathbf{r},t)$ and $\hat{\mathbf{D}}(\mathbf{r},t)$ are, respectively, the magnetic intensity and electric displacement vectors, which are often related to the EM field vectors $\hat{\mathbf{B}}(\mathbf{r},t)$ and $\hat{\mathbf{E}}(\mathbf{r},t)$ via the so-called constitutive equations [4-17, 20-24, 74-77].In such a linear, isotropic and non-magnetic dielectric medium, the vectors $\mathbf{k}$, $\hat{\mathbf{E}}$, and $\hat{\mathbf{B}}$ (*in that order*) form a *right-handed orthogonal set*and one can getthe simple*transversedispersionrelation*$k = \sqrt{\epsilon_r}\omega/c = n\omega/c$.

It deserves noting here that the vanishing of the *dot product*of the two *complex*electromagnetic field vectors$\hat{\mathbf{E}}(\mathbf{r},t)$and $\hat{\mathbf{B}}(\mathbf{r},t)$does not in general mean that their real parts are perpendicular. But, for a monochromatic electromagnetic *plane*wave propagating throughouta ponderable material medium, it can be shown that, at a certain spatial point$\mathbf{r}$ and at a time instant *t*, Re $\hat{\mathbf{E}}(\mathbf{r},t)$. Re $\hat{\mathbf{B}}(\mathbf{r},t) = 0$and Re $\hat{\mathbf{E}}$. Re $\hat{\mathbf{B}} = 0$, irrespective of the state of polarization of the plane wave entering the medium [74-76].

On the other hand, when a monochromatic time-harmonic plane EM wave is travelling througha linear, homogeneous, nonmagnetic *conducting* medium characterized by a complex index of refraction $\hat{n} \equiv n - j\,\kappa$, the propagation vector of the plane wave is, however, a *complex* vector quantity with its magnitude$\hat{k}$being related, via Equation (3), to$\hat{n}$as$\hat{k} \equiv \hat{n}\omega/c$. Thus, the time-harmonic and spatially-dependent complex electric- and magnetic- vector fields $\hat{\mathbf{E}}(\mathbf{r},t)$ and $\hat{\mathbf{B}}(\mathbf{r},t)$ of amonochromatic plane EM wave propagating inside such a conducting medium will be generally described by Equation (1).

In the case of the above-specified conducting medium and when $\mathbf{k}_r$ and $\mathbf{k}_i$ have different directions,$\hat{\mathbf{E}}(\mathbf{r},t)$ and $\hat{\mathbf{B}}(\mathbf{r},t)$ are not in phase with each other and their real parts do not have to be perpendicular to neither the vector $\mathbf{k}_r$, along which the wave is propagating, nor to the vector $\mathbf{k}_i$, along which the wave amplitude decreases most rapidly. Still, the wave is still called *transverse*(if $\hat{\epsilon}_r \neq 0$) in the sense $\hat{\mathbf{k}}.\hat{\mathbf{E}} = 0$ and $\hat{\mathbf{k}}.\hat{\mathbf{B}}$, where $\hat{\mathbf{E}}$ and $\hat{\mathbf{B}}$ are the complex vector amplitudes of $\hat{\mathbf{E}}(\mathbf{r},t)$ and $\hat{\mathbf{B}}(\mathbf{r},t)$, the real parts of whichare *not* perpendicular to each other, *except* for monochromatic *linearly-polarized* plane EM waves [76].Its complex vector amplitudes $\hat{\mathbf{E}}$ and $\hat{\mathbf{B}}$should be inter-linked by$\hat{\mathbf{B}} = \big(\hat{\mathbf{k}} \times \hat{\mathbf{E}}\big)/\omega$ or, by making use of both of the relation $\hat{\mathbf{k}}.\hat{\mathbf{E}} = 0$ and of the BAC-CAB rule, these $\hat{\mathbf{E}}$- and $\hat{\mathbf{B}}$- complex vector amplitudes can now be inter-connected by the formulas: $\hat{\mathbf{E}} = -[\omega/(\hat{\mathbf{k}}.\hat{\mathbf{k}})](\hat{\mathbf{k}} \times \hat{\mathbf{B}}) = -(c/\omega\,\hat{n}^2)(\hat{\mathbf{k}} \times \hat{\mathbf{B}})$[74-76].

Now, let us assume that the propagation vectors $\mathbf{k}_r$and $\mathbf{k}_i$to have the same spatial directionsuch that $\hat{\mathbf{k}} = (\mathrm{k_r} - j\mathrm{k_i})\ \mathbf{u} = \hat{\mathrm{k}}\,\mathbf{u}$, where $\mathbf{u}$ is an arbitrary *real* unit-vector along which the light plane wave is propagating insidemedium. This assumption *does hold* for the case of a light plane wave entering a linear, isotropic, homogeneous and nonmagnetic conducting medium at *normal incidence* to a plane boundary. In this special and practical situation, one can show that $\mathrm{k_r} = n\omega/c$and $\mathrm{k_i} = \kappa\omega/c$, and the respective electromagnetic plane wave field vectors $\hat{\mathbf{E}}(\mathbf{r},t)$ and $\hat{\mathbf{B}}(\mathbf{r},t)$are not in phase with each other,yetthe plane wave is still *transverse*if $\hat{\epsilon}_r \neq 0$such that $\mathbf{u}.\hat{\mathbf{E}}(\mathbf{r},t) = 0 = \mathbf{u}.\hat{\mathbf{B}}(\mathbf{r},t)$.These electromagnetic plane wave complex vector fields$\hat{\mathbf{E}}(\mathbf{r},t)$ and $\hat{\mathbf{B}}(\mathbf{r},t)$are now interrelated as

$$\hat{\mathbf{B}}(\mathbf{r},t) = \frac{\hat{n}}{c}\{\mathbf{u} \times \hat{\mathbf{E}}(\mathbf{r},t)\} \tag{5}$$

Furthermore, the quantities Re $\hat{\mathbf{E}}(0,t)$ and Re$\hat{\mathbf{B}}(0,t)$ are still *notperpendicular* to each other, *except for monochromatic linearly-polarized electromagnetic plane waves*.

Now, put $\mathbf{u}\,.\,\mathbf{r} = \zeta$ in Equation (1) and $\hat{\mathbf{k}}$=$(\mathrm{k_r} - j\mathrm{k_i})\mathbf{u}$to get the formula

$$\hat{\mathbf{E}}(\mathbf{r},t) = \{\hat{\mathbf{E}}_0 \exp(-\kappa\omega\zeta/c)\} \exp[-j\omega(n\zeta/c - t)] \tag{6}$$

Equation (6) implies that amonochromatic lightplane wave of original wavelength $\lambda_0$propagating along the $\mathbf{u}$-direction through alinear, isotropic, homogeneous, and nonmagnetic conductorhas a phase velocity $\mathbf{v}_\mathrm{p} = \mathbf{u}\,c/n$, a wavelength $\lambda = 2\pi/\mathrm{k_r} = \lambda_0/n$and its electric-field vector amplitude is most rapidly attenuated (exponentially) into the conductor. Also, one can show, using Equations (2) and (5) in conjunction with the BAC-CAB rule, that the *time-averaged* Poynting vector $\langle\hat{\mathbf{S}}(\mathbf{r},t)\rangle_t$of a light plane wave propagating in a conducting medium is given by

$$\langle\hat{\mathbf{S}}(\mathbf{r},t)\rangle_t = \mathbf{u}\left\{\frac{1}{2}\ \mathrm{Re}\ (\hat{n}^*/c)\left|\hat{\mathbf{E}}(\mathbf{r},t)\right|^2\right\}, \tag{7}$$

where$\hat{n}^*$ is the complex conjugate of $\hat{n}$.

As imposed by Maxwell's Equations and their time-harmonic plane wave solutions, the optical and dielectric properties of alinear and nonmagneticsubstanceiscompletely determined by its own wavelength-dependent (dispersive) complex index of refraction$\hat{n}(\lambda) \equiv n(\lambda) - j\kappa(\lambda)$, which also governsthe reflection and transmission of plane waves at aphysical boundary (interface) of two dissimilar media. The direction of specularly reflected and refracted (transmitted) plane light waves at the interface of suchoptical media can be determined by the common laws of specular reflection and refraction (Snell's law), which is modified in case of conducting media[9-17,20-31,74-76]. Let a monochromatic light plane wave propagating in a medium$l$of a complex index of refraction $\hat{n}_l$to hit obliquely, at an angle of incidence θ_l, a smooth and homogeneous interface of another medium $m$of a complex index of refraction $\hat{n}_m$. Thewave will be specularly reflected back into the incident medium at an angle of reflection $\theta_l' \equiv \theta_l$and refracts into the adjacent medium $m$at an angle of refraction θ_m.As the indices of refraction of the two media are complex, both the reflection and refraction angles arealso complex quantities, designated by $\hat{\theta}_l$and $\hat{\theta}_m$, the geometric meaning of which is not understood in the usual way. These complex angles of reflection and refraction can still be connected by a modified form of the ordinary (usual)Snell's law valid in case of dielectric media as$\hat{n}_l \sin\hat{\theta}_1 = \hat{n}_2 \sin\hat{\theta}_2$ [9-17, 20-31,74-76].It is worth noting here that the algebraic derivation of the mathematical formulations describing vector-/scalar- field quantities related to the reflection and refraction of plane monochromatic electromagnetic waves at the interface of two dissimilar media does not appeal to geometry, with the results, nevertheless,remain formally correct in both obliquely- ornormal- incidence cases.

The amount of reflected and transmitted electromagnetic energy flux per unit area (intensity) at an interface of two dissimilar media *cannot*, however, be extracted from the laws of specular reflection and refraction and can only be determined quantitatively from Fresnel's electric- or magnetic-field amplitude reflection and transmission coefficients[9-17, 20-31,74-76].The Fresnel's reflection and transmission coefficients at an interface of twomedia are in general complex functions inthe refractive indices of these media as well as in theangles of incidence and refraction of the plane wave incidentattheircommon interface. Also, the magnitudes of Fresnel's reflection and transmission coefficientsare dependent on thespectral wavelengthof this electromagnetic plane wave as well as on its state of polarization (in case of oblique incidence).

A planeelectromagnetic wave is called *p*-or *s*- linearly polarizedwhen its associated electric-field vectoris, respectively,parallel or perpendicular to the plane of incidence containing the propagation vectors of the incident, reflected, and transmitted waves at an interface of two mediaas well as to areal unit-vector **n**normal to such interface, which is perpendicular to the plane of incidence.The Fresnel's reflection and transmission coefficients for *p*- or *s*- linearly polarized plane electromagnetic waves having the complex electric-field vector amplitudes$\hat{E}_{\rm ip}$ or $\hat{E}_{\rm is}$ and are obliquely incident at the interface of two media l and $m$are defined as: $(\hat{r}_{lm})_{\rm p} \equiv (\hat{E}_{\rm rp}/\hat{E}_{\rm ip})$, $(\hat{t}_{lm})_{\rm p} \equiv (\hat{E}_{\rm tp}/\hat{E}_{\rm ip})$,$(\hat{r}_{lm})_{\rm s} \equiv (\hat{E}_{\rm rs}/\hat{E}_{\rm is})$ and $(\hat{t}_{lm})_{\rm s} \equiv (\hat{E}_{\rm ts}/\hat{E}_{\rm is})$,where$\hat{E}_{\rm rp}$, $\hat{E}_{\rm tp}$, $\hat{E}_{\rm rs}$, and$\hat{E}_{\rm ts}$ are complex amplitudes of the electric-field vectors of plane waves reflected and transmitted at that interface[9-17, 20-31,74-76].At normal-incidence, however, the distinction between the two types of light wave polarization in the formulations of reflection and transmission becomes irrelevant.

Now, consider aplane light wave propagating in a linear, isotropic, homogenous and nonmagnetic *conducting*medium $l$is incident normally ($\theta_l = 0^{\rm o}$) atits interface to anotheradjacent linearisotropic, homogenous and nonmagnetic *conducting*medium m. In the normal-incidence case, thefour independent expressions ofobliquely-incidence Fresnel's complex electric-field amplitude reflection and transmission coefficients, which are given in *Appendix* A and discussed in a variety of books of classical optics and electrodynamics [9-17, 20-31,74-76], reduce to a couple of independent general expressions of the form

$$\hat{r}_{lm} = \frac{\hat{n}_l - \hat{n}_m}{\hat{n}_l + \hat{n}_m} = \frac{(n_l - j\kappa_l) - (n_m - j\kappa_m)}{(n_l - j\kappa_l) + (n_m - j\kappa_m)} = \frac{(n_l - n_m) - j(\kappa_l - \kappa_m)}{(n_l + n_m) - j(\kappa_l + \kappa_m)} \tag{8}$$

$$\hat{t}_{lm} = \frac{2\,\hat{n}_l}{\hat{n}_l + \hat{n}_m} = \frac{2\,(n_l - j\kappa_l)}{(n_l - j\kappa_l) + (n_m - j\kappa_m)} = \frac{2\,(n_l - j\kappa_l)}{(n_l + n_m) - j(\kappa_l + \kappa_m)} \tag{9}$$

Equations (8) and (9) are generally valid for normal-incidence light reflection and transmission that occur ateither side of each interface of two adjacent linear, isotropic,homogeneous and nonmagnetic conducting or non-conducting media. The formulae of obliquely-/normal- incidence Fresnel's reflection and transmission coefficients for monochromatic *p*- and *s*- plane light waves satisfy two interrelated identities,which can be described as a set of expressions that are useful when one medium is conducting (or optically absorbing)

$$\hat{r}_{lm} = -\hat{r}_{ml}\hat{r}_{lm}^2 + \hat{t}_{lm}\hat{t}_{ml} = 1 \qquad (10)$$

The general expressions that describethe obliquely-incident Fresnel's reflection and transmission coefficients or the corresponding normal-incidence counterparts, which are cited in Equations (8) and (9), and the general identities given in Equation (10) form the basis of any general analysis to derive the formulations for the optical response of ideal multi-layered structures. This mathematical treatment can easily take into account possible multiple internal reflections and/or optical absorption that may occurinside one or more of the structure's layers of finite thicknesses and having smooth, homogeneous and plane parallel surfaces. However, we do not measure directly the Fresnel's reflection and transmission coefficients of a plane electromagnetic wave at an interface of two dissimilar adjacent media but we do measure the reflected and transmitted light intensity (or the time-averaged Poynting vector component normal to a detector surface) relative to its incidentintensity.By making use of Equation (7) and **n** = **u**, the expressions that describe the intensity reflection coefficient R_{lm} and transmission coefficient $T_{lm}$for a plane lightwavetravelling in medium l and hitting*normally* the interfaceto themedium m can readily be shown to have the following forms[9-17, 20-31,74-76]

$$R_{lm} \equiv \frac{\langle \hat{\mathbf{S}}_{\mathrm{r}}(\mathbf{r},t)\rangle_t . \mathbf{n}}{\langle \hat{\mathbf{S}}_{\mathrm{i}}(\mathbf{r},t)\rangle_t . \mathbf{n}} = \left|\frac{\hat{\mathbf{E}}_{\mathrm{r}}(\mathbf{r},t)}{\hat{\mathbf{E}}_{\mathrm{i}}(\mathbf{r},t)}\right|^2 = \hat{r}_{lm}\hat{r}_{lm}^* \qquad (11)$$

$$T_{lm} \equiv \frac{\langle \hat{\mathbf{S}}_{\mathrm{t}}(\mathbf{r},t)\rangle_t . \mathbf{n}}{\langle \hat{\mathbf{S}}_{\mathrm{i}}(\mathbf{r},t)\rangle_t . \mathbf{n}} = \mathrm{Re}\,(\hat{n}_m^*/\hat{n}_l^*)\left|\frac{\hat{\mathbf{E}}_{\mathrm{t}}(\mathbf{r},t)}{\hat{\mathbf{E}}_{\mathrm{i}}(\mathbf{r},t)}\right|^2 = [\mathrm{Re}\,(\hat{n}_m^*/\hat{n}_l^*)]\hat{t}_{lm}\hat{t}_{lm}^* \qquad (12)$$

The coefficients$\hat{r}_{lm}^*$ and $\hat{t}_{lm}^*$appearing in Equations (11) and (12) are, respectively, the complex conjugates of the complex Fresnel's reflection and transmission coefficients$\hat{r}_{lm}$ and $\hat{t}_{lm}$at such specified$l$-m interface which are already given in Equations (8) and (9).

However, the final formulae ofR_{lm} and T_{lm}at the interface between the l- and m- media will be tangled if are expressed in terms of the real and imaginary parts of $\hat{n}_l$ and $\hat{n}_m$upon explicit substitution of these complex Fresnel's coefficients$\hat{r}_{lm}$ and $\hat{t}_{lm}$ and their complex conjugates appear in Equations (11) and (12). For an obliquely-incident light plane wave, it can be shown that the T_{lm}-expression of Equation (12) should also includethe proper real ratio of the complex angles of refraction and incidence at the interfacewhich are expressed, via the modified Snell's law, in terms of $\hat{n}_l$ and $\hat{n}_m$[9-17, 20-31,74-76]. But, it ismathematically more convenient and handy to writeR_{lm} and T_{lm} at the interface ofthe l- and m- mediain much neater forms. This isattainedby re-writing Equations (8) and (9) in a way that $\hat{r}_{lm}$and $\hat{t}_{lm}$will beexpressedin terms of *realscalar* reflection and transmission coefficientsρ_{lm} and τ_{lm}defined as$\hat{r}_{lm} \equiv \rho_{lm}\exp(j\phi_{lm})$and $\hat{t}_{lm} \equiv \tau_{lm}\exp(j\chi_{lm})$, where$\phi_{lm}$ and χ_{lm} are the associated *real* phase changes on wave reflection and transmission atthat interface [9]. Simple mathematical handling of Equations (8) and (9) enables one to express ρ_{lm}, τ_{lm}, ϕ_{lm} and$\chi_{lm}$for the normal-incidence case as well as ρ_{ml}, τ_{ml}, $\phi_{ml}(= \phi_{lm})$ and χ_{ml}foropposite direction of light propagation,in terms ofthe refractive indices and extinction coefficients of the l- and m-mediaas

$$\rho_{lm}^2 \equiv \rho_{ml}^2 = \frac{(n_l - n_m)^2 + (\kappa_l - \kappa_m)^2}{(n_l + n_m)^2 + (\kappa_l + \kappa_m)^2} \tan\phi_{lm} = \frac{2\,(n_l\kappa_m - n_m\kappa_l)}{(n_l^2 + \kappa_l^2) - (n_m^2 + \kappa_m^2)} \qquad (13)$$

$$\tau_{lm}^2 = \frac{4\,(n_l^2+\kappa_l^2)}{(n_l+n_m)^2+(\kappa_l+\kappa_m)^2} \tan\chi_{lm} = \frac{(n_l\kappa_m - n_m\kappa_l)}{(n_l^2+\kappa_l^2)+n_l n_m+\kappa_l\kappa_m} \qquad (14)$$

$$\tau_{ml}^2 = \frac{4\,(n_m^2+\kappa_m^2)}{(n_l+n_m)^2+(\kappa_l+\kappa_m)^2} \tan\chi_{ml} = \frac{-(n_l\kappa_m - n_m\kappa_l)}{(n_m^2+\kappa_m^2)+n_l n_m+\kappa_l\kappa_m} \qquad (15)$$

As will be seen later, Equations (13) – (15) will also be valuablein the formulations obtained from the coherent superposition of complex electric-field vector amplitudes of multiple internal reflections taking place within a thin layer of a multi-layered structure. Using the above-stated formulaeofρ_{lm}^2, τ_{lm}^2, and τ_{ml}^2, I can now write the expressions that describe the normal-incidence intensity reflection coefficient$R_{lm}(= R_{ml})$ and transmission coefficients$T_{lm}$and T_{ml}at the interface of the two contiguous$l$- and m- media as

$$R_{lm} = R_{ml} \equiv \rho_{lm}^2 = \frac{(n_l-n_m)^2+(\kappa_l-\kappa_m)^2}{(n_l+n_m)^2+(\kappa_l+\kappa_m)^2} \qquad (16)$$

$$T_{lm} \equiv \left(\frac{n_m}{n_l}\right)\tau_{lm}^2 = \left(\frac{n_m}{n_l}\right)\frac{4\,(n_l^2+\kappa_l^2)}{(n_l+n_m)^2+(\kappa_l+\kappa_m)^2} \qquad (17)$$

$$T_{ml} \equiv \left(\frac{n_l}{n_m}\right)\tau_{ml}^2 = \left(\frac{n_l}{n_m}\right)\frac{4\,(n_m^2+\kappa_m^2)}{(n_l+n_m)^2+(\kappa_l+\kappa_m)^2} \qquad (18)$$

3. Optical response of multi-layered structures

Normally, multi-layered optical systems consist of various dissimilar solid layers; with each layer has two interfacestoits adjacent layers. Theoretical expressions describing theoverall oblique-incidence reflection and transmission of a multi-layered structure arein general intricate,particularly when the structure hasstrongly absorbing layers that may also suffer from non-ideal features such as anisotropy, non-uniformity (in thickness and composition), andsurface roughness.The problem is less complicated when one deals withsimple optical systems such asthree- and four- layered structures,whereinthe layer from which light hits the structure isusually asemi-infinite transparent (dielectric) mediumand other layers may be madefrom optically absorbing and/or dielectric materials [9-17, 74-76]. Also, the problem becomesmuch lesstangled if a monochromatic light plane wave is incident *normally* on such layered structures and if the materials of their adjoining layers are linear, isotropic, homogeneous, and non-magnetic. Such a multi-layered structure isoften regarded to be an “ideal” optical system, the subject of our concern here.

When the layers composing the structure are made from not strongly absorbing materials and surfaces are smoothand planeparallel, the light waves propagating insideeach layer willexecute multiple back and forth reflections from itsinternal opposite surfaces before exiting to nearby layers. Thus, to arrive atthe proper formulas that describe the total transmittance and specular reflectance of such a multi-layered optical system, oneought to take into account the contribution of the multiple internal reflections taking place in one or more of its layers. That is, one should sum over all the electric-field vectors and/or intensities of the individual reflected and transmitted light plane waves crossing the interfaces between the structure’s layers, in addition to taking into account the effect of possible light absorption inside individual layers.

To achieve the proper summation electric-field vectors or intensities of the multiple internally reflected light wave-fronts within a layer (of a complex index of refraction $\hat{n} \equiv n - j\,\kappa$) in a multi-layered structure, the *coherencelength*$l_c$of the incident beam of light having the wavelength λhas to be, however,considered.

The light coherence length is determined by the light source and the finite spectral bandwidth (SBW)$\Delta\lambda$ of the monochromator used in the spectrophotometer giving the light beamstriking the structure. The coherence length of the light of the monochromator can be given by the relation: $l_c = \lambda^2/2\pi\Delta\lambda$ [70, 71].

If the optical path length nd through a layer of ageometricalthickness $d$and an index of refraction $n$is much less than $l_c$or $\Delta\lambda \ll \lambda^2/2\pi nd$,the incident light beam issaid to be monochromatic with respect to the thin layer. Thus, interference between the light plane waves of internal multiple reflections taking place inside the layer cannot be ignored and their electric-field vectors shouldthen be summed to give the *net* reflection and transmission to its adjacent layers. This procedure is referred to as the *coherent* description of optical response of a multi-layered structure having a “coherent” thin layer.

On the other hand, when the bandwidth $\Delta\lambda$is finiteand the optical thickness $nd$of a layer is greater than the coherence length of the output light beamsuch that $\Delta\lambda \gg \lambda^2/2\pi nd$, the phases of the multiple internal reflected waves are randomized and cannot interfere with each other. Hence, interference effects in this case can be completely tolerated, so light intensities and not electric field amplitudes of the multiple internally reflected/exiting light plane waves inside/outside this thick layer have to be summed to obtain its net reflection and transmission. This procedure is often known as the *incoherent* treatment of the optical response of thick layers. When a plane-parallel optically thin/thick layer is made from a conductive or optical absorbing material, the attenuation of each internally reflected light wave insidethe layer is significant and the contribution of light absorption to the overall optical response of the structure should thus be taken into consideration.

To arrive at the fullformulaefor the total normal-incidence optical transmittance and specular reflectance of a four-layered structure, I should discussfirst the respectiveoptical reflection and transmission ofthree-layered structures. Depending on the optical thickness of its middle layer relative to the coherence length of incident light, the optical response of a three-layered structurehas to be treatedeither incoherentlyor coherently[4-17,20-31,73-76]. Moreover, if multiple reflections inside the middle layer of an “ideal” three-layered structureoccur, one should get the expressions that describeitstotal transmittanceand specular reflectance when a monochromatic lightbeam entering its mid layeralong the opposite directions of light propagation. This will be useful in the treatment of optical response of structures comprising of four or more stacked layers.

3.1 Optical transmittance and specular reflectance of three-layered structures

In practice, three-layered structures areeither composed of aslabthat is fully embedded in air(freely air-supported slab) or of a film laid onto a too thick absorbing substrate so thatno light will be transmitted out of the other substrate’sside or is reflected back towardits interface with the film. In the latter case, the front surface of the film is usually in contact with a semi-infinite layer of air. Such kind of three-layered structures can be imitatedas {layer 1/layer 2/ layer 3}-configuration, in which layer 1 is semi-infinite with a real index of refraction$n_1$and a vanishing extinction coefficientκ_1; that is- $\hat{n}_1 = n_1 - j0$. Layer 2 is aslab (or film)having a finite geometrical thickness d_2 and a complex index of refraction $\hat{n}_2 = n_2 - j\kappa_2$, whereas layer 3 is too thick or semi-infinite with a complex index of refraction $\hat{n}_3 = n_3 - j\kappa_3$. This three-layered structure is schematically illustrated in Figure 1.

Note that, in case of the dielectric-film-substrate optical unit, if layer 3(substrate) is plane-parallel transparent ($\kappa_3 = 0$)or is made of very weakly-absorbing material($\kappa_3 \sim 0$) with a finite thickness, part of the light beam may be specularly reflected from its other surface backwardto layer 2 (film) and light absorption may also take place repeatedly insideit before the remaining part of the light beam is transmitted through its back surface to itsneighboring medium (assumed to be semi-infinite and thus no back reflections from the far side of this last medium will take place); In view of the terminology of multi-layered structures in the present article,this multi-layered structureis anoptical four-layered structure and not a three-layered one.

In the nextsub-section, I shall treat the normal-incidence optical response of an "ideal" three-layered structure (Figure 1), wherein layer 1 is semi-infinite and transparent and layer 3 is optically absorbing and thick enough,while itslayer 2 (slab or film) is optically thick (i.e., *incoherent layer*) and whose surfaces are smooth, homogeneous and plane parallel. In this case, the effect of interference between the back and forth internal specular reflections of the monochromatic light plane waves propagating through this layer will be neglected;hence, one canapply the *incoherent*optical treatmentto obtain the *net*spectral transmittance and specular reflectance of this three-layered structure by adding the appropriate intensity reflection and transmission coefficients associated with all these multiple internally reflected light plane waves.

When layer 2 of the above-specified "ideal" three-layered structure is instead optically thin (i.e., coherent film), the interference between the multiply reflected light waves occurring inside it ought to be taken into account and the problem should then be analyzed *coherently*, as will be discussed in later sections. Optical absorption taking place within layer 2 of the above-specified three-layered structure shouldbe accounted for in both of these incoherent and coherent treatments, in which the film or slab will be presumed linear, stationary, isotropic, inhomogeneity-free (both in composition and thickness) and non-magnetic.

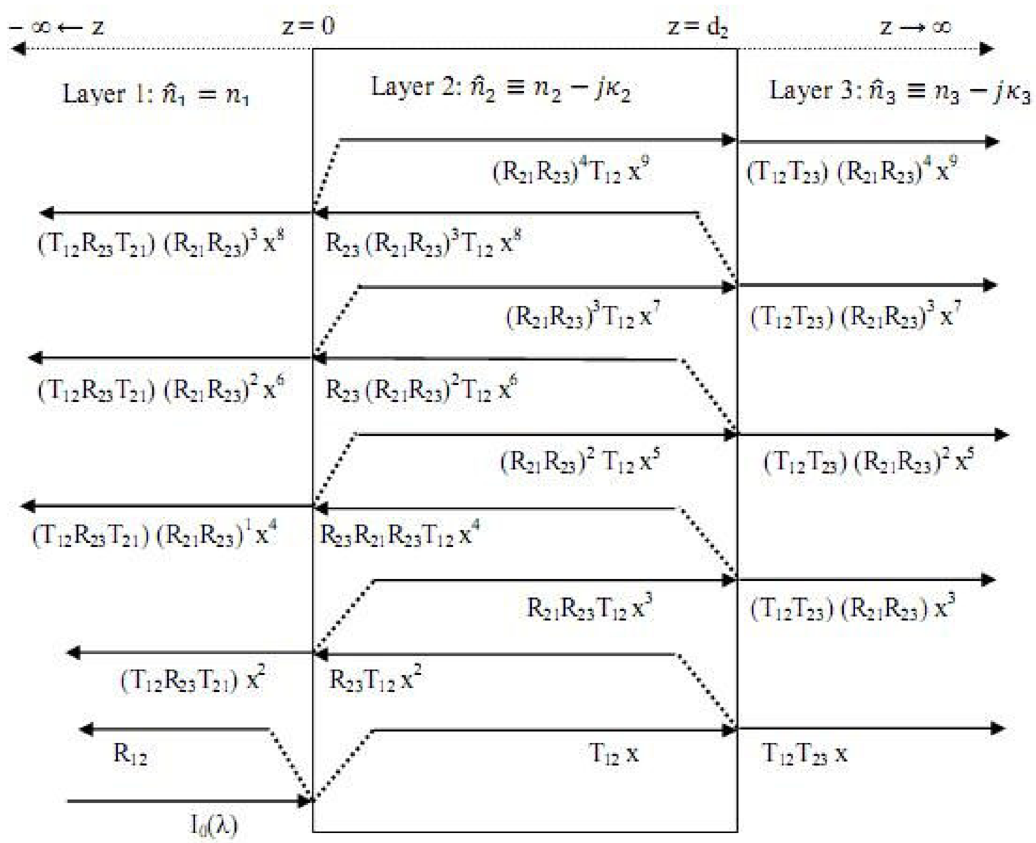

Figure 1: Incoherent superposition of intensity reflection and transmission coefficients at the interfaces of an "ideal" three-layered structure. A collimated beam of light plane wavestravelling (from left) insidethe semi-infinite dielectric (transparent) layer 1 and having a single spectral wavelength λ and an initial intensity $I_0(\lambda)$is incident normally onto the front surface of layer 2, with $x \equiv \exp(-\,\alpha_2 d_2)$, where α_2 and $d_2$are the optical absorption coefficient and the geometrical thickness of layer 2, respectively.

3.1.1 Incoherent normal-incidencereflection and transmission of an optically absorbing thick layersandwiched between semi-infinite transparentand absorbing media

A materialmedium whose optical, electrical,dielectric and magnetic properties are constant (homogeneous) throughout each plane perpendicular to a fixed direction (say the *z*-axis of a Cartesian coordinate system) is called a *stratified* medium [9]. Now, considerthree dissimilar stratified media (layers) that are piled in close contact with each other to form the"ideal" three-layered structure illustrated in Figure 1. Let layer 2 to be optically thick with a geometrical thickness d_2 and to have smooth,homogeneous, and plane-parallel surfaces. Assume a beam of monochromatic plane light wave of a spectral wavelength λ and initial intensity $I_0(\lambda)$travellingfrom left insidethe semi-infinite transparent layer 1 to be incident normally onto the *front* surface of layer 2. Neglectthe interference between the plane light waves of the back and forth reflections taking place at the internal surfaces of layer 2 and employ the incoherent analysis to treat itsoptical reflection and transmissionto its neighboring media (layers 1 and 3). It will be assumed that the plane light waves entering the semi-infinite layers 1 and 3 do not reflect back from their far sides.

Taking into account optical absorption within layer 2, characterized by the absorption parameter $x \equiv \exp(-\alpha_2 d_2)$, the algebraic sum of light intensities of all multiply reflected and transmitted waves from the "ideal" three-layered (123-) structure of Figure 1 will yieldtwo compact formulae that describe fully its total normal-incidence transmittance $T_{123}(\lambda)$and specular reflectance $R_{123}(\lambda)$given by [13, 21, 24, 26, 76]

$$\begin{aligned} R_{123}(\lambda) &= R_{12} + (T_{12}x)(R_{23}x)(T_{21}) + (T_{12}x)(R_{23}x)(R_{21}x)(R_{23}x)(T_{21}) \\ &\quad + (T_{12}x)(R_{23}x)(R_{21}R_{23}x^2)^2(T_{21}) + \cdots \\ &= R_{12} + (T_{12}R_{23}T_{21})\sum_{p=1}^{p=\infty} x^{2p}(R_{21}R_{23})^{p-1} = R_{12} + \frac{(T_{12}R_{23}T_{21})x^2}{1-R_{21}R_{23}x^2} \end{aligned} \tag{19}$$

$$\begin{aligned} T_{123}(\lambda) &= (T_{12}x)(T_{23}) + (T_{12}x)(R_{23}x)(R_{21}x)(T_{23}) + (T_{12}x)(R_{21}R_{23}x^2)^2(T_{23}) + \ldots \\ &= (T_{12}T_{23})\sum_{p=1}^{p=\infty} x^{2p-1}(R_{21}R_{23})^{m-1} = \frac{(T_{12}T_{23})\,x}{1-R_{21}R_{23}x^2} \end{aligned} \tag{20}$$

For light propagating in the medium *l* (*m*) and normally incident at the interface to the medium *m* (*l*), the intensity reflection coefficients$R_{lm}$and R_{ml} and the intensity transmission coefficients $T_{lm}$and T_{ml}can be evaluated from Equations (16) – (18).

For the three-layered structure specified in Figure 1, Equations (13) – (15) can be used to obtain the real scalar reflection and transmission coefficients ρ_{lm}, ρ_{ml}, τ_{lm}, τ_{ml} and associated phase changes on reflection and transmission $\phi_{lm}(=\phi_{ml})$, χ_{lm}, and χ_{ml} along opposite directions of light propagation through the interfacesof each pair of its adjacent layers. The required formulasthat describe these parameters are given below

$$\rho_{12}^2 = \rho_{21}^2 = \frac{(n_1-n_2)^2+\kappa_2^2}{(n_1+n_2)^2+\kappa_2^2} \tan\phi_{12} = -\frac{2\,n_1\kappa_2}{(n_2^2+\kappa_2^2)-n_1^2} \tag{21}$$

$$\tau_{12}^2 = \frac{4\,n_1^2}{(n_1+n_2)^2+\kappa_2^2} \tan\chi_{12} = \frac{\kappa_2}{n_1+n_2} \tag{22}$$

$$\rho_{23}^2 = \rho_{32}^2 = \frac{(n_2 - n_3)^2 + (\kappa_2 - \kappa_3)^2}{(n_2 + n_3)^2 + (\kappa_2 + \kappa_3)^2} \tan\phi_{23} = \frac{2\,(n_2\kappa_3 - n_3\kappa_2)}{(n_2^2 + \kappa_2^2) - (n_3^2 + \kappa_3^2)} \qquad (23)$$

$$\tau_{23}^2 = \frac{4\,(n_2^2 + \kappa_2^2)}{(n_2 + n_3)^2 + (\kappa_2 + \kappa_3)^2} \tan\chi_{23} = \frac{n_2\kappa_3 - n_3\kappa_2}{(n_2^2 + \kappa_2^2) + n_2 n_3 + \kappa_2\kappa_3} \qquad (24)$$

$$\tau_{21}^2 = \frac{4\,(n_2^2 + \kappa_2^2)}{(n_1 + n_2)^2 + \kappa_2^2} \tan\chi_{21} = -\frac{n_1\kappa_2}{(n_2^2 + \kappa_2^2) + n_1 n_2} \qquad (25)$$

$$\tau_{32}^2 = \frac{4\,(n_3^2 + \kappa_3^2)}{(n_2 + n_3)^2 + (\kappa_2 + \kappa_3)^2} \tan\chi_{32} = -\frac{n_2\kappa_3 - n_3\kappa_2}{(n_3^2 + \kappa_3^2) + n_2 n_3 + \kappa_2\kappa_3} \qquad (26)$$

It will be seen later that the formulas expressed in Equations (21) – (26) areof great value in the discussion of the optical response of "ideal"*four-layered structures* incorporating an optically transparent or absorbing thin "coherent" film laid onto atransparent or partially-absorbing "incoherent" substrate of finite thickness.

In practice, it is common to express the general Equations (19) and (20) that describe the *netincoherent*normal-incidence spectral transmittance and specular reflectance of the "ideal" three-layered structure illustrated in Figure 1 in terms of the optical constants of its three layers, viz., n_1, n_2, n_3, κ_2, α_2 and κ_3. This can be accomplished by re-writing Equations (16) – (18)explicitly in terms of these optical constants to obtain the respective expressions thatdescribethe *incoherent*normal-incidence intensity reflection and transmission coefficients at each interface of its layers for both directions of light propagation through each interface as given below.

$$R_{12} = R_{21} \equiv \rho_{12}^2 = \frac{(n_1 - n_2)^2 + \kappa_2^2}{(n_1 + n_2)^2 + \kappa_2^2} \qquad (27)$$

$$R_{23} = R_{32} \equiv \rho_{23}^2 = \frac{(n_2 - n_3)^2 + (\kappa_2 - \kappa_3)^2}{(n_2 + n_3)^2 + (\kappa_2 + \kappa_3)^2} \qquad (28)$$

$$T_{12} \equiv \left(\frac{n_2}{n_1}\right) \tau_{12}^2 = 1 - R_{12} = \frac{4\,n_1 n_2}{(n_1 + n_2)^2 + \kappa_2^2} \qquad (29)$$

$$T_{21} \equiv \left(\frac{n_1}{n_2}\right) \tau_{21}^2 = \left(\frac{n_1}{n_2}\right) \frac{4\,(n_2^2 + \kappa_2^2)}{(n_1 + n_2)^2 + \kappa_2^2} \qquad (30)$$

$$T_{23} \equiv \left(\frac{n_3}{n_2}\right) \tau_{23}^2 = \left(\frac{n_3}{n_2}\right) \frac{4\,(n_2^2 + \kappa_2^2)}{(n_2 + n_3)^2 + (\kappa_2 + \kappa_3)^2} \qquad (31)$$

$$T_{32} \equiv \left(\frac{n_2}{n_3}\right) \tau_{32}^2 = \left(\frac{n_2}{n_3}\right) \frac{4\,(n_3^2 + \kappa_3^2)}{(n_2 + n_3)^2 + (\kappa_2 + \kappa_3)^2} \qquad (32)$$

Now, let us utilize Equations (19) and (20) to express the total*incoherentnormal-incidence*spectral transmittance and specular reflectance of some meek three-layered structures of practical importance in the field of optics. This is discussed in the following sub-sections.

3.1.1.1Optically thick dielectricslab bounded by twosemi-infinite transparent media

A three-layered structure of practical interestis whoselayers 1 and 3 of Figure 1 are semi-infinite, transparent ($\kappa_1 = \kappa_3 = 0$)and haverefractive indices$n_1$and n_3, and whose layer 2is a dielectric slab ($\hat{n}_2 = n_2$, $\kappa_2 = 0$, $\alpha_2 = 0$and $x = 1$), which is thick enough for the multiple reflected plane light waves within it to interfere coherently. Then $T_{12} = T_{21} = 1 - R_{12}$, $R_{21} = R_{12}$and Equations (19) and (20) reducetothe much simpler forms

$$R_{123}(\lambda) = \frac{R_{12} + R_{23} - 2\,R_{21}R_{23}}{1 - R_{21}R_{23}} \quad (33a)$$

$$T_{123}(\lambda) = \frac{T_{23}(1 - R_{12})}{1 - R_{21}R_{23}} \quad (33b)$$

An example of this simple three-layered structureis an opticallythick dielectric slab sandwiched between two *identical*transparent media,like, for instant, a thick unsupported (free-standing) glass slide placed in air ($n_1 = n_3 = 1$). Then, in thefree-absorption ($\kappa_2 = 0$) spectral region of such an air-supported glass slide, Equations (27) – (32) yield$R_{21} = R_{23} = R_{32} = R_{12}$and $T_{21} = T_{23} = T_{32} = T_{12} = 1 - R_{12}$, with R_{12} and T_{12}given by

$$R_{12} = \frac{(1 - n_2)^2}{(1 + n_2)^2} T_{12} = \frac{4n_2}{(1 + n_2)^2} \quad (34)$$

Consider an optically thick transparent, homogeneousglass slab of aconstant real index of refraction n_2 = 1.5and vanishing extinction coefficient ($\kappa_2 = 0$),with smooth (polished) andplane-parallel surfaces. Equation (34) tells us that the intensity reflection and transmissioncoefficients$R_{12}$and T_{12}at any of its interfaces to the neighboring semi-infinite air layers are equal to 0.04 and 0.96, respectively, which sum to unityprovided that scatter losses are being ignored.

Inserting the intensity reflection and transmission coefficients given in Equation (34) into Equations (33a) and (33b), the total normal-incidence *incoherent*spectral transmittance $T_{123}(\lambda) = T_{\mathrm{vgv}}(\lambda)$and specular reflectance $R_{123}(\lambda) = R_{\mathrm{vgv}}(\lambda)$of a transparent glass (g)slide standing freely in vacuum (v)and having an index of refraction $n_{\mathrm{g}}(\lambda)$ can now be givenby the simple formulae

$$R_{\mathrm{vgv}}(\lambda) = \frac{2R_{12}}{1 + R_{12}} = \frac{\left(n_{\mathrm{g}} - 1\right)^2}{n_{\mathrm{g}}^2 + 1} \quad (35a)$$

$$T_{\mathrm{vgv}}(\lambda) = \frac{T_{12}^2}{1 - R_{12}^2} = \frac{1 - R_{12}}{1 + R_{12}} = \frac{2n_{\mathrm{g}}}{n_{\mathrm{g}}^2 + 1} \quad (35b)$$

Equations (35a) and (35b) yieldthe numerical values 0.07692 and 0.92308for the $T_{\mathrm{vgv}}(\lambda)$and $R_{\mathrm{vgv}}(\lambda)$for an *air-supported transparent glass slide*with a*constant*index of refraction $n_{\mathrm{g}}(\lambda) = 1.5$, implying that $R_{\mathrm{vgv}}(\lambda) + T_{\mathrm{vgv}}(\lambda) = 1$ as conservation of total energy tells us, provided that losses due to scatter at its surfaces and optical absorption throughout its bulk are insignificant.In the spectral wavelength range where they are feasible,Equations (35a) and (35b) are usually employedto determine the values of $n_{\mathrm{g}}(\lambda)$ of a free-standing non-absorbing optically thick glass slide from the measured values of its total normal-incidence transmittance $T(\lambda)$or specular reflectance $R(\lambda)$at the wavelength $\lambda$by making use the following simple expression

$$n_{\mathrm{g}}(\lambda) = \frac{1}{T(\lambda)} + \sqrt{\left[\frac{1}{T(\lambda)}\right]^2 - 1} = \frac{1 + \sqrt{R(\lambda)[2 - R(\lambda)]}}{1 - R(\lambda)} \quad (36)$$

Equations (35a), (35b)and(36) can also be applied to many "ideal" three-layered structures having the {air/thick dielectricslab/air}-pilingat each spectral wavelength λin wavelength range of the electromagnetic spectrum over which the slab's material is highly optically transparent. In the case of a thick dielectric slab made from weakly dispersive material, the measured values of its$T_{\mathrm{vgv}}(\lambda)$and $R_{\mathrm{vgv}}(\lambda)$ would be almost constant at all spectral wavelengths in the transparent region and itsreal index of refraction is thustaken to be wavelength-independent; that is, $n_2(\lambda)$ =constant. This might be workablefforordinary thick glass slides and quartz wafers over the upper part of the ultraviolet (UV) radiation, in addition to the visible (VIS) and near infrared (NIR) spectral regions. Theseexpressions cannot, however, be used for describing the optical response of a three-layered structure in the spectral region where the optical absorption and dispersion of the slab's material aresignificant. In such cases, one must thereforerevert to the full formulaecontaining all material's optical parameters underlined in Equations (19) and (20), in conjunction with Equations (27) – (32), to attain a complete description of the problem, as is detailed below.

3.1.1.2Air-supported optically thick absorbing layers

Another practical three-layered structure is a thick layer made from a linear, isotropic, homogeneous, nonmagnetic and optically absorbing material, with the layer being standing freely in air; that is- an air-supported slab of*complex*refractive index $\hat{n}_2 = n_2 - j\kappa_2$,an absorption coefficient $\alpha_2 \equiv 4\pi\kappa_2/\lambda$and a geometrical thickness d_2 larger than the coherence length of the monochromatic light beam striking it.Assuming that the surfaces of this free-standing absorbing layerare smooth and plane parallel, the completetheoretical expressions that describe itstotal *incoherent*normal-incidence spectral transmittance$T_{123}(\lambda)$andspecular reflectance $R_{123}(\lambda)$caneasily be shown, by making use of the general Equations (19) and (20), to have the following forms

$$R_{123}(\lambda) = R_{12} + \frac{T_{12}R_{23}T_{21}e^{-2\alpha_2 d_2}}{1 - R_{21}R_{23}e^{-2\alpha_2 d_2}} \quad (37a)$$

$$T_{123}(\lambda) = \frac{T_{12}T_{23}e^{-\alpha_2 d_2}}{1 - R_{21}R_{23}e^{-2\alpha_2 d_2}} \quad (37b)$$

Since, for the above-described {*air*/*thick absorbing slab*/*air*}-structure $n_1 = n_3 = 1$, $\kappa_1 = \kappa_3 = 0$, $n_2 > 1$, and $\kappa_2 \neq 0$, the intensity reflection and transmission coefficients at the two plane-parallel slab's interfaces to air, for both directions of normal-incidence light propagation, will now have, by making use of Equations (27) – (32), the following forms

$$R_{12} = R_{21} = R_{23} = R_{32} = \frac{(n_2 - 1)^2 + \kappa_2^2}{(n_2 + 1)^2 + \kappa_2^2} \quad (38a)$$

$$T_{12} = T_{32} = 1 - R_{12} = \frac{4\, n_2}{(n_2 + 1)^2 + \kappa_2^2} \tag{38b}$$

$$T_{21} = T_{23} = \left(\frac{1}{n_2}\right) \frac{4\,(n_2^2 + \kappa_2^2)}{(n_2 + 1)^2 + \kappa_2^2} = T_{12}\left(1 + \frac{\kappa_2^2}{n_2^2}\right) \neq T_{12} \tag{38c}$$

For the sake of convenient later discussion, let us designate the slab (film) by the symbol (f) and air (vacuum) by the symbol (v). Further, renaming the reflection and transmission coefficients R_{12} and T_{12} at the air-film (vf-) interface by R_{vf} and T_{vf} $(= 1 - R_{\mathrm{vf}})$, respectively, and the slab's total spectral normal-incidence transmittance $T_{123}(\lambda)$ and specular reflectance $R_{123}(\lambda)$ by $T_{\mathrm{vfv}}(\lambda)$and $R_{\mathrm{vfv}}(\lambda)$, respectively. Insert Equations (38a) – (38c) into Equations (37a) and (37b) to get more compact and informative forms as follows

$$R_{\mathrm{vfv}}(\lambda) = R_{\mathrm{vf}} + \frac{R_{\mathrm{vf}}\,(1 - R_{\mathrm{vf}})^2 \left(1 + \frac{\kappa_2^2}{n_2^2}\right) e^{-2\alpha_2 d_2}}{1 - R_{\mathrm{vf}}^2 e^{-2\alpha_2 d_2}} \tag{39}$$

$$T_{\mathrm{vfv}}(\lambda) = \frac{(1 - R_{\mathrm{vf}})^2 \left(1 + \frac{\kappa_2^2}{n_2^2}\right) e^{-\alpha_2 d_2}}{1 - R_{\mathrm{vf}}^2 e^{-2\alpha_2 d_2}} \tag{40}$$

The intensity reflection coefficient R_{vf} at the air-film interface upon which light is normally incident from the *left*of the air-supported film (hereafter is called the *forward* direction of light propagation) is now given, in terms of the film's optical constants $n_2$and κ_2, by the expression

$$R_{\mathrm{vf}} = \frac{(n_2 - 1)^2 + \kappa_2^2}{(n_2 + 1)^2 + \kappa_2^2} \tag{41}$$

It should be emphasized here that Equations (39) and (40) were rarely reported [11, 78] and are often cited in the literature without the term $(1 + \kappa_2^2/n_2^2)$ [24, 26, 79, 80], implicitly assumed that the thick slab is weakly absorbing such that $\kappa_2^2/n_2^2 \ll 1$in certain range of spectral wavelengths.

As a consequence of the multiple internal reflections within an air-supported thick plane-parallel slab made from optically-absorbing material, the slab's transmittance given by Equation (40) is *not*in direct proportion to exp $(-\alpha_2 d_2)$as implied bythe simple Lambert-Beer law,which describes the attenuation of light travelling *once* throughan isotropic,homogenous sample. This is because part of the incident light wave traverses the slab back and forth several times corresponding to an increase in the effective optical path length inside it, and hence gives rise to more optical absorption within its bulk.Lambert-Beer behavior of transmittance is only expected for a free-standing absorbing slab if the intensity reflection coefficients at its interfaces to air (hereR_{vf}) are low.One should retain here, that the Lambert-Beer law applies well to bulky liquid samples, but is generally notvalid forabsorbing films of smooth, homogenous and plane-parallelsurfaces; it has, however, to be stated that Lambert-Beer law is oftenusedin the analysis of most literature work on filmswith fairly largeintensity reflection coefficients at itsinterfaces.

An issue that deserves to mention at this stage is the procedure of analyzing the measured data of the total spectral transmittance $T(\lambda)$and specular reflectance $R(\lambda)$of three-layered and other multi-layered structures to determine their optical parameters, namely their $n(\lambda)$ and $\kappa(\lambda)$ or $\alpha(\lambda)$ $(\equiv 4\pi\kappa(\lambda)/\lambda)$. In general, $n(\lambda)$ and $\kappa(\lambda)$ of a semiconducting or dielectric film (slab) are varying functions of wavelengthλ;

that is- the material of a slab or film possessing this feature is regarded to be dispersive, and the dependence of its optical parameters on the wavelength of the electromagnetic radiation incident onto it must be taken into account in a complete optical analysis.

In the simplest case of a thick air-supported dielectric slab ($\kappa_2(\lambda) = 0$), Equation (36) in the spectral range of its validity can directly be applied to calculate$n_2(\lambda)$ from the respective values of its $T(\lambda)$or$R(\lambda)$. In contrast, however, for a thick air-supported optically-absorbing slab, analytical solution of the normal-incidence theoretical $T(\lambda)$- and $R(\lambda)$- expressions given in Equations (39) and (40) to calculate directly its optical constants $n_2(\lambda)$ and $\kappa_2(\lambda)$ at each individual wavelength λis not possible. Only in partially-absorbing slabs standing freely in air where the approximation $\{\kappa_2(\lambda)/n_2(\lambda) < 1\}$ is valid, one can be able to solve analytically these formulas to express $n_2(\lambda)$ and $\kappa_2(\lambda)$ in terms of $T(\lambda)$ and $R(\lambda)$ [80]. Otherwise, one has to revert to conventional numerical and curve-fitting procedures to determine the dispersion of optical constants of a thick air-supported optically-absorbing slab. To implement such kind of numerical analysis,the proper expressions which represent thewavelength dependence of its optical parameters $n_2(\lambda)$ and $\kappa_2(\lambda)$[1-25, 74-77] over the spectral range studied should be inserted into Equations (39)- (41).

However, conventional curve-fitting procedures often result in multiple solutions with differentandsometimes illusiveor unrealistic values for the fitting parameters, unless a global solution is being accomplished; thus, complementary results (from different measurements) are needed for comparison purposes. Nevertheless, one normallyemploys some justified approximations tofacilitate analysis and numerical computation of the measured optical data without losing the physical meaning and accuracy of the output results. Even so, accurate analysis of experimental optical data of multi-layered structures will be much intricate such that reliable and sophisticated computational and curve-fitting procedures should thus to be employed.

3.1.2 Coherent transmissivity and specular reflectivityof multi-layered structures

The above-discussed optical approach based on the *incoherent superposition* of transmitted and specularly reflected light intensities of multiple back and forth internal reflections at the opposite interfaces of a homogeneous plane-parallel layer with its surrounding media (layers) is correct only if the optical thickness of the layer is *much greater than the coherence length of light*l_c or $\Delta\lambda \gg \lambda^2/2\pi nd$. However, the light interference between such internally reflected waves inside the layer cannot be tolerated whenthelayer is sufficiently thin such that itsoptical thickness is smaller than or comparable to the coherence length of incident light incident-that is, $\Delta\lambda \ll \lambda^2/2\pi nd$. In this case, one cannot simply add the scalar intensities of the individual light plane waves reflected back and forth from its opposite surfaces to get itstotal transmissivity (transmittance) and specular reflectivity (reflectance).

In principle, interference-fringe maxima and minima are supposed to be superimposed on the fringe-free transmittance and specular reflectance spectra for ideal optical structures composed of any number of stacked thin stratified layers. Only under some conditions, however, interference-fringe fingerprints (maxima and minima) may be seen in the measured overall transmitted and specularly reflectedlight signals of an optically thin film (slab) standing freely in air. Moreover, the above-cited interference features are frequentlyobserved in experimental transmission and specular reflection of ahomogeneous plane-parallel dielectric or semiconducting optically thin film which is deposited ontoa thick optically transparent or absorbing substrate of different physical properties.

In the following sub-sections, I shall therefore discuss in detail the *coherent treatment* of the optical response of sufficiently thin layers to enlighten most of the expected characteristics of their transmittance and reflectance spectra. Later, I will combine both of the incoherent and coherent analysis to obtain the full formulae that describe the spectral transmittance and specular reflectance of thin dielectric or optically absorbing films which is intimately laid onto much thicker substrates.

3.1.2.1Theoretical approaches for coherent description of transmittance and specular reflectance of optically thin layers

A variety of mathematical approaches were proposed to derive the general formulae forthe transmittance and specular reflectanceof multi-layered structureshaving a stack of thin dielectric or conducting layers onto which amonochromaticlinearly-polarized light plane wave is incident. One of these methods is entailed in the comprehensive optical theorythat is described by Born and Wolf [9] and by Potter [10]. Some features of this theory are relevant in the discussion given in the present article and hence will be briefly outlined here. Thisoptical theory is based on the reduction of the coupled Maxwell first-order partial differential equations in $\hat{\mathbf{E}}(\mathbf{r},t)$- and $\hat{\mathbf{B}}(\mathbf{r},t)$- fields of thelight wave propagating into a stratified mediumto a set of second-order partial differential equations, which can be solved, subjected to apt boundary conditions, to get a general*two-by-two* unimodular matrixcharacteristic of the medium.Its basic formulations are equally applicable to both transverse electric (TE) and transverse magnetic (TM) lightplane waves propagating into the medium along aroutemaking a non-zero angle to its direction of stratification.

It deserves noting here that the*s*- and *p*- polarized light plane waves whose $\hat{\mathbf{E}}(\mathbf{r},t)$-vectors be everywhere perpendicular and parallel to the plane of incidenceare referred to asTE and TM waves, but in the sense we still describe a *transverse*light wave travelling through *open* regions. Consequently, any arbitrarily polarized lightplane wave incident on a multi-layered structure can be resolved into two plane waves: one is TE and the other isTM. This classification of TE and TM light plane waves is *not*as that adoptedin *guided* regions of specific structures (wave guides), where these acronyms have totally different meanings [4-10, 20-24, 74-76].

Moreover, as boundary conditions at a discontinuity surface for the $\hat{\mathbf{E}}(\mathbf{r},t)$- and $\hat{\mathbf{B}}(\mathbf{r},t)$- componentsperpendicular and parallel to the plane of incidence are independent of each other; the TE and TM light plane waves will also be mutually independent. Further, for a linear and isotropic medium deficient of external electric charges and current sources, Maxwell's equations remain unchanged when one re-writes them with$\hat{\mathbf{E}}(\mathbf{r},t)=\hat{\mathbf{D}}(\mathbf{r},t)/\epsilon$ and$\hat{\mathbf{H}}(\mathbf{r},t)=\hat{\mathbf{B}}(\mathbf{r},t)/\mu$ and simultaneouslyϵ and$-\mu$are interchanged, where ϵ and $\mu$are, respectively, the permittivity and permeability of the medium.Therefore, any optical theorem for TM plane waves may be deduced from the results obtained for TE plane waves using this change, and soit willbe sufficient to discuss the case of TE lightplane waves only. Thegeneral matrix optical theory [9, 10]can be utilized to derivethe expressions that describe transmittance and specular reflectance of multi-layered structures of linear, thindissimilar stratified layers onto which amonochromatic TE- or TM-lightplane wave is impinging at an arbitrary angle of incidence.

A less involved but informative methodology,called the *coherent*amplitude superposition treatment of the oblique-/normal-incidence multiple internal reflections taking place inside a linear, isotropic, and homogeneous plane-parallel thin film, is also discussed in some articles and books of electrodynamics and optics [16, 17, 20-24, 26, 28,74-76]. In this optical approach, one can add the Fresnel'scomplex amplitude reflection and transmission coefficients of all individual reflected and transmitted light plane waves modulated by a phase change upon its each single traversal inside the film. The obtained*net* complex amplitude reflection and transmission coefficients canthen be used to find the formulaeofthe overallstructure's transmissivity and specular reflectivity.Though this *coherent description*works well for multi-layered structures incorporating a quite small number of piled thin layers, it is, however, cumbersome and inconvenient for analysis of the optical response of a stack of a large number of thin layers, and I will not here discuss this approach further.

Other less popular approaches like, for example, those based on the so-called single graph theory [51-53] and impedance-matching method [11] have been also recommended for treating the optical response of coherent layers. Also, these optical approaches will not be discussed here further and interested readers can consult the above-cited references for more details.

Anotherinstructiveapproach, called the method of resultant plane waves or the $E^{+}- E^{-}$transfermatrix method[20-23, 29-33, 76], can efficiently be employed to treat the optical response of multi-layered structures of any number of thin films tomonochromatic TE (or TM)light plane waves, whether the wave is obliquely- or normally- incident on the structure.Based on some constructivepoints relevant to this $E^{+}- E^{-}$transfer matrix method, which will be of prime concern of the present article, the general formulas that describe the optical response of multi-layered structures can be derived in a directand simple manner.

The essential features of the$E^{+}- E^{-}$transfer matrixmethod and the relatedgeneral formulations that lead to the derivation ofthe totalspectral transmittance $T(\lambda)$and specular reflectance $R(\lambda)$of a stack of severaldissimilar thin layers bounded by semi-infinite transparent mediaare detailed in *Appendix B*. In the next sub-sections, I shall discuss in some detail the basic formulations and physics behind this optical method that enable one to derive the normal-incidence$T(\lambda)$ and $R(\lambda)$expressions for describing the optical response of an "ideal" three-layered structure composing of athin dielectric (transparent) or optically-absorbing layer (film) sandwiched between two optically thick layers. The obtained formulations of $T(\lambda)$ and $R(\lambda)$ of the latter specified three-layered structure will be compared with their respective expressions that are reported in the literature or derived by others using different approaches.

3.1.2.2The general E^{+}-E^{-} transfer matrix formulae for the total transmission and specular reflectioncoefficientsof multi-layered structures

For a monochromatic *s*-polarized (TE) plane light wave,the $E^{+}- E^{-}$ transfer matrix method for a stack of j successive layers separated by $j-1$smoothand plane-parallel interfaces, with the layers index $j = 1, 2, 3, 4$ ……,can be described by asingle general matrix equation of the form

$$\begin{pmatrix} \hat{E}_{+}^{1} \\ \hat{E}_{-}^{1} \end{pmatrix} = \boldsymbol{M} \begin{pmatrix} \hat{E}_{+}^{f} \\ \hat{E}_{-}^{f} \end{pmatrix} \tag{42}$$

As discussed in Figure B1, the fields $\hat{E}_{+}^{1}$ and $\hat{E}_{-}^{1}$are, respectively, the electric field components of the TE plane light wave travelling through and reflected from the first (incident) layer. The fields $\hat{E}_{+}^{f}$ and $\hat{E}_{-}^{f}$are, in that order, the electric field components of light plane waves transmitted into and reflected from the final (last) layer of the j-layered stack.

The transfer matrix $\boldsymbol{M}$, which is referred to as the *characteristicmatrix* of the*wholej-layered structure*, for the$(j-1)$ interfaces separating thesuccessive j layers, including the *first* layer from which the light plane wave is being incident onto the front surface of the*second* layer ($j = 2$), can be evaluated from the general expression

$$\boldsymbol{M} \equiv \hat{\boldsymbol{I}}_{1} \circ \hat{\boldsymbol{T}}_{2} \circ \hat{\boldsymbol{I}}_{2} \circ \hat{\boldsymbol{T}}_{3} \circ \hat{\boldsymbol{I}}_{3} \circ \hat{\boldsymbol{T}}_{4} \circ \hat{\boldsymbol{I}}_{4} \circ \dots\dots\dots\dots\dots\dots \circ \hat{\boldsymbol{I}}_{j-2} \circ \hat{\boldsymbol{T}}_{j-1} \circ \hat{\boldsymbol{I}}_{j-1} \tag{43}$$

The matrix $\hat{\boldsymbol{I}}_i$, which involves the complex Fresnel's reflection and transmission amplitude coefficients $\hat{r}_{\rm i}(\equiv \hat{r}_{\rm i,\ i+1})$ and $\hat{t}_{\rm i}$ $(\equiv \hat{t}_{\rm i,\ i+1})$at the ith-interface of the dissimilar $i$and $i+1$stacked layersis given by the formula

$$\hat{\boldsymbol{I}}_{\rm i} = \left(\frac{1}{\hat{t}_{\rm i}}\right)\begin{pmatrix} 1 & \hat{r}_{\rm i} \\ \hat{r}_{\rm i} & 1 \end{pmatrix} \tag{44}$$

Further, the transmission matrix $\widehat{\boldsymbol{T}}_i$ is defined as

$$\widehat{\boldsymbol{T}}_{\rm i} = \begin{pmatrix} e^{j\widehat{\delta}_{\rm i}} & 0 \\ 0 & e^{-j\widehat{\delta}_{\rm i}} \end{pmatrix} \tag{45}$$

Now, let a monochromatic light plane wave of a specificangular frequency ω (or a discrete wavelength λ) to imping onto an interface of the ith-layer at an angle of incidence $\widehat{\theta}_i$, and this layer has a geometric thickness d_i and a complex index of refraction $\hat{n}_i = n_i - j\kappa_i$. The *complexphase-change angle*$\hat{\delta}_i$ produced upon a single traversal of the light plane wave in this$i$th-layer is given by $\hat{\delta}_i \equiv \mathrm{Re}\,\hat{\delta}_{\rm i} - j\,\mathrm{Im}\,\hat{\delta}_{\rm i} = d_i(\omega/c)\hat{n}_i\cos\widehat{\theta}_i = (2\,\pi d_i/\lambda).\,(u_i - jv_i)$, where u_i and v_i are real quantities that are, respectively, equal to n_i and κ_i at *normal incidence* $(\widehat{\theta}_i = 0)$[9, 76].The imaginary part Im $\hat{\delta}_{\rm i}$ of $\hat{\delta}_{\rm i}$ is often incorporated in the optical absorption terms appearing in the formulas of the total intensity of reflected and transmitted light signals produced by an optical system. On the other hand, the real part Re $\hat{\delta}_{\rm i}$of $\hat{\delta}_{\rm i}$will be responsible for both the interference between the light plane waves reflected back and forth from the internal surfaces of the layer andoptical absorptionof these internally reflected waves.

To proceed further in the derivation of the total transmittance and specular reflectance of acertain$j$-layered stack, it may be more convenient to evaluate separately the specific *characteristic* matrix $\boldsymbol{C} \equiv \boldsymbol{T} * \boldsymbol{I}$for each of its$(j-1)$ layers following the *first* (*incident*) layer. The first layer of most practical multi-layered structures is normally a semi-infinite linear, isotropic, stationary, homogeneous, nonmagnetic, and non-absorbing (transparent) mediumwith a real constant index of refraction, or in particular air. To be more precise, Ishall assign the *characteristicmatrix*$\boldsymbol{C}_{\rm i} \equiv \boldsymbol{T}_{\rm i}\boldsymbol{I}_{\rm i}$for each ith-layer of this j-layered stackwith the index$i \geq 2$; thus, thematrix$\boldsymbol{C}_{\rm i}$can be expressed in a simple manner as described below

$$\boldsymbol{C}_{\rm i} \equiv \boldsymbol{T}_{\rm i}\boldsymbol{I}_{\rm i} = \left(\frac{1}{\hat{t}_{\rm i}}\right)\begin{pmatrix} e^{j\widehat{\delta}_{\rm i}} & \hat{r}_{\rm i}e^{j\widehat{\delta}_{\rm i}} \\ \hat{r}_{\rm i}e^{-j\widehat{\delta}_{\rm i}} & e^{-j\widehat{\delta}_{\rm i}} \end{pmatrix} \tag{46}$$

In terms of the characteristic matrix $\boldsymbol{C}_{\rm i}$of the *individual mid layers* of the j-layered structure, Equation (43) of the characteristic matrix $\boldsymbol{M}$ of the *entirestructure* can now be re-written as

$$\boldsymbol{M} \equiv \hat{\boldsymbol{I}}_1 \circ \prod_{i\,=\,2}^{i\,=\,j-1} \boldsymbol{C}_i \tag{47}$$

Now, let us define the *net* electric-field amplitude specular *reflection*$\hat{r}_{\rm net}$and *transmission*$\hat{t}_{\rm net}$coefficients, relative to the amplitude of the electric-field of the incident monochromatic TE light plane wave,of the *whole*multi-layered stack by the following identities

$$\hat{r}_{\text{net}} \equiv \frac{\hat{E}_-^1}{\hat{E}_+^1} \tag{48}$$

$$\hat{t}_{\text{net}} \equiv \frac{\hat{E}_+^f}{\hat{E}_+^1} \tag{49}$$

It is worth noting here that the general matrix formula given in Equation (42),together with Equations (46) and (47), can now be employed to derivethe full expressions that describe these complex specular reflection $\hat{r}_{\text{net}}$and transmission $\hat{t}_{\text{net}}$ coefficients of a j-layeredstructure, and hence its *total*spectral transmittance and specular reflectance.

3.1.3 Derivation of the total transmittance and specular reflectance of stratified three-layered structures from the $E^+ - E^-$ matrix formulations

Let usnow apply the above-discussed results of the $E^+ - E^-$matrix method to evaluate the matrix formulae for a*three-layered* stack of dissimilar ponderable linear, isotropic, homogeneous and nonmagnetic media. The first medium is presumed to be a semi-infinite and transparent dielectric ($\kappa_1 = 0$)layer of a purely real index of refraction $\hat{n}_1 = n_1$. To simplify the problem further without affecting the desired formulations,letthe *lastlayer* of thestructure to be thick enough, so one can put $\hat{E}_-^f \equiv 0$ in the matrix Equation (42).In the next sub-sections, the $E^+ - E^-$ matrix formulations will be utilized to derivethe proper formulasfor the total *normal-incidence*transmittance and specular reflectance of three-layered structures made from a plane-parallel stratified dielectric (orabsorbing)thin filmsandwiched between two dielectric (transparent) media.

The $E^+ - E^-$ matrix analysis will be carried out for *both directions of the light plane wave propagatingin the film*and the attainedexpressions, combined with the above-derived formulae of incoherent normal-incidence reflectivity and transmissivity of a thick slab, will then be used to derive the fullformulaethat describe optical response of a *four-layered* structure made of a thin film laid onto a thick dielectric substrate, with this film-substrate unit being immersed in air.

3.1.3.1 Case of a thin dielectric film sandwiched between two semi-infinite dielectric media

Let a stratified dielectric thin slab (layer 2), of finite thickness d_2 and a real index of refraction n_2, to be sandwiched between two dissimilar semi-infinite dielectric media (layers 1 and 3)with the real indices of refractionn_1 and n_3 ($\neq n_1$ or n_2), respectively. In the terminology of the above-described $E^+ - E^-$ matrix optical approach, $j = 3$ (two interfaces)for such athree-layered structureand its characteristics matrix $\boldsymbol{M} = \boldsymbol{I}_1\boldsymbol{C}_2$, with$\boldsymbol{C}_2 \equiv \boldsymbol{T}_2\boldsymbol{I}_2$. As layer 2 is a thin transparent film, inside which interference of multiply reflected light plane waves occurs with no light absorption upon traversing back and forth, $\hat{\delta}_2$ is a *purereal* quantity. Thus, for a*normal-incidence* TE plane wave of wavelength λ, $\hat{\delta}_2 = \text{Re}\,\hat{\delta}_2 = 2\pi n_2 d_2/\lambda \equiv \phi_2/2$ as $u_2 = n_2$ and $v_2 = \kappa_2 = 0$ for $\theta_1 = 0^{\text{o}}$[9, 76] and the terms $e^{\pm j\,\text{Re}\,\hat{\delta}_2}$in Equation (46) haveno damping effect on light intensity. Further, as*no* light waves are *reflected* from the *backside* of the *semi-infinitelayer* 3, one can set$\hat{E}_-^f = 0$and $\hat{E}_+^f = \hat{E}_+^3$; So, Equations (42) and (47) yielda simple matrix equation of the form

$$\begin{pmatrix} \hat{E}_+^1 \\ \hat{E}_-^1 \end{pmatrix} = \boldsymbol{I}_1\boldsymbol{C}_2 \begin{pmatrix} \hat{E}_+^f \\ 0 \end{pmatrix} = \left(\frac{1}{\hat{t}_1\hat{t}_2}\right)\begin{pmatrix} 1 & \hat{r}_1 \\ \hat{r}_1 & 1 \end{pmatrix}\begin{pmatrix} e^{j\hat{\delta}_2} & \hat{r}_2 e^{j\hat{\delta}_2} \\ \hat{r}_2 e^{-j\hat{\delta}_2} & e^{-j\hat{\delta}_2} \end{pmatrix}\begin{pmatrix} \hat{E}_+^3 \\ 0 \end{pmatrix}$$

$$= \frac{1}{\hat{t}_1 \hat{t}_2} \begin{pmatrix} \left[e^{j\phi_2/2} + \hat{r}_1 \hat{r}_2 e^{-j\phi_2/2}\right] & \left[\hat{r}_1 e^{-j\phi_2/2} + \hat{r}_2 e^{j\phi_2/2}\right] \\ \left[\hat{r}_1 e^{j\phi_2/2} + \hat{r}_2 e^{-j\phi_2/2}\right] & \left[e^{-j\phi_2/2} + \hat{r}_1 \hat{r}_2 e^{j\phi_2/2}\right] \end{pmatrix} \begin{pmatrix} \hat{E}_+^3 \\ 0 \end{pmatrix}$$

$$= \left(\frac{1}{\hat{t}_1 \hat{t}_2}\right) \begin{pmatrix} \left[e^{j\phi_2/2} + \hat{r}_1 \hat{r}_2 e^{-j\phi_2/2}\right] \hat{E}_+^3 \\ \left[\hat{r}_1 e^{j\phi_2/2} + \hat{r}_2 e^{-j\phi_2/2}\right] \hat{E}_+^3 \end{pmatrix} \tag{50}$$

Equations (48) - (50) can now be utilizedto find the expressions that describe the*net*complex reflection and transmission coefficients$\hat{r}_{\text{net}}$ and$\hat{t}_{\text{net}}$of this all-dielectric 123-layered structure, when a monochromatic TE light plane waveis propagating in the semi-infinite transparent layer 1 and is normally incident(from left) onto the thin mid layer 2 (dielectric slab). The required results are expressed by the following formulae

$$\hat{r}_{\text{net}} = \hat{r}_{123} \equiv \frac{\hat{E}_-^1}{\hat{E}_+^1} = \frac{\hat{r}_1 + \hat{r}_2 e^{-j\phi_2}}{1 + \hat{r}_1 \hat{r}_2 e^{-j\phi_2}} \tag{51}$$

$$\hat{t}_{\text{net}} = \hat{t}_{123} \equiv \frac{\hat{E}_+^3}{\hat{E}_+^1} = \frac{\hat{t}_1 \hat{t}_2 e^{-j\phi_2/2}}{1 + \hat{r}_1 \hat{r}_2 e^{-j\phi_2}} \tag{52}$$

Other workers [29-33] have briefly implemented the $E^+ - E^-$ matrix approach, which is, however, discussed in more shining auxiliary details by the present author.As implied in Equations (A3) and (A4), the notation given to the complex reflection and transmission coefficients at the respective interfaces of this simple three-layered structure means explicitly that $\hat{r}_1 \equiv \hat{r}_{12}$, $\hat{r}_2 \equiv \hat{r}_{23}$, $\hat{t}_1 \equiv \hat{t}_{12}$, and $\hat{t}_2 \equiv \hat{t}_{23}$. Further, from the previous discussion given in section (2), one can re-cite here that $\hat{r}_{lm} \equiv \rho_{lm} \exp(j\phi_{lm})$ and $\hat{t}_{lm} \equiv \tau_{lm} \exp(j\chi_{lm})$, with the parameters$\rho_{lm}$, τ_{lm}, ϕ_{lm}, and χ_{lm} have their usual meaning defined there; So, $\hat{r}_{12}\hat{r}_{12}^* \equiv \rho_{12}^2$, $\hat{t}_{12}\hat{t}_{12}^* \equiv \tau_{12}^2$, where $\hat{r}_{lm}^*$and $\hat{t}_{lm}^*$ are the complex conjugates of $\hat{r}_{lm}$ and $\hat{t}_{lm}$, respectively.

By putting$\kappa_2 = \kappa_3 = 0$and$\phi_{12} = \phi_{23} = \chi_{12} = \chi_{23} = 0$ in Equations (21) – (24) and inserting ρ_{12}, ρ_{23}, τ_{12} and τ_{23} in Equations (51) and (52), onecan then geta couple of formulaefor the *totalnormal-incidence*TE transmittance $T(\lambda)$and specular reflectance $R(\lambda)$of thisall-dielectric three-layered (123-) structureof the forms[9]

$$R(\lambda) = R_{123} \equiv \hat{r}_{123}\hat{r}_{123}^* = \frac{\rho_{12}^2 + \rho_{23}^2 + 2\rho_{12}\rho_{23}\cos\phi_2}{1 + \rho_{12}^2\rho_{23}^2 + 2\rho_{12}\rho_{23}\cos\phi_2} \tag{53}$$

$$T(\lambda) = T_{123} \equiv \frac{n_3}{n_1}\hat{t}_{123}\hat{t}_{123}^* = \frac{n_3}{n_1}\frac{\tau_{12}^2\tau_{23}^2}{1 + \rho_{12}^2\rho_{23}^2 + 2\rho_{12}\rho_{23}\cos\phi_2} \tag{54}$$

Recall that the angle parameterϕ_2 is given by $\phi_2 = 2\,\mathrm{Re}\,\hat{\delta}_2 = 4\pi n_2 d_2/\lambda$.

Now, let us find the expressions for the corresponding overall phase changes on reflectionφ_{r} and transmission φ_{t} from such a thin parallel-plane dielectricfilm bounded by two dissimilar semi-infinite dielectricmedia. These expressions can readily be achievedfrom Equations (51) and (52) by re-writing the *complex*specular reflection coefficient $\hat{r}_{123}$ and the*complex* transmission coefficient $\hat{t}_{123}$in a polar form as$\hat{r}_{123} \equiv |\hat{r}_{123}|e^{j\varphi_{\mathrm{r}}}$ and $\hat{t}_{123} \equiv |\hat{t}_{123}|e^{j\varphi_{\mathrm{t}}}$[9] and using the above-cited notation for $\hat{r}_1$, $\hat{r}_2$ etc., viz.

$$\tan\varphi_{\mathrm{r}} \equiv \tan(\arg\hat{r}_{123}) = -\frac{\rho_{23}(1-\rho_{12}^2)\sin\phi_2}{\rho_{12}(1+\rho_{23}^2)+\rho_{23}(1+\rho_{12}^2)\cos\phi_2} \tag{55}$$

$$\tan\varphi_{\mathrm{t}} \equiv \tan(\arg\hat{t}_{123}) = -\frac{1-\rho_{12}\rho_{23}}{1+\rho_{12}\rho_{23}}\tan(\phi_2/2) \tag{56}$$

Equations (53) and (54) that describe the*normal-incidence* TE transmittance and reflectance of an all-dielectric three-layered structurecan also bereached by using the *coherent* description of its optical response via summing the Fresnel's complex amplitude reflection and transmission coefficients of light waves reflected and transmitted fromthe opposite internal interfaces of its mid layer to adjacent media [20-23, 26-33, 76] and from other different approaches [9-11, 51-53].

3.1.3.2Case of a thin optically-absorbingslabbounded by two semi-infinite dielectric media

Now, consider the case of a three-layered structure composing of an optically*thin* stratifiednonmagnetic optically-absorbingslab (layer 2)that has smooth, homogeneous and plane-parallel surfaces and a *complex* index of refraction $\hat{n}_2 \equiv n_2 - j\kappa_2$ and which is bounded by *semi-infinite*stratifiednonmagnetic dielectriclayers 1 and 3 having unlikereal indices of refraction n_1 and $n_3(\neq n_2)$ and vanishing extinction coefficients ($\kappa_1 = \kappa_3 = 0$).

The optical analysiscarried out for the*all-dielectric* 123-structuredescribed in the previous sub-section shouldnow be modified to take into consideration not only the effect of interference between the reflected light plane waves inside the thin absorbing slab (film) of the above-specified {*semi-infinite dielectric layer* 1/*thin absorbing layer* 2/*semi-infinite dielectric layer* 3}-structureand but also the light absorption upon each singletraversal of these reflected waves.

To analyze the optical response of the latter three-layered structure case, let a monochromatic TE light plane wave to be incident *normally* from the semi-infinite dielectric layer 1 atits interface to the absorbing thin layer 2 (film),inside which back and forth light wave reflections and optical absorption are both taking place and from which part of these multiply intensity-decayed reflected waves will eventually be transmitted to both of the semi-infinite dielectric layers1 and 3. These two phenomena are embodied in the phase change produced upon the passage of the light wave inside the film. The resulting complex phasechange upon each *single traversal* of such reflected light wavesis now given by $\hat{\delta}_2 \equiv \hat{\beta}_2/2 \equiv \mathrm{Re}\,\hat{\delta}_2 - j\,\mathrm{Im}\,\hat{\delta}_2 = (2\pi n_2 d_2/\lambda) - j\,(2\pi\kappa_2 d_2/\lambda) = (\phi_2/2) - j\,(\alpha_2 d_2/2)$, where $\alpha_2 \equiv 4\pi\kappa_2/\lambda$ is the absorption coefficient of the film's material.

In this case, the general matrix formula relating the electric-field amplitudesof the transmitted and specularly reflectedTE light plane waves to the electric-field amplitude of the normally-incidentTE light plane wave will be then modified as below

$$\begin{pmatrix}\hat{E}_+^1\\ \hat{E}_-^1\end{pmatrix} = \boldsymbol{I}_1\boldsymbol{C}_2\begin{pmatrix}\hat{E}_+^f\\ 0\end{pmatrix} = \left(\frac{1}{\hat{t}_1\hat{t}_2}\right)\begin{pmatrix}1 & \hat{r}_1\\ \hat{r}_1 & 1\end{pmatrix}\begin{pmatrix}e^{j\hat{\delta}_2} & \hat{r}_2e^{j\hat{\delta}_2}\\ \hat{r}_2e^{-j\hat{\delta}_2} & e^{-j\hat{\delta}_2}\end{pmatrix}\begin{pmatrix}\hat{E}_+^3\\ 0\end{pmatrix}$$

$$= \left(\frac{1}{\hat{t}_1\hat{t}_2}\right)\begin{pmatrix}\left[e^{j\hat{\delta}_2}+\hat{r}_1\hat{r}_2e^{-j\hat{\delta}_2}\right] & \left[\hat{r}_1e^{-j\hat{\delta}_2}+\hat{r}_2e^{j\hat{\delta}_2}\right]\\ \left[\hat{r}_1e^{j\hat{\delta}_2}+\hat{r}_2e^{-j\hat{\delta}_2}\right] & \left[e^{-j\hat{\delta}_2}+\hat{r}_1\hat{r}_2e^{j\hat{\delta}_2}\right]\end{pmatrix}\begin{pmatrix}\hat{E}_+^3\\ 0\end{pmatrix}$$

$$= \left(\frac{1}{\hat{t}_1\hat{t}_2}\right)\begin{pmatrix}\left[e^{j\hat{\delta}_2}+\hat{r}_1\hat{r}_2e^{-j\hat{\delta}_2}\right]\hat{E}_+^3\\ \left[\hat{r}_1e^{j\hat{\delta}_2}+\hat{r}_2e^{-j\hat{\delta}_2}\right]\hat{E}_+^3\end{pmatrix} \tag{57}$$

Using the afore-adoptednotation forwriting the complex phase angle $\hat{\delta}_2$ and $\hat{r}_1$, $\hat{r}_2$, $\hat{t}_1$, and $\hat{t}_2$, the Fresnel'scomplex-amplitude reflection and transmission coefficients at the interfaces of the {*semi-infinite dielectric layer* 1/*thin absorbinglayer* 2/*semi-infinite dielectric layer* 3}-structure, one can acquirefrom Equation (57) a couple of neat complex expressions for its*total normal-incidence*TE-wave specular reflection coefficient $\hat{r}_{123}$ and transmission coefficient $\hat{t}_{123}$ as described below in Equations (58) and (59)

$$\hat{r}_{123} \equiv \frac{\hat{E}_-^1}{\hat{E}_+^1} = \frac{\hat{r}_{12} + \hat{r}_{23} e^{-j\hat{\beta}_2}}{1 + \hat{r}_{12}\hat{r}_{23} e^{-j\hat{\beta}_2}} \tag{58}$$

$$\hat{t}_{123} \equiv \frac{\hat{E}_+^3}{\hat{E}_+^1} = \frac{\hat{t}_{12}\hat{t}_{23} e^{-j(\hat{\beta}_2/2)}}{1 + \hat{r}_{12}\hat{r}_{23} e^{-j\hat{\beta}_2}} \tag{59}$$

Equations (58) and (59) are *exclusivelyidentical* to thoseattained from a *coherent*superposition of the Fresnel's complex amplitude reflection and transmission coefficients of the individual reflected and transmitted TE waves at the internal smooth surfaces of a thin plane-parallel conducting film sandwiched between two semi-infinite dielectric media [20-24, 26-33, 76] and to those that have been accomplishedby other different approaches [9-11, 51-53].

It is worth noting here that Equations (58) and (59)equally describe the *net*specular reflection $\hat{r}_{123}$and transmission $\hat{t}_{123}$coefficients ofan "ideal" three-layered structure having the{*semi-infinite dielectric layer* 1/*thin absorbing layer* 2/*semi-infinite dielectric layer* 3}-pilingwhether the monochromatic lightplane waveis normally-incident($\theta_1 = 0^{o}$) or obliquely-incidentonto such an optical structure. In the oblique-incidence case, the Fresnel's reflection and transmission coefficients $\hat{r}_{12}$, $\hat{r}_{23}$, $\hat{t}_{12}$, and $\hat{t}_{23}$ at the respective interfaces of this structure should be evaluated from Equations (A3) and (A4) for the *s*-polarized (TE) light plane waves or from Equations (A9) and (A11) for the *p*-polarized (TM) light plane waves, with $\hat{n}_2 \cos \hat{\theta}_2$being replaced by $u_2 - jv_2$in either polarization case[9].

However, the *normal-incidence* case is rather simpler as the Fresnel's complex amplitude transmission and specular reflection coefficients $\hat{r}_{12}$, $\hat{r}_{23}$, $\hat{t}_{12}$, $\hat{t}_{23}$etc. for the above-described "ideal" {*semi-infinite dielectric layer*/*thin absorbingfilm*/*semi-infinite dielectric layer*}-structureare identical for bothmonochromatic TE and TM light plane waves. The distinction between the two optical treatments is irrelevant, from both the physics and the mathematical viewpoints. To proceed further, let us assume that the TE or TM light plane wave that is incident normally onto the above-specified three-layered structure has a free-space spectral wavelength λ_0 and the thin optically-absorbing film (layer 2) of such a structure has a finite geometric thickness d_2. Further, let us use the following notation:$\hat{\beta}_2 \equiv (4\pi n_2 d_2/\lambda_0) - j\,(4\pi\kappa_2 d_2/\lambda_0) \equiv \phi_2 - j\,\gamma_2$, with$\gamma_2 \equiv \alpha_2 d_2$,$\hat{r}_{lm} \equiv \rho_{lm} \exp\,(j\phi_{lm})$ and $\hat{t}_{lm} \equiv \tau_{lm} \exp\,(j\chi_{lm})$in order to re-write Equations (58) and (59) in more manageable mathematical forms as described below

$$\hat{r} \equiv \hat{r}_{123} \equiv \rho e^{j\delta_r} = \frac{\rho_{12} e^{j\phi_{12}} + \rho_{23} e^{-\gamma_2} e^{j(\phi_{23} - \phi_2)}}{1 + \rho_{12}\rho_{23} e^{-\gamma_2} e^{j(\phi_{12} + \phi_{23} - \phi_2)}} \tag{60}$$

$$\hat{t} \equiv \hat{t}_{123} \equiv \tau e^{j\delta_t} = \frac{\tau_{12}\tau_{23} e^{-\frac{\gamma_2}{2}} e^{j\left(\chi_{12} + \chi_{23} - \frac{\phi_2}{2}\right)}}{1 + \rho_{12}\rho_{23} e^{-\gamma_2} e^{j(\phi_{12} + \phi_{23} - \phi_2)}} \tag{61}$$

The required normal-incidence real scalar reflection and transmission coefficients ρ_{12}, ρ_{12}, τ_{12} and τ_{23} and the associated real phase changesϕ_{12}, ϕ_{23}, χ_{12} and χ_{23} that take place upon the wave transmission and specular reflection at the respective layer interfaces(boundaries) of the above-specified{*semi-infinite dielectric layer*/*thin absorbing film*/*semi-infinite dielectric layer*}-structurecan be found, in terms of the optical constants of its layers, from the set of expressions given previously in Equations (21) - (26).

It is not difficult to performa somewhat lengthy mathematical manipulation of the complex terms appearing in Equation (60) togetthe full expressions for the*totalnormal-incidence*specular reflectance $R_{123} \equiv |\hat{r}|^2 \equiv \hat{r}_{123}\hat{r}^*_{123}$of the above-specified{*thick dielectric layer*/*thinabsorbingfilm*/*thick dielectric layer*}-structure, as well as the associated *total*phase-change angle δ_{r} on reflection [9]. These expressions can be tidily given by the formulae below

$$R_{123} = \frac{\rho_{12}^2 + \rho_{23}^2 \mathrm{e}^{-2\gamma_2} + 2\,\rho_{12}\rho_{23}\mathrm{e}^{-\gamma_2}\cos(-\phi_{12} + \phi_{23} - \varphi_2)}{1 + \rho_{12}^2\rho_{23}^2 \mathrm{e}^{-2\gamma_2} + 2\,\rho_{12}\rho_{23}\mathrm{e}^{-\gamma_2}\cos\,(\phi_{12} + \phi_{23} - \varphi_2)} \qquad (62)$$

$$\tan\delta_{\mathrm{r}} = \frac{\rho_{23}(1 - \rho_{12}^2)\sin(\varphi_2 + \phi_{23}) + \rho_{12}(\mathrm{e}^{\gamma_2} - \rho_{23}^2\mathrm{e}^{-\gamma_2})\sin\phi_{12}}{\rho_{23}(1 + \rho_{12}^2)\cos\,(\varphi_2 + \phi_{23}) + \rho_{12}(\mathrm{e}^{\gamma_2} + \rho_{23}^2\mathrm{e}^{-\gamma_2})\cos\phi_{12}} \qquad (63)$$

By the same token, for the different semi-infinite dielectric layers 1 and 3 of the above-specified {*semi-infinitedielectric layer*1/*thinabsorbingfilm*/*thick dielectric layer*3}-structure,onecan readily find fromEquation (61) the corresponding expressionsthat describe the *totalnormal-incidence*spectral transmittance $T_{123} \equiv (n_3/n_1)|\hat{t}|^2 \equiv (n_3/n_1) * (\hat{t}_{123}\hat{t}^*_{123})$ and the associated*total*phase-change angle δ_{t}on transmission through itsthin filmas given below [9]

$$T_{123} = \frac{n_3}{n_1}\frac{\tau_{12}^2\tau_{23}^2\mathrm{e}^{-\gamma_2}}{1 + \rho_{12}^2\rho_{23}^2\mathrm{e}^{-2\gamma_2} + 2\,\rho_{12}\rho_{23}\mathrm{e}^{-\gamma_2}\cos(\phi_{12} + \phi_{23} - \varphi_2)} \qquad (64)$$

$$\tan\left[\delta_{\mathrm{t}} - \chi_{12} - \chi_{23} + \frac{\varphi_2}{2}\right] = \frac{\mathrm{e}^{\gamma_2}\sin\varphi_2 - \rho_{12}\rho_{23}\sin(\phi_{12} + \phi_{23})}{\mathrm{e}^{\gamma_2}\cos\varphi_2 + \rho_{12}\rho_{23}\cos(\phi_{12} + \phi_{23})} \qquad (65)$$

It deserves noting here that Equations (62) – (65) are equally valid for the *normally-incident* TE- or TM- light plane wavebut with the substitution of the suitablevalues of thescalar transmission and specular reflection coefficients$\tau_{lm}$and $\rho_{lm}$and the associated phase-angle changes $\phi_{lm}$and χ_{lm} at the respective l-m interfacesforeither linearly-polarized plane wave [9].

It is also valuable to recall here that the overall phase-angle change δ_{r}on reflection is referred to the *first* boundary of the thin film with the dielectric layer 1 (i.e., the 1-2 interface), while the totalphase-angle change δ_{t}on transmission through this film is referred to its *second* boundary with the thick weakly-absorbing layer 3 (or the 2-3 interface). Further, these δ_{r}- and δ_{t}- formulae are equally applicable whether the dielectric layer 3 is thick enough (semi-infinite) or when its *back surface* is utterly rough or blackened, so in both optical situations no light is being specularly reflected from or transmitted through this non-smooth and inhomogeneous surface.

For the purposes of future treatment of the optical behavior of simple four-layered structures, it is valuable to discuss in some detail the normal-incidence optical response of the three-layered structure having the {*semi-infinite air layer*/*thin absorbingfilm*/*semi-infinite substrate*}-piling. To distinguish between the optical responses of such a three-layered structure for opposite directionsof propagation of the light plane wave incident onto it,I shall designate the air (vacuum), film, and substrate layers by the letters v, f, and s, respectively, instead of using the 123-sequence.

Now, let a monochromatic TE(s-polarized) light plane waveof free-spectral wavelength $\lambda_0$to be normally incident onto the above-specifiedthree-layered structure from the side of the semi-infinite*air*layer 1 ($n_1 = 1$), and refer to this 123-direction along which the wave is propagating through the film as the *vfs-route*. Further, assume the layer 3to be sufficiently thick *weakly-absorbing* substrate($\kappa_s \neq 0$) and having a real index of refraction n_s, while the film (layer 2) to be optically thin with a geometric thickness d_2 and is made of a material having a complex index of refraction $\hat{n}_2 = n_2 - j\kappa_2$, with its extinction coefficient $\kappa_2$being related to its optical absorption coefficient$\alpha_2$as $\alpha_2 \equiv 4\pi\kappa_2/\lambda_0$.Temporarily, the number 2 is still, however, being used as a subscript for the various optical constants of the film (layer 2), but will be replaced by the letter *f* in all other symbolic designations.

By making use of Equations (21) - (24),the*normal-incidence*scalar specular reflection and transmission coefficients andthe associated phase changes on reflection and transmission at the respective interfaces of such {*semi-infinite air layer/thin absorbingfilm/semi-infinite substrate*}-structurecan now be re-written as follows

$$\rho_{12}^2 = \rho_{\mathrm{vf}}^2 = \frac{(1-n_2)^2 + \kappa_2^2}{(1+n_2)^2 + \kappa_2^2} \tan\phi_{12} = \tan\phi_{\mathrm{vf}} = -\frac{2\,\kappa_2}{n_2^2 + \kappa_2^2 - 1} \tag{66}$$

$$\rho_{23}^2 = \rho_{\mathrm{fs}}^2 = \frac{(n_2-n_s)^2 + (\kappa_2-\kappa_s)^2}{(n_2+n_s)^2 + (\kappa_2+\kappa_s)^2} \tan\phi_{23} = \tan\phi_{\mathrm{fs}} = -\frac{2(n_s\kappa_2 - n_2\kappa_s)}{(n_2^2+\kappa_2^2) - (n_s^2+\kappa_s^2)} \tag{67}$$

$$\tau_{12}^2 = \tau_{\mathrm{vf}}^2 = \frac{4}{(1+n_2)^2 + \kappa_2^2} \tan\chi_{12} = \tan\chi_{\mathrm{vf}} = \frac{\kappa_2}{1+n_2} \tag{68}$$

$$\tau_{23}^2 = \tau_{\mathrm{fs}}^2 = \frac{4(n_2^2+\kappa_2^2)}{(n_2+n_s)^2 + (\kappa_2+\kappa_s)^2} \tan\chi_{23} = \tan\chi_{\mathrm{fs}} = -\frac{n_s\kappa_2 - n_2\kappa_s}{n_2^2+\kappa_2^2+n_2n_s+\kappa_2\kappa_s} \tag{69}$$

Moreover, for future convenience and neatnessof the required final formulations ofthe specular reflectance R_{123} and transmittance T_{123} given in Equations (62)and (64),I shalladopt the following symbolicnotation:$x_2 \equiv \exp\,(-\gamma_2)$, $\Delta_1 \equiv -\phi_{12} + \phi_{23} - \phi_2 \equiv -\phi_{\mathrm{vf}} + \phi_{\mathrm{fs}} - \phi_2$ and $\Delta_2 \equiv \phi_{12} + \phi_{23} - \phi_2 \equiv \phi_{\mathrm{vf}} + \phi_{\mathrm{fs}} - \phi_2$, with the angles being expressed in *radians*.It is worth noting here that this tidy notation terminology has been also used, but in different symbolic forms, in the treatment of optical response of multi-layered structures [51-53].

Let us first obtain, in view of this new symbolization,some well-ordered expressionsthat describeboth the spectral *normal-incidence* transmittance$T_{\mathrm{vfs}} \equiv T_{123}$and specular reflectance $R_{\mathrm{vfs}} \equiv R_{123}$of this{*air/thin absorbing film/thick weakly-absorbing substrate*}-structure when the light plane wave is travelling through the film along the *vfs-route*. This can be achieved by replacing the coefficients ρ_{12}^2, τ_{12}^2, ρ_{23}^2 and τ_{23}^2byρ_{vf}^2, τ_{vf}^2, ρ_{fs}^2 and τ_{fs}^2, respectively, which are already describedin Equations (66) – (69), and also by making use of the above-cited replacements of their associated phase angles on transmission and specular reflection at the respective interfaces of the film to its neighboring layers.

Then, when the TE light plane wave is propagating through the film of such an "ideal" three-layered structure along the *vfs-route*, one can keenly re-write Equations (62) and (64) to obtain a couple of much more compact expressions to describe its*totalnormal-incidence*specular reflectance R_{vfs} and transmittance T_{vfs}, viz.

$$R_{\mathrm{vfs}} = \frac{\rho_{\mathrm{vf}}^2 + \rho_{\mathrm{fs}}^2 x_2^2 + 2x_2\rho_{\mathrm{vf}}\rho_{\mathrm{fs}} \cos \Delta_1}{1 + \rho_{\mathrm{vf}}^2\rho_{\mathrm{fs}}^2 x_2^2 + 2x_2\rho_{\mathrm{vf}}\rho_{\mathrm{fs}} \cos \Delta_2} \tag{70}$$

$$T_{\mathrm{vfs}} = \frac{n_{\mathrm{s}}\tau_{\mathrm{vf}}^2\tau_{\mathrm{fs}}^2 x_2}{1 + \rho_{\mathrm{vf}}^2\rho_{\mathrm{fs}}^2 x_2^2 + 2x_2\rho_{\mathrm{vf}}\rho_{\mathrm{fs}} \cos \Delta_2} \tag{71}$$

Now, let the monochromatic TE light plane wave to be incident normally uponthe thin filmof this{*air/thin absorbing film/thick weakly-absorbingsubstrate*}-structure from the side of the thick substrate (layer 3) and travelling through the film (layer 1) towards the semi-infinite air medium (layer1). Designate this 321-direction of light wave propagation through the film (f)by the *sfv-route*, for which the normal-incidence scalar specular reflection and transmission coefficients at the respective interfaces of such a three-layered structureρ_{12}^2, τ_{12}^2, ρ_{23}^2 and τ_{23}^2have to be then replaced by ρ_{sf}^2 , τ_{sf}^2 , ρ_{fv}^2 and τ_{fv}^2, respectively, with the correspondingly phase-angle changes ϕ_{12},ϕ_{23}, χ_{12}, and χ_{23}being replaced by ϕ_{sf},ϕ_{fv}, χ_{sf}, and χ_{fv}. For this *sfv-route*, Equations (21) – (24) become

$$\rho_{12}^2 = \rho_{\mathrm{sf}}^2 = \frac{(n_{\mathrm{s}} - n_2)^2 + (\kappa_{\mathrm{s}} - \kappa_2)^2}{(n_{\mathrm{s}} + n_2)^2 + (\kappa_{\mathrm{s}} + \kappa_2)^2} \tan \phi_{12} = \tan \phi_{\mathrm{sf}} = -\frac{2\,(n_{\mathrm{s}}\kappa_2 - n_2\kappa_{\mathrm{s}})}{(n_2^2 + \kappa_2^2) - (n_{\mathrm{s}}^2 + \kappa_{\mathrm{s}}^2)} \tag{72}$$

$$\rho_{23}^2 = \rho_{\mathrm{fv}}^2 = \frac{(n_2 - 1)^2 + \kappa_2^2}{(n_2 + 1)^2 + \kappa_2^2} \tan \phi_{23} = \tan \phi_{\mathrm{fv}} = -\frac{2\kappa_2}{n_2^2 + \kappa_2^2 - 1} \tag{73}$$

$$\tau_{12}^2 = \tau_{\mathrm{sf}}^2 = \frac{4(n_{\mathrm{s}}^2 + \kappa_{\mathrm{s}}^2)}{(n_{\mathrm{s}} + n_2)^2 + (\kappa_{\mathrm{s}} + \kappa_2)^2} \tan \chi_{12} = \tan \chi_{\mathrm{sf}} = \frac{n_{\mathrm{s}}\kappa_2 - n_2\kappa_{\mathrm{s}}}{n_{\mathrm{s}}^2 + \kappa_{\mathrm{s}}^2 + n_2 n_{\mathrm{s}} + \kappa_2\kappa_{\mathrm{s}}} \tag{74}$$

$$\tau_{23}^2 = \tau_{\mathrm{fv}}^2 = \frac{4(n_2^2 + \kappa_2^2)}{(n_2 + 1)^2 + \kappa_2^2} \tan \chi_{23} = \tan \chi_{\mathrm{fv}} = -\frac{\kappa_2}{n_2^2 + \kappa_2^2 + n_2} \tag{75}$$

Furthermore, for theplane wave propagation through the film along the *sfv-route*, Ishall nowuse the following notation:$\Delta_1' \equiv -\phi_{12} + \phi_{23} - \phi_2 \equiv -\phi_{\mathrm{sf}} + \phi_{\mathrm{fv}} - \phi_2$ and, since $\phi_{\mathrm{vf}} \equiv \phi_{\mathrm{fv}}$ and $\phi_{\mathrm{sf}} \equiv \phi_{\mathrm{fs}}$ (evenforan absorbingsubstrate), $\Delta_2 \equiv \phi_{12} + \phi_{23} - \phi_2 \equiv \phi_{\mathrm{sf}} + \phi_{\mathrm{fv}} - \phi_2$. For this case also, note that different notation has been used by other researchers [51-53]. Accordingly, the expressions that describe the total normal-incidence specular reflectance $R_{\mathrm{sfv}} = R_{123}$ and transmittance $T_{\mathrm{sfv}} = T_{123}$ of this{*air/thin absorbing film/thick weakly-absorbingsubstrate*}-pilingwhen the TE light plane wave is propagating along the *sfv-route*should now re-written as

$$R_{\mathrm{sfv}} = \frac{\rho_{\mathrm{sf}}^2 + \rho_{\mathrm{fv}}^2 x_2^2 + 2x_2\rho_{\mathrm{sf}}\rho_{\mathrm{fv}} \cos \Delta_1'}{1 + \rho_{\mathrm{sf}}^2\rho_{\mathrm{fv}}^2 x_2^2 + 2x_2\rho_{\mathrm{sf}}\rho_{\mathrm{fv}} \cos \Delta_2} \tag{76}$$

$$T_{\mathrm{sfv}} = \frac{1}{n_{\mathrm{s}}} \frac{\tau_{\mathrm{sf}}^2\tau_{\mathrm{fv}}^2 x_2}{1 + \rho_{\mathrm{sf}}^2\rho_{\mathrm{fv}}^2 x_2^2 + 2x_2\rho_{\mathrm{sf}}\rho_{\mathrm{fv}} \cos \Delta_2} \tag{77}$$

At this stage, it is worthwhile to point out a couple of features regarding the mathematical forms and applicability of the general well-ordered formulae given in Equations (70), (71), (76) and (77) that describe the *total* transmittance and specular reflectance of the above-specified"ideal" {*semi-infiniteair layer/thin absorbing film/thick weakly-absorbingsubstrate*}-structure.

Firstly, the respective expressions that were just cited, in different symbolic forms, by other workers [27, 28, 43, 51-53] can be shown to reconcile with these equations if one performs some mathematical manipulation and uses analogous definitions for the variousreflection and transmission coefficients and phase-angle parameters involved in their reported formulas.

Secondly, in case of dissimilar dielectric layers 1 (air) and 3 (substrate), the R_{vfs}- and R_{sfv}- formulae are totally different, while the T_{vfs}- and T_{sfv}- formulae are identical under some practical conditions, as will be clear shortly and as weredeliberated by many researchers [27, 28, 43, 51-53], but in contrary with the findings reported in a more recent work [58]. This issue will now be discussed in detail.One can, using Equations (66) – (69) and (72) – (75), explicitly express Equations (70), (71), (76), and (77) that describe *totalnormal-incidence*specular reflectance and transmittance of the {*air*/thin *conducting film*/*thick dielectricsubstrate*}-structure in terms of its layers optical constants n_2, κ_2, $n_{\mathrm{s}}$and κ_{s}as

$$R_{\mathrm{vfs}}=\frac{\left[\frac{(1-n_2)^2+\kappa_2^2}{(1+n_2)^2+\kappa_2^2}\right]+\left[\frac{(n_2-n_{\mathrm{s}})^2+(\kappa_2-\kappa_{\mathrm{s}})^2}{(n_2+n_{\mathrm{s}})^2+(\kappa_2+\kappa_{\mathrm{s}})^2}\right]x_2^2+2x_2\left\{\left[\frac{(1-n_2)^2+\kappa_2^2}{(1+n_2)^2+\kappa_2^2}\right]\left[\frac{(n_2-n_{\mathrm{s}})^2+(\kappa_2-\kappa_{\mathrm{s}})^2}{(n_2+n_{\mathrm{s}})^2+(\kappa_2+\kappa_{\mathrm{s}})^2}\right]\right\}^{\frac{1}{2}}\cos\Delta_1}{1+\left[\frac{(1-n_2)^2+\kappa_2^2}{(1+n_2)^2+\kappa_2^2}\right]\left[\frac{(n_2-n_{\mathrm{s}})^2+(\kappa_2-\kappa_{\mathrm{s}})^2}{(n_2+n_{\mathrm{s}})^2+(\kappa_2+\kappa_{\mathrm{s}})^2}\right]x_2^2+2x_2\left\{\left[\frac{(1-n_2)^2+\kappa_2^2}{(1+n_2)^2+\kappa_2^2}\right]\left[\frac{(n_2-n_{\mathrm{s}})^2+(\kappa_2-\kappa_{\mathrm{s}})^2}{(n_2+n_{\mathrm{s}})^2+(\kappa_2+\kappa_{\mathrm{s}})^2}\right]\right\}^{\frac{1}{2}}\cos\Delta_2}$$

$$=\frac{[(n_2+n_{\mathrm{s}})^2+(\kappa_2+\kappa_{\mathrm{s}})^2][(1-n_2)^2+\kappa_2^2]+[(1+n_2)^2+\kappa_2^2][(n_2-n_{\mathrm{s}})^2+(\kappa_2-\kappa_{\mathrm{s}})^2]x_2^2+2x_2A_{\mathrm{com}}\cos\Delta_1}{[(1+n_2)^2+\kappa_2^2][(n_2+n_{\mathrm{s}})^2+(\kappa_2+\kappa_{\mathrm{s}})^2]+[(1-n_2)^2+\kappa_2^2][(n_2-n_{\mathrm{s}})^2+(\kappa_2-\kappa_{\mathrm{s}})^2]x_2^2+2x_2A_{\mathrm{com}}\cos\Delta_2}$$

(78)

$$T_{\mathrm{vfs}}=\frac{\left[\frac{4n_{\mathrm{s}}}{(1+n_2)^2+\kappa_2^2}\right]\left[\frac{4\left(n_2^2+\kappa_2^2\right)}{(n_2+n_{\mathrm{s}})^2+(\kappa_2+\kappa_{\mathrm{s}})^2}\right]x_2}{1+\left[\frac{(1-n_2)^2+\kappa_2^2}{(1+n_2)^2+\kappa_2^2}\right]\left[\frac{(n_2-n_{\mathrm{s}})^2+(\kappa_2-\kappa_{\mathrm{s}})^2}{(n_2+n_{\mathrm{s}})^2+(\kappa_2+\kappa_{\mathrm{s}})^2}\right]x_2^2+2x_2\left\{\left[\frac{(1-n_2)^2+\kappa_2^2}{(1+n_2)^2+\kappa_2^2}\right]\left[\frac{(n_2-n_{\mathrm{s}})^2+(\kappa_2-\kappa_{\mathrm{s}})^2}{(n_2+n_{\mathrm{s}})^2+(\kappa_2+\kappa_{\mathrm{s}})^2}\right]\right\}^{\frac{1}{2}}\cos\Delta_2}$$

$$=\frac{16n_{\mathrm{s}}(n_2^2+\kappa_2^2)x_2}{[(1+n_2)^2+\kappa_2^2][(n_2+n_{\mathrm{s}})^2+(\kappa_2+\kappa_{\mathrm{s}})^2]+[(1-n_2)^2+\kappa_2^2][(n_2-n_{\mathrm{s}})^2+(\kappa_2-\kappa_{\mathrm{s}})^2]x_2^2+2x_2A_{\mathrm{com}}\cos\Delta_2}$$

(79)

$$R_{\mathrm{sfv}}=\frac{\left[\frac{(n_{\mathrm{s}}-n_2)^2+(\kappa_{\mathrm{s}}-\kappa_2)^2}{(n_{\mathrm{s}}+n_2)^2+(\kappa_{\mathrm{s}}+\kappa_2)^2}\right]+\left[\frac{(n_2-1)^2+\kappa_2^2}{(n_2+1)^2+\kappa_2^2}\right]x_2^2+2x_2\left\{\left[\frac{(n_{\mathrm{s}}-n_2)^2+(\kappa_{\mathrm{s}}-\kappa_2)^2}{(n_{\mathrm{s}}+n_2)^2+(\kappa_{\mathrm{s}}+\kappa_2)^2}\right]\left[\frac{(n_2-1)^2+\kappa_2^2}{(n_2+1)^2+\kappa_2^2}\right]\right\}^{\frac{1}{2}}\cos\Delta_1'}{1+\left[\frac{(n_{\mathrm{s}}-n_2)^2+(\kappa_{\mathrm{s}}-\kappa_2)^2}{(n_{\mathrm{s}}+n_2)^2+(\kappa_{\mathrm{s}}+\kappa_2)^2}\right]+\left[\frac{(n_2-1)^2+\kappa_2^2}{(n_2+1)^2+\kappa_2^2}\right]x_2^2+2x_2\left\{\left[\frac{(n_{\mathrm{s}}-n_2)^2+(\kappa_{\mathrm{s}}-\kappa_2)^2}{(n_{\mathrm{s}}+n_2)^2+(\kappa_{\mathrm{s}}+\kappa_2)^2}\right]\left[\frac{(n_2-1)^2+\kappa_2^2}{(n_2+1)^2+\kappa_2^2}\right]\right\}^{\frac{1}{2}}\cos\Delta_2}$$

$$=\frac{[(n_2+1)^2+\kappa_2^2][(n_{\mathrm{s}}-n_2)^2+(\kappa_{\mathrm{s}}-\kappa_2)^2]+[(n_{\mathrm{s}}+n_2)^2+(\kappa_{\mathrm{s}}+\kappa_2)^2][(n_2-1)^2+\kappa_2^2]x_2^2+2x_2A_{\mathrm{com}}\cos\Delta_1'}{[(n_2+1)^2+\kappa_2^2][(n_{\mathrm{s}}+n_2)^2+(\kappa_{\mathrm{s}}+\kappa_2)^2]+[(n_2-1)^2+\kappa_2^2][(n_{\mathrm{s}}-n_2)^2+(\kappa_{\mathrm{s}}-\kappa_2)^2]x_2^2+2x_2A_{\mathrm{com}}\cos\Delta_2}$$

(80)

$$T_{\mathrm{sfv}} = \frac{\frac{1}{n_{\mathrm{s}}}\left[\frac{4\left(n_{\mathrm{s}}^2+\kappa_{\mathrm{s}}^2\right)}{(n_{\mathrm{s}}+n_2)^2+(\kappa_{\mathrm{s}}+\kappa_2)^2}\right]\left[\frac{4\left(n_2^2+\kappa_2^2\right)}{(n_2+1)^2+\kappa_2^2}\right]x_2}{1+\left[\frac{(n_{\mathrm{s}}-n_2)^2+(\kappa_{\mathrm{s}}-\kappa_2)^2}{(n_{\mathrm{s}}+n_2)^2+(\kappa_{\mathrm{s}}+\kappa_2)^2}\right]+\left[\frac{(n_2-1)^2+\kappa_2^2}{(n_2+1)^2+\kappa_2^2}\right]x_2^2+2x_2\left\{\left[\frac{(n_{\mathrm{s}}-n_2)^2+(\kappa_{\mathrm{s}}-\kappa_2)^2}{(n_{\mathrm{s}}+n_2)^2+(\kappa_{\mathrm{s}}+\kappa_2)^2}\right]\left[\frac{(n_2-1)^2+\kappa_2^2}{(n_2+1)^2+\kappa_2^2}\right]\right\}^{\frac{1}{2}}\cos\Delta_2}$$

$$= \frac{\frac{16}{n_{\mathrm{s}}}(n_{\mathrm{s}}^2+\kappa_{\mathrm{s}}^2)(n_2^2+\kappa_2^2)x_2}{[(n_2+1)^2+\kappa_2^2][(n_{\mathrm{s}}+n_2)^2+(\kappa_{\mathrm{s}}+\kappa_2)^2]+[(n_2-1)^2+\kappa_2^2][(n_{\mathrm{s}}-n_2)^2+(\kappa_{\mathrm{s}}-\kappa_2)^2]x_2^2+2x_2A_{\mathrm{com}}\cos\Delta_2}$$

(81)

The parameter A_{com} is described by the expression given below

$$A_{\mathrm{com}} = \{[(1+n_2)^2+\kappa_2^2][(1-n_2)^2+\kappa_2^2][(n_2+n_{\mathrm{s}})^2+(\kappa_2+\kappa_{\mathrm{s}})^2][(n_2-n_{\mathrm{s}})^2 + (\kappa_2-\kappa_{\mathrm{s}})^2]\}^{\frac{1}{2}} \quad (82)$$

Several conclusions of practical interest can be inferred from the set of Equations (78) – (81) as briefly discussed below.

Consider an $\{$*air/film/substrate*$\}$-structure composed of a semi-infinite dielectric substrate onto which a thick *metallic film*sothat $\kappa_2 d_2 \geqslant 0.37\lambda_0$, these equations reduce, using $\kappa_{\mathrm{s}} = 0$and setting the terms containingx_2 and $x_2^2$to zero,tothe following simple forms [9]

$$R_{\mathrm{vfs}} \cong \frac{(1-n_2)^2+\kappa_2^2}{(1+n_2)^2+\kappa_2^2} R_{\mathrm{sfv}} = \frac{(n_{\mathrm{s}}-n_2)^2+\kappa_2^2}{(n_{\mathrm{s}}+n_2)^2+\kappa_2^2} \quad (83)$$

$$T_{\mathrm{vfs}} = T_{\mathrm{sfv}} = \frac{16n_{\mathrm{s}}(n_2^2+\kappa_2^2)}{[(n_2+1)^2+\kappa_2^2][(n_2+n_{\mathrm{s}})^2+\kappa_2^2]}x_2 \sim \exp\left(-\frac{4\pi\kappa_2 d_2}{\lambda_0}\right) \cong 0 \quad (84)$$

Equations (83) and (84) tell us that the *normal-incidence*optical response of a “thick”*metallicfilm*laid on an infinite transparent substrate placed in air is dominated by reflection at the *air-film*(or *substrate-film*)interface when light is incident from the air (or substrate) side on the film. For both directions of light wave propagation, little light is transmitted to the substrate (or air), implying that in a “thick” *metallic film* the effect of interference of the multiple light beam reflections inside the metallic film is of little importance.

Further, one can inspect from Equations (79) and (81) that the normal-incidence transmittances $T_{\mathrm{vfs}}$and T_{sfv} of an “ideal”$\{$*air layer/thinabsorbing film/semi-infinite dielectricsubstrate*$\}$-structure are indeed equal to each other provided that the dielectric substrate is transparent ($\kappa_{\mathrm{s}} = 0$) or weakly-absorbing with $\kappa_{\mathrm{s}} < \kappa_2$and $\kappa_{\mathrm{s}}^2 \ll n_{\mathrm{s}}^2$. This is true whether the thin film bounded by air and substrate is made from a non-absorbing (transparent) dielectric material or optically-absorbingsubstance with $\kappa_2 \neq 0$and $x_2 < 1$. This fact has a special significance in the derivation of the full formulae describing the total specular reflectance and transmittance of four-layered optical systems having the {air/thin film/dielectric substrate/air}-stacking.

Moreover, R_{vfs} given in Equation (78) does not equal toR_{sfv}given in Equation (80) unless $\kappa_{\mathrm{s}} = 0$and $\kappa_2 = 0$,so$x_2 \equiv 1$ and $\Delta_1 = \Delta_1' = -\varphi_2$ as $\phi_{12} = \phi_{23} = 0$;a situation that is justified only for a perfectly dielectric thin film bounded by two semi-infinite transparent media. This “strict” condition may be fulfilled for a thin semi-conducting layerover a limited portion of its transmission spectrum for wavelengths aboveits fundamental absorptionedge (that is, in its fully transparent region).Just for an {*air/thin dielectric film/thick dielectric substrate*}-structureone can only allege that thetotal normal-incidence reflectance $R_{\mathrm{vfs}} = R_{\mathrm{sfv}} = 1 - T_{\mathrm{vfs}} = 1 - T_{\mathrm{sfv}}$[58], but *not*for an absorbing thin film ($\kappa_2 \neq 0$). Consequently,if one put $\kappa_2 = 0$and $\kappa_{\mathrm{s}} = 0, x_2 \equiv 1,\ \phi_{12} = \phi_{23} = 0$ and $\Delta_1 = \Delta_2 = \Delta_1' = -\phi_2$ intoEquation (78) – (82), the formulae of the spectral normal-incidence specular reflectance $R_{\mathrm{vfs}} = R_{\mathrm{sfv}}$ and transmittance $T_{\mathrm{vfs}} = T_{\mathrm{sfv}}$of this later three-layered structure are, for $n_2 > n_{\mathrm{s}}$, given by the following expressions

$$R_{\mathrm{vfs}} = \frac{(n_2-1)^2(n_2+n_{\mathrm{s}})^2 + (n_2+1)^2(n_2-n_{\mathrm{s}})^2 - 2\,(n_2^2-1)(n_2^2-n_{\mathrm{s}}^2)\cos\left(\frac{4\pi n_2 d_2}{\lambda_0}\right)}{(n_2+1)^2(n_2+n_{\mathrm{s}})^2 + (n_2-1)^2(n_2-n_{\mathrm{s}})^2 - 2\,(n_2^2-1)(n_2^2-n_{\mathrm{s}}^2)\cos\left(\frac{4\pi n_2 d_2}{\lambda_0}\right)}$$

$$= \frac{(n_2^2-n_{\mathrm{s}})^2 + n_2^2(n_{\mathrm{s}}-1)^2 - (n_2^2-1)(n_2^2-n_{\mathrm{s}}^2)\cos\left(\frac{4\pi n_2 d_2}{\lambda_0}\right)}{(n_2^2+n_{\mathrm{s}})^2 + n_2^2(n_{\mathrm{s}}+1)^2 - (n_2^2-1)(n_2^2-n_{\mathrm{s}}^2)\cos\left(\frac{4\pi n_2 d_2}{\lambda_0}\right)} \tag{85}$$

$$T_{\mathrm{vfs}} = \frac{16\, n_{\mathrm{s}} n_2^2}{(n_2+1)^2(n_2+n_{\mathrm{s}})^2 + (n_2-1)^2(n_2-n_{\mathrm{s}})^2 - 2\,(n_2^2-1)(n_2^2-n_{\mathrm{s}}^2)\cos\left(\frac{4\pi n_2 d_2}{\lambda_0}\right)}$$

$$= \frac{8\, n_2^2 n_{\mathrm{s}}}{(n_2^2+n_{\mathrm{s}})^2 + n_2^2(n_{\mathrm{s}}+1)^2 - (n_2^2-1)(n_2^2-n_{\mathrm{s}}^2)\cos\left(\frac{4\pi n_2 d_2}{\lambda_0}\right)} \tag{86}$$

4. Optical response of four-layered structures

Consider asolid thin film with a geometric thickness d_2 and a complex index of refraction $\hat{n}_2 \equiv n_2 - j\kappa_2$that is deposited onto an optically-thick non-magneticand homogeneous substrate with a finite geometric thickness d_{s} ($\gg d_2$) and a complex index of refraction $\hat{n}_{\mathrm{s}} \equiv n_{\mathrm{s}} - j\kappa_{\mathrm{s}}$.Let thisfilm-substrate unitto be bounded by two semi-infinite transparentmedia of constant indices of refraction n_1 and n_4 ($\neq n_1$); thus,in the terminology of the present article, one has in hand a four-layered optical structure. Most literaturetreatments of the optical response of four-layered structuresoften presuppose isotropic, non-magnetic and homogenous (in thickness and composition) film and presume that the interfaces of layers in contact with each other are smooth and plane-parallel, that is- an"ideal" four-layered system.

Further, depending on the spectral region of interest, $\kappa_{\mathrm{s}} = 0$ (transparent substrate), or $\kappa_{\mathrm{s}} \ll \kappa_2$ and $\kappa_{\mathrm{s}}^2 \ll n_{\mathrm{s}}^2$(weakly-absorbing substrate), or the substrate may be optically absorbing over a wide range of spectral wavelengths below and/or above the optical absorption edge of the film. Accordingly, the influence of the optical absorption inthe substrate ought to be taken into account in any rigorous mathematical treatment or experimental analysis of the optical response of an "ideal" four-layered structure containing such a partially- or strongly- absorbing substrate.

The formulae of the *total*transmittance $T(\lambda)$and specular reflectance $R(\lambda)$, at a certain light wavelength λ, of "ideal" four-layered structures might not be suitable when one deals with optical four-layered structures including*real*solid films [59-73]. This is because such films may wellsuffer from inhomogeneities of different types (thickness variation, and/or surface roughnessand/or refractive- index fluctuations due to variations in the film composition orclustering), depending on the film material andon the technique employed for its deposition. Thedeparture from "ideal"multi-layered structureshas the effect of altering markedly theirmeasured spectra and optical parameters when are computed from the $R(\lambda)$- and $T(\lambda)$- formulae developed for their counterpart "ideal" systems [35, 59]. Though, the as-measured transmission or specular reflection spectra of "non-ideal" filmscan be analyzed using modified$R(\lambda)$- and $T(\lambda)$- formulae that take into account thesenuisancesto get reliable values for theiroptical parameters. This issue will not be discussed here furtherand intent readers are advised to consult the pertinent literature articles[35, 59-73].

Further, material films of technological importance are usuallydeposited onto thick transparent glass slides or on top of the much more exorbitant quartz or sapphire wafers. Also, among other crystalline substrates, crystalline silicon (Si) wafers are occasionally used as substrates for some films of interest. However, a*thininterfacial* layer of *native* silicon dioxide (SiO_2) is consistentlydeveloped on the surface of the Si-wafer in contact with the deposited film (also on the Si-wafer outer surface), unless extremely precautionary practical measures are taken prior to film deposition. Accordingly, one has to deal with a *five*- or *six-layered* structure, for which the mathematical formulations and analysis of their total transmittance and specular reflectance are much more involved and complicated [26, 28, 81] and the optical response of such multi-layered structures will not be discussed further in the present article.

Additional complication arises when one employs the $R(\lambda)$- and $T(\lambda)$- formulae of an "ideal" four-layered structure to analyze its optical spectra obtained by the use of a *large non-zerospectrophotometer'sspectral bandwidth*(SBW), yet the effect of this could be eliminated in practiceby choosingreasonably small SBWs and/or by correcting its weight in the analysis of the as-measured optical transmission/reflection spectra of the structure under investigation [35].

To proceed further, let us enlighten some other aspects of practically encountered "ideal" optical {layer 1/film/substrate/layer 4}-structures. From a geometrical point of view, the substrate (layer 3) of this four-layered structure now introducesa front interface(boundary) tolayer 4(usually a semi-infinite air medium)besides its back boundarywithone of the surfaces of the film (layer 2), the other (front) surface of which possesses another interface to layer 1, which is typicallya semi-infinite layer of air. In this air/film/thick substrate/air structure, the film and substrate will be assumed to be made of linear, isotropic, homogeneous, non-magnetic and optically transparent and/or partially-absorbing materials.

If all surfaces of the film and substrate of the above-specified four-layered structure are plane-parallel, smooth, homogeneous, and of high optical quality,the light plane waves propagating through each of them will execute multiple back and forth reflections at their respective internal surfaces. Therefore, one hasto include the effect of multiple internal reflections occurring insideboth the film and the substrate in a complete derivation of the spectral $R(\lambda)$- and $T(\lambda)$- formulaedescribing the optical response of such an "ideal" four-layered structure.

For an optically *thin*film deposited on top of a sufficiently *thick* substrate, both having smooth and plane-parallel surfaces, the interference between the internal specularly reflected plane waves travelling inside the thin film cannot be tolerated and its optical behavior must be treated in the framework of the *coherent* formulations already discussed in preceding sections. However, the internal light wave specular reflections taking place inside the thick substrate had tobe treated *incoherently*, as the interference between these reflected waves is normal considered to be of little importance (see details in previous sections). On the other hand, when the film and substrate are both*thick*and have smooth, homogenous and plane-parallel surfaces, the internal light plane wave specular reflections taking place inside each of them willbe treated *incoherently*.

In contrast, if the backside of the substrate in contact with the layer 4 of a four-layered optical structure is rough and unpolished or blackened, the light wave reflections at this back surface will be random; thus, specular reflections from this surface can be neglected and the light beam transmitted from the film into the thick optically absorbing substrate will only suffer from optical absorption during its passage inthe substrate before it emerges to layer 4. So, the formulations of the optical responseof a structure containing such kind of substrate turn out to be relatively less involved than the case when the substrate's backside is of high optical quality.

The complete mathematical formulations of the total *normal-incidence*spectral transmittance and specular reflectance of only two particular types of "ideal" four-layered structures will shortly be derived in detail. The first is that consists of an optically *thin* film deposited onto a thick dielectric substrate, with their free outer surfaces being surrounded by semi-infinite air media, that is- an "ideal"{*air*/*thin film*/*thick dielectric substrate*/*air*}-structure, while the second is that comprising ofa film and a dielectric substrate which are *both*optically thick, that is- having the {*air*/*thick film*/*thick dielectric substrate*/*air*}-piling. In the present article, the former structure will be referred to as *type-A* optical system, while the latter will be termed as *type-B* optical system.

4.1 Spectral transmittance and specular reflectance of {*air*/*film*/thick *substrate*/*air*}-structures

A simple model of the four-layered optical structures is that entails a partially-absorbing or non-absorbing film deposited ontoan opticallythick transparent or weakly-absorbingsubstrate, with the film-substrate unit being immersed in normal air, that is-an {*air*/*film*/*thick substrate*/*air*}-structure. Of course, the materials of both the solid film and the substrate will be presumed to be linear, isotropic, homogeneous (in thickness and composition) and nonmagnetic ($\mu = \mu_0$) and their surfaces in contact with each other to be smooth, homogeneousand plane-parallel. Now, let us derive in detail the full analytical expressions that describe the spectral transmittance and specular reflectance for such an "ideal" {*air*/*film*/*thick substrate*/*air*}-structure.

Before proceeding further, it is important to point out here that as multiple internal specular reflections occur in both of the film and thick substrate of this particular four-layered structure, there are in practice two directions of light propagation and transmission through the film, and this should therefore be equally considered in a complete and rigorous analysis of its optical response.To be more specific, when a light beam is incident on the smooth *air-film* interface from the side of the semi-infinite air medium (layer 1), it will be partially reflected back intothis air layer and the other part will be refractedinto the film (layer 2) toward the opposite internal *film-substrate* interface. This refracted (transmitted) light beam in turn will be partly reflected back inside the film and partly transmitted into the substrate (layer 3). The light beam transmitted inside the substrate will continue propagating towards its internal backside surface that is in contact with the other semi-infinite air medium (layer 4), through which a part of this light plane wave will be transmitted (without backward reflections), while the other part will be reflected back into the substrate, propagating through it in the opposite direction and heading towards its internal boundary with the film. This back reflected light beam will in turn be partly transmitted inside the film and partly reflected back again into the substrate, and so on.

In effect, a monochromatic light plane wave hitting an "ideal" {*air*/*film*/*thick substrate*/*air*}-structure will be repeatedly reflected back and forth at the inner smooth andplane-parallel surfaces of both the film and the substrate. The result is that a fraction of thesespecularly reflected light plane waves will progressively propagate in the forward spatial direction along the path sequence: vacuum (layer 1) → film (layer 2) → substrate (layer 3), called the 123- (*vfs*-) *route*, and will eventually transmitted into the last air medium (layer 4) without being reflected backward, giving rise to the total spectral transmission $T_{1234}$for this four-layered structure. The other fraction of all multiply reflected light beams will return back to the incident air medium (layer 1), also without being reflected backward, via the opposite direction of propagation alongside the path sequence: substrate (layer 3) → film (layer 2) → vacuum (layer 1), labeled by the 321- (*sfv*-) *route*, with asystem's total specular reflectance R_{1234}. The geometrical and symbolical details of the overall optical response of an{*air*/*film*/*thick substrate*/*air*}-structure is clarified in Figure 2shownbelow.

As already discussed the forward and backward specular reflections and transmissions at the interfaces of the film to airand substratealong the opposite *vfs*- and *sfv*- *routes*of lightpropagation give identical values for the transmittancesT_{vfs} and T_{sfv}. In contrast, the specular reflectancesR_{vfs} and R_{sfv}werepreviously shown to be described by *entirelydifferent* expressions and only have the same values if both the film and substrate are transparent in the sense that $\kappa_2 = 0$and $\kappa_{\mathrm{s}} = 0$. When a TE light plane wave of wavelength λ_0 is incident onan {*air*/*film*/*thick substrate*/*air*}-structure from left (see Figure 2), the total transmittance $T_{\mathrm{tot}}(\lambda_0)$ or T_{1234} and specular reflectance $R_{\mathrm{tot}}(\lambda_0)$or $R_{1234}$are determined by a couple of general expressions, which are equally valid whether the TE light plane wave is normally or obliquely incident onto the *air-film* interface, viz.

$$R_{1234} \equiv R_{123} + R_{34}T_{123}T_{321}x_3^2 + R_{34}^2R_{321}T_{123}T_{321}x_3^4 + R_{34}^3R_{321}^2T_{123}T_{321}x_3^6 + R_{34}^4R_{321}^3T_{123}T_{321}x_3^8 + \ldots.$$

$$= R_{123} + R_{34}T_{123}T_{321}x_3^2[1 + R_{34}R_{321}x_3^2 + R_{34}^2R_{321}^2x_3^4 + R_{34}^3R_{321}^3x_3^6 + \ldots.]$$

$$= R_{123} + \frac{R_{34}T_{123}T_{321}x_3^2}{1 - R_{34}R_{321}x_3^2} = R_{123} + \frac{R_{34}T_{123}T_{321}\exp(-2\alpha_{\mathrm{s}}d_{\mathrm{s}})}{1 - R_{34}R_{321}\exp(-2\alpha_{\mathrm{s}}d_{\mathrm{s}})} \qquad (87)$$

$$T_{1234} \equiv T_{34}T_{123}x_3 + T_{34}T_{123}(R_{34}R_{321})x_3^3 + T_{34}T_{123}(R_{34}R_{321})^2x_3^5 + T_{34}T_{123}(R_{34}R_{321})^3x_3^7 + T_{34}T_{123}(R_{34}R_{321})^4x_3^9 + \cdots$$

$$= T_{34}T_{123}x_3[1 + (R_{34}R_{321})x_3^2 + (R_{34}R_{321})^2x_3^4 + (R_{34}R_{321})^3x_3^6 + \ldots.]$$

$$= \frac{T_{34}T_{123}x_3}{1 - R_{34}R_{321}x_3^2} = \frac{T_{123}T_{34}\exp(-\alpha_{\mathrm{s}}d_{\mathrm{s}})}{1 - R_{34}R_{321}\exp(-2\alpha_{\mathrm{s}}d_{\mathrm{s}})} \qquad (88)$$

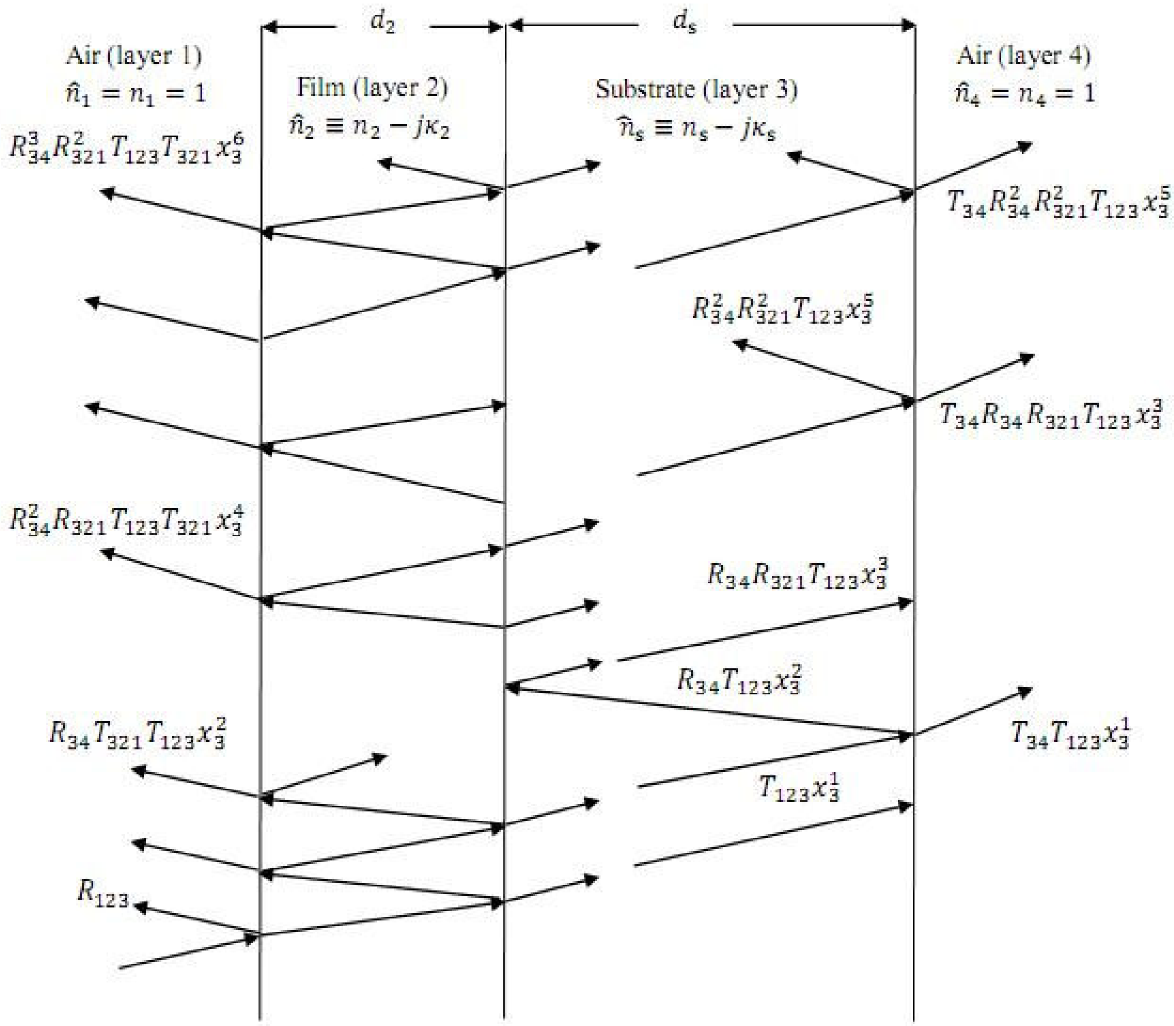

Figure 2:A sketch showing the optical response of an "ideal" {*air*/*film*/*thick substrate*/*air*}-structure. C*oherent* interference between the multiple specular reflections occurring inside the film will be only important if the film is optically thin; otherwise the multiple internal reflections inside both of the film and substrate had to be treated *incoherently*. Here, the absorption parameter$x_3 \equiv \exp\,(-\alpha_s d_s)$and $\alpha_s \equiv 4\pi\kappa_s/\lambda_0$, which is the substrate's optical absorption coefficient. The corresponding parameters for the film will appear explicitly in the derivation details (see text).

The general expressions described by Equations (87) and (88) are identical to the respective mathematical formulationsreferred to in a number of literature articles and advanced books of optics [10, 11, 26-28, 42-44, 51-53] for the total specular reflectance and transmittance of"ideal" four-layered structures similar to that described in Figure 2.Nevertheless, one may notice that there are some differences in writing the final forms of the T_{1234}- and R_{1234}- expressions quoted in such references, and most of these marginally diverse results originate from the different definitions and analytical approaches being adopted to accomplish them. But, it is worth noting here that the numerator of the R_{1234}-expression quoted by Minkov in two of his published papers [51, 52] contains the symbol ρ_{34} to represent the reflectance of the *back substrate-air interface*. Further, the R_{1234}-expressions appear in his papers [51, 52] do not match with each other in the sense that the symbol τ_{34}^2 is quoted in one of them [51] and τ_{34}^4 is written in the other [52].

These discrepancy seems to originate from mistyping and the symbol ρ_{34} in the numerator of Minkov's R_{1234}- expressions should be replaced by ρ_{34}^2 ($\equiv R_{34}$). In the same Minkov publications [51-53], however, these supposed misprints do not appear neither in Minkov's T_{1234}-expressions nor in the denominator of the corresponding R_{1234}-expressions.

4.1.1Normal-incidence transmittance and specular reflectance of type-A optical structures

Now, let us specialize the generalR_{1234}- and T_{1234}- formulasgiven in Equations (87) and (88) to get the full formulae that describe the total reflectance $R_{1234} = R_{\mathrm{vfsv}}$ and transmittance $T_{1234} = T_{\mathrm{vfsv}}$ of an "ideal"$\{$*air*/*thin conductingfilm*/*thick dielectricsubstrate*/*air*$\}$-structure (or the so-called type-A four-layered optical system) when a monochromatic TE light plane wave is incident normally at its*air-film* interface. To realize this, onefirst needsto find the explicit expressions that describe the intensity reflection and transmission coefficients $R_{34} = R_{\mathrm{sv}}$ and $T_{34} = T_{\mathrm{sv}}$ at the back *substrate-air* interface. By making using of Equations (16) - (18) and inserting $\kappa_{\mathrm{s}} = \kappa_{\mathrm{v}} = 0$ and $n_{\mathrm{v}} = 1$ into these equations, onecan readily get the following expressions

$$R_{\mathrm{sv}} = \frac{(n_{\mathrm{s}} - 1)^2}{(n_{\mathrm{s}} + 1)^2} \qquad (89)$$

$$T_{\mathrm{sv}} = 1 - R_{\mathrm{sv}} = \frac{4n_{\mathrm{s}}}{(n_{\mathrm{s}} + 1)^2} \qquad (90)$$

Further, let us put$\kappa_{\mathrm{s}} = \alpha_{\mathrm{s}} = 0$ and $R_{123} = R_{\mathrm{vfs}}$, $R_{321} = R_{\mathrm{sfv}}$, $T_{123} = T_{\mathrm{vfs}} = T_{321} = T_{\mathrm{sfv}}$, which are already derived in Equations (78) – (82) for the "ideal" $\{$*air*/*film*/*thicksubstrate*$\}$-structureonto which monochromatic TE light plane waves are *incidentnormally*, and insertthe values of R_{sv} and T_{sv}described by the above-obtained Equations (89)and (90)into Equations (87) and (88). The resulting formulations will be a couple of comprehensiveexpressions that describe the overall*normal-incidence*specular reflectanceR_{vfsv} and transmittance $T_{\mathrm{vfsv}}$of an "ideal"$\{$*air*/*thinconductingfilm*/*thickdielectricsubstrate*/*air*$\}$-structure and which can be written in a simple symbolic form as shown below

$$R_{\mathrm{vfsv}} = R_{\mathrm{vfs}} + \frac{R_{\mathrm{sv}} T_{\mathrm{vfs}}^2}{1 - R_{\mathrm{sv}} R_{\mathrm{sfv}}} = R_{\mathrm{vfs}} + \frac{(n_{\mathrm{s}} - 1)^2 T_{\mathrm{vfs}}^2}{(n_{\mathrm{s}} + 1)^2 - (n_{\mathrm{s}} - 1)^2 R_{\mathrm{sfv}}} \qquad (91)$$

$$T_{\mathrm{vfsv}} = \frac{T_{\mathrm{sv}} T_{\mathrm{vfs}}}{1 - R_{\mathrm{sv}} R_{\mathrm{sfv}}} = \frac{4\, n_{\mathrm{s}} T_{\mathrm{vfs}}}{(n_{\mathrm{s}} + 1)^2 - (n_{\mathrm{s}} - 1)^2 R_{\mathrm{sfv}}} \qquad (92)$$

Next, let us evaluate the formulae for the total normal-incidence specular reflectance R_{vfsv} and transmittance T_{vfsv} of the type-A four-layered structure in terms of the optical parameters n_2, κ_2, andα_2 or $x_2 \equiv \exp{(-\alpha_2 d_2)}$of its film and n_{s} of its substrate ($\kappa_{\mathrm{s}} = 0$). To accomplish this goal, I have to revert to the original R_{vfs}-, R_{sfv}-and T_{vfs}- expressions given in Equations (78) – (82).

Careful inspection of these expressions ensures that their denominators are all identical and both of their numerators and denominators can be enthusiasticallyhandledtocontain thefactor $\{[(1 + n_2)^2 + \kappa_2^2] * [(n_2 + n_{\mathrm{s}})^2 + \kappa_2^2]\}$, which can then be dropped out from theirfinal forms. The squareroot isalso the same in their numerators and denominators and its argument can be written in a neaterform. Then, Equations (78) –

(81) can be, by making use of somewhat dull mathematical manipulations,shown to reduce to the compact forms as given below

$$R_{\mathrm{vfs}} = \frac{C_1 + C_2 x_2^2 \ + C_{\mathrm{com}}(2x_2 \cos\Delta_1)}{C_3 + C_4 x_2^2 \ + C_{\mathrm{com}}(2x_2 \cos\Delta_2)} \tag{93}$$

$$R_{\mathrm{sfv}} = \frac{C_2 + C_1 x_2^2 \ + C_{\mathrm{com}}(2x_2 \cos\Delta_1')}{C_3 + C_4 x_2^2 \ + C_{\mathrm{com}}(2x_2 \cos\Delta_2)} \tag{94}$$

$$T_{\mathrm{vfs}} = \frac{[16\, n_{\mathrm{s}}(n_2^2 + \kappa_2^2)]x_2}{C_3 + C_4 x_2^2 \ + C_{\mathrm{com}}(2x_2 \cos\Delta_2)} \tag{95}$$

$$C_1 = [(n_2 - 1)^2 + \kappa_2^2] * [(n_2 + n_{\mathrm{s}})^2 + \kappa_2^2] \tag{96}$$

$$C_2 = [(n_2 + 1)^2 + \kappa_2^2] * [(n_2 - n_{\mathrm{s}})^2 + \kappa_2^2] \tag{97}$$

$$C_3 = [(n_2 + 1)^2 + \kappa_2^2] * [(n_2 + n_{\mathrm{s}})^2 + \kappa_2^2] \tag{98}$$

$$C_4 = [(n_2 - 1)^2 + \kappa_2^2] * [(n_2 - n_{\mathrm{s}})^2 + \kappa_2^2] \tag{99}$$

$$C_{\mathrm{com}} = \sqrt{[(n_2^2 + \kappa_2^2 - 1)^2 + 4\kappa_2^2] * [(n_2^2 + \kappa_2^2 - n_{\mathrm{s}}^2)^2 + 4\, n_{\mathrm{s}}^2 \kappa_2^2]} \tag{100}$$

Equations (93) – (95) can readily be simplified further by expressing theircosine termscos Δ_1, $\cos\Delta_1'$, and $\cos\Delta_2$, which are effusivelydescribed in Equations (C1) – (C6) of *AppendixC*, in terms of the above-mentioned optical parameters of the Type-A four-layered structure. The obtained results can then be re-written in the following forms

$$\cos\Delta_1 = \left[\frac{C_1' + 4\, n_{\mathrm{s}}\kappa_2^2}{C_{\mathrm{com}}}\right]\cos\varphi_2 + 2\kappa_2\left[\frac{(n_2^2 + \kappa_2^2 - n_{\mathrm{s}}^2) - n_{\mathrm{s}}(n_2^2 + \kappa_2^2 - 1)}{C_{\mathrm{com}}}\right]\sin\varphi_2 \tag{101}$$

$$\cos\Delta_1' = \left[\frac{C_1' + 4\, n_{\mathrm{s}}\kappa_2^2}{C_{\mathrm{com}}}\right]\cos\varphi_2 \ + 2\kappa_2\left[\frac{n_{\mathrm{s}}(n_2^2 + \kappa_2^2 - 1) - (n_2^2 + \kappa_2^2 - n_{\mathrm{s}}^2)}{C_{\mathrm{com}}}\right]\sin\varphi_2 \tag{102}$$

$$\cos\Delta_2 = \left[\frac{C_1' - 4\, n_{\mathrm{s}}\kappa_2^2}{C_{\mathrm{com}}}\right]\cos\varphi_2 \ - 2\kappa_2\left[\frac{n_{\mathrm{s}}(n_2^2 + \kappa_2^2 - 1) + (n_2^2 + \kappa_2^2 - n_{\mathrm{s}}^2)}{C_{\mathrm{com}}}\right]\sin\varphi_2 \tag{103}$$

It is not difficult to show that the parameter C_1'that appears in the above Equations (101) – (103) can be written in the following expression

$$C_1' = (n_2^2 + \kappa_2^2 - 1) * (n_2^2 + \kappa_2^2 - n_{\mathrm{s}}^2) \tag{104}$$

Armed with the set of Equations (93) – (104), I amnow in a position to re-manipulate Equations (91) and (92) to acquire the formulae that fully describe the total *normal-incidence* specular reflectance R_{vfsv} and transmittance T_{vfsv} of the type-A four-layered structure having the {*air/thin conductingfilm/thick dielectricsubstrate/air*}-piling. Let us first start with the relatively simple T_{vfsv}-expression, which can now be re-written in the form

$$T_{\mathrm{vfsv}} = \frac{\frac{4\, n_{\mathrm{s}}[16\, n_{\mathrm{s}}(n_2^2+\kappa_2^2)]x_2}{C_3+C_4x_2^2+C_{\mathrm{com}}(2x_2\cos\Delta_2)}}{(n_{\mathrm{s}}+1)^2-(n_{\mathrm{s}}-1)^2\left\{\frac{C_2+C_1x_2^2+C_{\mathrm{com}}(2x_2\cos\Delta_1')}{C_3+C_4x_2^2+C_{\mathrm{com}}(2x_2\cos\Delta_2)}\right\}}$$

$$= \frac{4\, n_{\mathrm{s}}[16\, n_{\mathrm{s}}(n_2^2+\kappa_2^2)]x_2}{[C_3(n_{\mathrm{s}}+1)^2-C_2(n_{\mathrm{s}}-1)^2]+x_2^2[C_4(n_{\mathrm{s}}+1)^2-C_1(n_{\mathrm{s}}-1)^2]-2x_2C_{\mathrm{com}}[(n_{\mathrm{s}}-1)^2\cos\Delta_1'-(n_{\mathrm{s}}+1)^2\cos\Delta_2]}$$

(105)

To avoid redundancy in writing the expressions describing the various terms appearing in the final forms of the R_{vfsv}- and T_{vfsv}- formulae of such an “ideal” four-layered structure, the expression given in Equation (105) will now be re-written in a neater form to get the final formula that determinesits totalTE*normal-incidence* transmittance T_{vfsv}[35], viz.

$$T_{\mathrm{vfsv}} = \frac{A_1x}{A_2-A_3x+A_4x^2} \qquad (106)$$

For organized handling of the problem, the absorption parameter $x_2 \equiv \exp(-\,\alpha_2 d_2) \equiv \exp(-\,4\pi\kappa_2 d_2/\lambda_0)$appears in Equation (105) is now inserted in the above-cited Equation (106) as $x \equiv \exp(-\,\alpha d)$. In addition, it will be also more convenient for later tidy writing of the transmittance and reflectance formulations to replace the previous symbols designating the film’s geometrical thickness d_2, its index of refraction n_2, extinction coefficient κ_2 and absorption coefficient α_2 by the following new symbols d, n, κ, and $\alpha(\equiv 4\pi\kappa/\lambda_0)$, respectively. The phase change $\varphi_2 \equiv 4\pi n_2 d_2/\lambda_0$ due to the optical path difference arising from a double traversal of the light wave inside the film is thus re-written as$\varphi \equiv 4\pi n\, d/\lambda_0$. In this new symbol notation, the parameter$A_1$is nowgiven by the following expression

$$A_1 = 16\, n_{\mathrm{s}}(n^2+\kappa^2) \qquad (107)$$

The final forms of the other parameters A_2, A_3, and A_4cited in Equation (106), however,can be only found by a bit tedious mathematical manipulation of the first, second, and third terms of the denominator of Equation (105).The first termcan be simplified, using the expressions of C_2 and C_3 given in Equations (97) and (98), to get the requiredA_2-formula

$$A_2 = [(n+1)^2+\kappa^2] * [(n+1) * (n+n_{\mathrm{s}}^2)+\kappa^2] \qquad (108)$$

Similarly, though the third term in the denominator of Equation (105) is more involved, careful mathematical handling, by making use of Equations (100), (102), and (103)that describe, respectively, the parameters C_{com}, $\cos\Delta_1'$, and $\cos\Delta_2$, enables one to arrive at a well-ordered expression for the term$A_3$of the form

$$A_3 = \{(n^2+\kappa^2-1)*(n^2+\kappa^2-n_s^2)-2\kappa^2(n_s^2+1)\}*(2\cos\varphi)$$
$$-\kappa\{2\,(n^2+\kappa^2-n_s^2)+(n_s^2+1)*(n^2+\kappa^2-1)\}(2\sin\varphi) \quad (109)$$

Lastly, the second term in the denominator of Equation (105) can be worked out, using Equations (96) and (99), to get a simple expression for the parameter $A_4$of Equation (106) as

$$A_4 = [(n-1)^2+\kappa^2]*[(n-1)*(n-n_s^2)+\kappa^2] \quad (110)$$

For purposes to be discussed later, let us at this stage to presume that the film is made from an optically non-absorbing (transparent) dielectricsubstance (at least over the wavelength portion of the electromagnetic spectrum that lies above its optical absorption edge). The resulting meek four-layered optical system has the$\{$*air*/*thin dielectricfilm*/*thick dielectricsubstrate*/*air*$\}$-stacking. Then,by putting$\kappa \cong 0$in the whole set of Equations (107) – (110) cited above, one will develop a somewhat simple form for the total spectral *normal-incidence* transmittance Tproduced bythe "ideal" $\{$*air*/*thin dielectricfilm*/*thick dielectricsubstrate*/*air*$\}$-structure as described below

$$T = \frac{16\,n_s n^2 x}{\{(n+1)^3*(n+n_s^2)\}-2x\cos\varphi\,\{(n^2-1)*(n^2-n_s^2)\}+x^2\{(n-1)^3(n-n_s^2)\}} \quad (111)$$

Next, let us derive the complete expression for the specular normal-incidence reflectance R_{vfsv} of the "ideal" optical system having the above-specified type-A four-layered structure- that is, the $\{$*air*/*thin conductingfilm*/*thick dielectricsubstrate*/*air*$\}$-structure. To attain this aim,one had to substitute the mathematical expressions of R_{vfs}, $R_{\mathrm{sfv}}$and T_{vfs}given above by Equations (93) – (95) into the general Equation (91) and adopt the new designation scheme of the film's parameters described above. After some dreary mathematical handling, it is not difficult to show that the required general R_{vfsv}-formula can be written in the following form

$$R_{\mathrm{vfsv}} = \frac{C_1+C_2x^2+C_{\mathrm{com}}(2x\cos\Delta_1)}{C_3+C_4x^2+C_{\mathrm{com}}(2x\cos\Delta_2)} + \frac{(n_s-1)^2\left\{\frac{[16\,n_s(n^2+\kappa^2)]x_2}{C_3+C_4x^2+C_{\mathrm{com}}(2x\cos\Delta_2)}\right\}^2}{(n_s+1)^2-(n_s-1)^2\left\{\frac{C_2+C_1x_2^2+C_{\mathrm{com}}(2x_2\cos\Delta_1')}{C_3+C_4x_2^2+C_{\mathrm{com}}(2x_2\cos\Delta_2)}\right\}}$$

$$= \left\{\frac{C_1+C_2x^2+C_{\mathrm{com}}(2x\cos\Delta_1)}{C_3+C_4x^2+C_{\mathrm{com}}(2x\cos\Delta_2)}\right\} + \left\{\frac{1}{C_3+C_4x^2+C_{\mathrm{com}}(2x\cos\Delta_2)}\right\}$$

$$*\left\{\frac{(n_s-1)^2[256\,\mathrm{n}_s^2(n^2+\kappa^2)^2x^2]}{(n_s+1)^2[C_3+C_4x_2^2+C_{\mathrm{com}}(2x_2\cos\Delta_2)]-(n_s-1)^2[C_2+C_1x_2^2+C_{\mathrm{com}}(2x_2\cos\Delta_1')]}\right\}$$

$$= D_1 + \left\{\frac{C_5x^2}{D_2}\right\}*\left\{\frac{1}{D_3}\right\} \quad (112)$$

The new parameter C_5appearing in Equation (112) has been chosen to have the form given below

$$C_5 = 64\, n_\mathrm{s}(n_\mathrm{s} - 1)^2(n^2 + \kappa^2)^2 \tag{113}$$

To find the term D_1 of Equation (112), a simple manipulation of the terms $\{C_\mathrm{com} \cos \Delta_1\}$ and $\{C_\mathrm{com} \cos \Delta_2\}$ ought to be made, by making use of Equations (100) - (103). The obtained results of these last terms can be represented bythe following relationships

$$C_\mathrm{com} \cos \Delta_1 = \{C_1' + 4n_\mathrm{s}\kappa^2\} \cos \varphi - 2\kappa\{-(n^2 + \kappa^2 - n_\mathrm{s}^2) + n_\mathrm{s}(n^2 + \kappa^2 - 1)\} \sin \varphi \tag{114}$$

$$C_\mathrm{com} \cos \Delta_2 = \{C_1' - 4n_\mathrm{s}\kappa^2\} \cos \varphi - 2\kappa\{(n^2 + \kappa^2 - n_\mathrm{s}^2) + n_\mathrm{s}(n^2 + \kappa^2 - 1)\} \sin \varphi \tag{115}$$

Thus, the D_1-term can be re-written in a neat form as below

$$D_1 = \frac{A_5 + \{A_6 \cos \varphi - A_7 \sin \varphi\}x + A_8 x^2}{A_9 + \{A_{10} \cos \varphi - A_{11} \sin \varphi\}x + A_{12} x^2} \tag{116}$$

The set of the new parameters A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11}, and A_{12}that appear in the previous Equation (116) are reasonably chosen to be typified by the following compact expressions

$$A_5 = [(n-1)^2 + \kappa^2] * [(n + n_\mathrm{s})^2 + \kappa^2] \tag{117}$$

$$A_6 = 2\{(n^2 + \kappa^2 - 1) * (n^2 + \kappa^2 - n_\mathrm{s}^2) + 4n_\mathrm{s}\kappa^2\} \tag{118}$$

$$A_7 = 4\kappa\{-(n^2 + \kappa^2 - n_\mathrm{s}^2) + n_\mathrm{s}(n^2 + \kappa^2 - 1)\} \tag{119}$$

$$A_8 = [(n+1)^2 + \kappa^2] * [(n - n_\mathrm{s})^2 + \kappa^2] \tag{120}$$

$$A_9 = [(n+1)^2 + \kappa^2] * [(n + n_\mathrm{s})^2 + \kappa^2] \tag{121}$$

$$A_{10} = 2\{(n^2 + \kappa^2 - 1) * (n^2 + \kappa^2 - n_\mathrm{s}^2) - 4n_\mathrm{s}\kappa^2\} \tag{122}$$

$$A_{11} = 4\kappa\{(n^2 + \kappa^2 - n_\mathrm{s}^2) + n_\mathrm{s}(n^2 + \kappa^2 - 1)\} \tag{123}$$

$$A_{12} = [(n-1)^2 + \kappa^2] * [(n - n_\mathrm{s})^2 + \kappa^2] \tag{124}$$

The term D_2 in the denominator of Equation (112) is already evaluated throughout the discussion of the corresponding normal-incidence spectral transmittance formula of the same structure and it is given by the value of the denominator of Equation (106), viz.

$$D_2 = A_2 - xA_3 + x^2 A_4 \tag{125}$$

Finally, the term D_3 of Equation (112) is simply the denominator of the D_1-expression already described in Equation (116) and is expressed by the following formula

$$D_3 = A_9 + \{A_{10} \cos \varphi - A_{11} \sin \varphi\}x + A_{12} x^2 \tag{126}$$

It deserves noting here that the expression for the total *normal-incidence*spectral reflectance R_{vfsv} of the above-specified{*air*/*thin conducting film*/*thick dielectric substrate*/*air*}-structure,and which is detailed inEquation (112),together with all of its parameters being expressed in Equation (113) – (126),is that just cited, in different style of writing the general formulations using diverseset of symbols and defined terms, in addition to some specific model approximations, in a number of literature articles [41, 51, 52, 54, 60, 61].

4.1.2Normal-incidence interference-free transmittance and specular reflectance ofthe type-B four-layered structure with multiple internal reflections insideboth the film and the substrate

Consider a type-B four-layered structure with the {air/thick film/thick dielectric substrate/air}-assembling,in which the dielectric (transparent) substrate(s) has an index of refraction n_{s} and an insignificant extinction coefficient ($\kappa_{\mathrm{s}} \cong 0$), while the optically-absorbing film (*f*) has a complex index of refraction $\hat{n} \equiv n - j\kappa$and an absorption coefficient α ($\equiv 4\pi\kappa/\lambda$), where λ is the wavelength of the monochromatic incident TE light plane wave.Further, assume that both of the substrate and the film are optically thickenough and possess smooth and plane-parallel surfaces. In this case, *incoherent*multiple internal back and forth specular reflections will take place inside the film as well as the substrate since no interference between these reflections is significant. Take into account the optical absorption that may occur within the film, of a geometrical thickness d, and then employthe *incoherent*formalism to derive the total *normal-incidenceinterference-free* transmittance and specular reflectance of such atype-B four-layered structure [28, 83].

Let us now denote,for a monochromaticTE light plane wave propagatingthrough such a structure along the *vfsv-route*, the intensity reflection coefficients at the *air-film*, *film-substrate*, and *substrate-air*interfaces by, in that order,R_{vf}, R_{fs} and R_{sv}, respectively.With the aid of Equation (16), one can write the formulae of these intensity reflection coefficients (for the case$n > n_{\mathrm{s}}$) as

$$R_{\mathrm{vf}} \equiv R_{\mathrm{fv}} = \frac{(n-1)^2 + \kappa^2}{(n+1)^2 + \kappa^2} \tag{127}$$

$$R_{\mathrm{fs}} \equiv R_{\mathrm{sf}} = \frac{(n-n_{\mathrm{s}})^2 + \kappa^2}{(n+n_{\mathrm{s}})^2 + \kappa^2} \tag{128}$$

$$R_{\mathrm{sv}} \equiv R_{\mathrm{vs}} = \frac{(n_{\mathrm{s}}-1)^2}{(n_{\mathrm{s}}+1)^2} \tag{129}$$

Note here that R_{fv}, R_{sf} and R_{vs} are the intensity reflection coefficients at the *air-film*, *film-substrate*, and *substrate-air* interfaces,but forthe light plane wave that is propagating along the opposite *vsfv-route*. The expressions that describe the respectiveintensity transmission coefficients T_{vf}, T_{fs}, T_{sv}, T_{fv}, T_{sf} and T_{vs}at the above-specified three interfaces of the type-B four-layered structure for thetwo opposite *vfsv*-and *vsfv-routes*of the light-wave propagation through the film can readily be shown, by making use of Equations (17) and (18), to have the following forms

$$T_{\mathrm{vf}} \equiv 1 - R_{\mathrm{vf}} = \frac{4\,n}{(n+1)^2 + \kappa^2} \tag{130}$$

$$T_{\mathrm{fv}} = \left(\frac{1}{n}\right)\frac{4\,(n^2+\kappa^2)}{(n+1)^2+\kappa^2} \tag{131}$$

$$T_{\mathrm{fs}} = \left(\frac{n_{\mathrm{s}}}{n}\right)\frac{4\,(n^2+\kappa^2)}{(n+n_{\mathrm{s}})^2+\kappa^2} \tag{132}$$

$$T_{\mathrm{sf}} \equiv 1 - R_{\mathrm{sf}} = \frac{4\,nn_{\mathrm{s}}}{(n+n_{\mathrm{s}})^2+\kappa^2} \tag{133}$$

$$T_{\mathrm{sv}} \equiv T_{\mathrm{vs}} \equiv 1 - R_{\mathrm{sv}} = \frac{4\,n_{\mathrm{s}}}{(n_{\mathrm{s}}+1)^2} \tag{134}$$

It deserves mentioning here that T_{fv} is generally*not* equal to T_{vf}andT_{fs} is *not* equal to T_{sf}except when $n^2 \gg \kappa^2$, a feasible approximation in the visible and near infrared spectral regions for many partially-absorbing or semiconducting solid filmslaid ontotransparent dielectric substrates.Based on this practical model approximation (i.e., $n^2 \gg \kappa^2$) and using the notations adopted for the intensity reflection and transmission coefficients, namely R_{vf}, R_{fs}, T_{vf}, …. R_{vfs}, R_{sfv}, T_{vfs}… etc. and $x \equiv \exp(-\alpha d)$, onecan re-write Equations (19) and (20) for the light plane wave propagating alongthe *vfs-route*and along the opposite *sfv-route* through the film asdescribed below [28]

$$R_{\mathrm{vfs}} = R_{\mathrm{vf}} + \frac{(T_{\mathrm{vf}}R_{\mathrm{fs}}T_{\mathrm{fv}})x^2}{1-(R_{\mathrm{fv}}R_{\mathrm{fs}})x^2} \cong \frac{R_{\mathrm{vf}} - 2R_{\mathrm{vf}}R_{\mathrm{fs}}x^2 + R_{\mathrm{fs}}x^2}{1-R_{\mathrm{vf}}R_{\mathrm{fs}}x^2} \tag{135}$$

$$T_{\mathrm{vfs}} = \frac{(T_{\mathrm{vf}}T_{\mathrm{fs}})x}{1-(R_{\mathrm{fv}}R_{\mathrm{fs}})x^2} \cong \frac{(1-R_{\mathrm{vf}})(1-R_{\mathrm{fs}})x}{1-R_{\mathrm{vf}}R_{\mathrm{fs}}x^2} \tag{136}$$

$$R_{\mathrm{sfv}} = R_{\mathrm{sf}} + \frac{(T_{\mathrm{sf}}R_{\mathrm{fv}}T_{\mathrm{fs}})x^2}{1-(R_{\mathrm{fv}}R_{\mathrm{fs}})x^2} \cong \frac{R_{\mathrm{fs}} - 2\,R_{\mathrm{vf}}R_{\mathrm{fs}}x^2 + R_{\mathrm{vf}}x^2}{1-R_{\mathrm{vf}}R_{\mathrm{fs}}x^2} \tag{137}$$

$$T_{\mathrm{sfv}} = \frac{(T_{\mathrm{sf}}T_{\mathrm{fv}})x}{1-(R_{\mathrm{fv}}R_{\mathrm{fs}})x^2} \cong T_{\mathrm{vfs}} \cong \frac{(1-R_{\mathrm{vf}})(1-R_{\mathrm{fs}})x}{1-R_{\mathrm{vf}}R_{\mathrm{fs}}x^2} \tag{138}$$

Finally, let us now find the total *normal-incidenceinterference-free*spectral transmittance $T_{1234} \Leftrightarrow T_{\mathrm{vfsv}}$and specular reflectance $R_{1234} \Leftrightarrow R_{\mathrm{vfsv}}$of the afore-specified "ideal" type-B four layered structure having the {air/thick weakly-absorbing film/thick dielectric substrate/air}-stacking, wherein *incoherent*multiple internal reflections inside both of the optically thick weakly-absorbing film and non-absorbing substrate being considered. Insert the expressions of R_{vfs}, R_{sfv}, T_{vfs}, and T_{sfv}given in Equations (135) – (138) into the expressions ofR_{vfsv} and T_{vfsv}, given by Equations (91) and (92), to express (for $n^2 \gg \kappa^2$)R_{vfsv} and T_{vfsv}in terms of $x \equiv \exp(-\alpha d)$, R_{vf}, R_{fs}, and R_{sv} by the following neat formulae

$$R_{\mathrm{vfsv}} \cong \frac{R_{\mathrm{vf}} - 2R_{\mathrm{vf}}R_{\mathrm{fs}}x^2 + R_{\mathrm{fs}}x^2}{1-R_{\mathrm{vf}}R_{\mathrm{fs}}x^2} + \frac{R_{\mathrm{sv}}\left[\frac{(1-R_{\mathrm{vf}})(1-R_{\mathrm{fs}})x}{1-R_{\mathrm{vf}}R_{\mathrm{fs}}x^2}\right]^2}{1-R_{\mathrm{sv}}\frac{R_{\mathrm{fs}}-2\,R_{\mathrm{vf}}R_{\mathrm{fs}}x^2+R_{\mathrm{vf}}x^2}{1-R_{\mathrm{vf}}R_{\mathrm{fs}}x^2}}$$

$$\cong \frac{R_{\text{vf}} - 2R_{\text{vf}}R_{\text{fs}}x^2 + R_{\text{fs}}x^2}{1 - R_{\text{vf}}R_{\text{fs}}x^2} + \frac{1}{(1 - R_{\text{vf}}R_{\text{fs}}x^2)} \frac{R_{\text{sv}}(1 - R_{\text{vf}})^2(1 - R_{\text{fs}})^2x^2}{[(1 - R_{\text{fs}}R_{\text{sv}}) - (R_{\text{vf}}R_{\text{fs}} + R_{\text{vf}}R_{\text{sv}} - 2\, R_{\text{vf}}R_{\text{fs}}R_{\text{sv}})x^2]}$$

$$\cong \left[\frac{R_{\text{vf}} + R_{\text{fs}}\,(1 - 2R_{\text{vf}})\,x^2}{1 - R_{\text{vf}}R_{\text{fs}}x^2}\right] + \left[\frac{R_{\text{sv}}}{(1 - R_{\text{sv}})^2(1 - R_{\text{vf}}R_{\text{fs}}x^2)}\right] * \left[\frac{M^2x^2}{U - Wx^2}\right] \qquad (139)$$

$$T_{\text{vfsv}} = \frac{T_{\text{sv}}T_{\text{vfs}}}{1 - R_{\text{sv}}R_{\text{sfv}}} \cong \frac{\frac{(1-R_{\text{vf}})(1-R_{\text{fs}})(1-R_{\text{sv}})x}{1-R_{\text{vf}}R_{\text{fs}}x^2}}{1 - R_{\text{sv}}\frac{R_{\text{fs}}-2\,R_{\text{vf}}R_{\text{fs}}x^2+R_{\text{vf}}x^2}{1-R_{\text{vf}}R_{\text{fs}}x^2}}$$

$$\cong \frac{(1 - R_{\text{vf}})(1 - R_{\text{fs}})(1 - R_{\text{sv}})x}{(1 - R_{\text{vf}}R_{\text{fs}}x^2) - (R_{\text{fs}}R_{\text{sv}} + R_{\text{vf}}R_{\text{sv}}x^2 - 2\, R_{\text{vf}}R_{\text{fs}}R_{\text{sv}}x^2)}$$

$$\cong \frac{(1 - R_{\text{vf}})(1 - R_{\text{fs}})(1 - R_{\text{sv}})x}{(1 - R_{\text{fs}}R_{\text{sv}}) - (R_{\text{vf}}R_{\text{fs}} + R_{\text{vf}}R_{\text{sv}} - 2\, R_{\text{vf}}R_{\text{fs}}R_{\text{sv}})x^2} \qquad (140a)$$

Equation (140a) can be alternatively reassembled to take the following neat form

$$T_{\text{vfsv}} \cong \frac{Mx}{U - Wx^2} \cong \frac{Me^{-\alpha d}}{U - We^{-2\,\alpha d}} \qquad (140b)$$

The parameters M, U, and Wthat appear in Equations (139) and (140) are given by expressions

$$M = (1 - R_{\text{vf}})(1 - R_{\text{fs}})(1 - R_{\text{sv}}) \qquad (141)$$

$$U = 1 - R_{\text{fs}}R_{\text{sv}} \qquad (142)$$

$$W = R_{\text{vf}}R_{\text{fs}} + R_{\text{vf}}R_{\text{sv}} - 2\, R_{\text{vf}}R_{\text{fs}}R_{\text{sv}} \qquad (143)$$

Equations (127) –(129) give the values of the intensity reflection coefficients R_{vf}, R_{fs}, R_{sv}at the *air-film*, *film-substrate*, and *substrate-air* interfaces of the type-B four-layered structure that enterinto its R- and T-formulae,given in Equations (139) and (140),in terms of the optical constants n, κ,$\alpha$and $n_{\text{s}}$of the film and substrate comprising this structure.

It is worth mentioning here that Equations (140a) and (140b) are exactly analogous to the*normal-incidence*T-formulae cited (or evaluated) by Maley [28] and others [82-88], using different symbols for the intensity reflection and transmission coefficients, to describe the optical response of "ideal" type-B four-layered structures. But,it should be pointed out that the *normal-incidence*R_{vfsv}- and T_{vfsv}- formulae given in Equations (139) and (140) can be reached*only*on the basis of the above-constrained model approximations. In other words, these formulae are only valid for an optically thick weakly-absorbing film, with the condition $n^2 \gg \kappa^2$ being fulfilled,which is intimately laid onto thick enough dielectric (transparent) substrate, inside both of which *incoherent* multiple internal specular reflections are taking place at their respective interfaces

It also deserves sayinghere that Equation (140) has been quotedby Swanepoel[35] for describing the total *normal-incidence interference-free transmittance*T_α of a semiconducting (or dielectric) film rested on a thick non-absorbing dielectric substrate. In addition, Swanepoel [35] has derived another form for T_α from the transmission curves displaying interference-fringe maxima T_{M} and minima T_{m} such that $T_\alpha = \sqrt{T_{\text{M}}T_{\text{m}}}$ by integrating, assuming a narrow integration region where all relevant optical parameters to be constant, the formula ofthe interference-fringe transmission curve between a maximum and an adjacent minimum. Nevertheless, the T_α-formula derived by Swanepoel [35] seems to be not usefulfor rigorous quantitative analysis of the transmission curves of these optical four-layered structures to deduce the absorption coefficient in spectral regions of dispersive optical constants of the film where transmission is low. Yet, reasonable consistency can be foundin the high-transmission region between the Swanepoel's*interference-free*T_α-formula and Equation (140)derived for the *normal-incidenceincoherent*transmission of type-B four-layered structures. The latter T-formulais viable over most of the ultraviolet-visible-near infrared (UV-Vis-NIR) portion of the electromagnetic spectrum, butonly when $n^2 \gg \kappa^2$. This is expected to be observed for an optically thick plane-parallel stratified film deposited onto a thick dielectric substrate, or when the thickness of the film is not uniform,or when the film suffers from composition and clustering inhomogeneities. In all these cases, interference fringes would be destroyed yielding smooth experimental transmission curves,which can be reasonably signifiedby the full T_α-formula given in Equation (140). The optical absorption coefficient $\alpha(\lambda)$of the weakly-absorbing film of such a type-B four-layered structure can be determined from the T_α-formula if values of its $n(\lambda)$ and $\kappa(\lambda)$ are known at each spectral wavelength λor, alternatively, when one has in hand their proper dispersion formulae that are effective over a broadspectral wavelength range.Both numerical analysis approaches can also be employed in case of type-A four-layered structures.

5. Conclusions

In the present article, the completemathematical formulae that describe the total *normal-incidence*spectral transmittance $T(\lambda)$and specular reflectance $R(\lambda)$of unsupported (free-standing in air) layers (three-layered structures) and of filmsdeposited on top ofsufficiently thick dielectric substrates immersed in atmospheric air (four-layered structures) have been expansivelyderived. These formulae are specifically valid for linear, isotropic, homogeneous and nonmagnetic and non-conducting (dielectric)or conducting layers,uponwhich monochromatic transverse electric (TE or *s*-polarized) or transverse magnetic (TM or *p*-polarized)light plane waves of specific spectral wavelength $\lambda$are *normally* incident.

In addition to exemplifyingthe main points relevant to the procedure employed in the present article to derive the $T(\lambda)$- and $R(\lambda)$- of "ideal" three- and four- layered structures, a number of arguments and approximations that are commonly implemented in the literature analysis of optical response of multi-layered structures is worth noting and ought to be elucidated here.

First, the *coherent* treatment based on the so-called $E^+ - E^-$ matrix analysis approach has been employed for multi-layered structures incorporating uniform *opticallythin*layers or films, while for *sufficiently thick* films or substrates the *incoherent* analysis of the scalar reflection and transmission intensities of the structure has been utilized instead.The former treatment should be effected when the condition $\Delta\lambda \ll (\lambda^2/2\pi \mathrm{ND})$is fulfilled or, equivalently, when the optical path length ND in the layer (film) is much less than the coherence length l_c ($\equiv \lambda^2/2\pi\Delta\lambda$) of the pseudo-monochromatic incident light beam whose spectral bandwidth (SBW) is $\Delta\lambda$. The symbols N and D designate, respectively, the index of refraction and geometrical thickness of the layer (film) under consideration, The latter approach is usually implemented when $\Delta\lambda \gg (\lambda^2/2\pi \mathrm{ND})$or $\mathrm{ND} \gg l_c$.

Second, the above-stated analytical optical approaches can readily be extended to derive the mathematical $T(\lambda)$- and $R(\lambda)$- formulae (although they are more intricate and involved) to the casesof pseudo-monochromaticTE- (or TM-)light plane waves that are *obliquely*incident onto non-magnetic (or magnetic) three-layered structures as well as films laid on optically transparent or absorbing substrates (four-layered structures) as discussed in [53], and, in principle, for "ideal" multi-layered optical structures with a large number of stacked thin and/or thick layers.

Third, foran "ideal"$\{air/film/thick\ dielectric\ substrate\}$-structurewhose film and substrate have, respectively, the geometrical thicknesses d and d_{s} ($\gg d$), the model approximation $R_{\mathrm{vfs}} \cong R_{\mathrm{sfv}} \cong 1 - T_{\mathrm{vfs}}$can be noted to be crudely valid if one put $x^2 = \exp(-2\alpha d) \sim 1$in the numerators of the R_{vfs}- and R_{sfv}-formulae, given in Equations (78) and (80), and leaving x and x^2 unchanged in their denominators and in the correspondingT_{vfs}- and T_{sfv}- formulae, given in Equations (79) and (81). This timid approximation is problematic if one uses the full Fabry-Pérot (FP) expression for the *normal-incidence* transmittanceT_{vfs} ($= T_{\mathrm{sfv}}$) without eliminating x and x^2[21-24, 58].

Fourth, let one toconsider that $\kappa_2 = \kappa \cong 0$, $\kappa_{\mathrm{s}} = 0$, $n_2 = n$, $\alpha_2 = \alpha = 4\pi\,\kappa/\lambda \neq 0$, $d_2 = d$and $x_2 = x \equiv \exp(-\alpha d) \neq 0$. Then, Equations (C6), (C7)and (82)give$\cos\Delta_2 \cong \cos\phi_2 \cong \cos\phi = \cos(4\pi\,nd/\lambda)$and $A_{\mathrm{com}} \cong (n^2-1)(n^2-n_{\mathrm{s}}^2)$. Insert these last parametersinto Equation (79)or (81) toacquirea less involved formula that describes the total*normal-incidence*transmittance T_{vfs} of an "ideal"type-A structure in terms of n, $n_{\mathrm{s}}(< n)$, ϕ, and x as given below

$$T_{\mathrm{vfs}} \equiv \frac{16n^2 n_{\mathrm{s}} x}{(n+1)^2(n+n_{\mathrm{s}})^2 + x^2(n-1)^2(n-n_{\mathrm{s}})^2 - 2x(n^2-1)(n^2-n_{\mathrm{s}}^2)\cos(4\pi\,nd/\lambda)} \qquad (144)$$

Equation (144) is exclusivelyidentical to the Manifacier's T-formula [45]used to describe the normal-incidence transmissionof a weakly-absorbing thin film sandwiched between two *semi-infinite non-absorbing* media. The Manifacier's T-formula, however, does not take into account the fact that the thickness of a dielectric substrate onto which a thin film is being laidis *notinfinite* but *finite*- that is, a four-layered structure and not a three-layered structure for which Equation (144) has been derived. Thus, it cannot be used, as suggested by Manifacier et al. [45], to describe properly the normal-incidence transmission of a thinuniform film deposited ontop of a thick dielectric substrate of smooth and plane-parallel surfaces. This is because interference-free(*incoherent*) multiple light wave specular reflections at the internal surfaces of the substrate have significant effect on the system's transmission (and reflection). Despite of its impractical constraint, Manifacier's T-formula is frequently employed to analyzethe measured normal-incidence transmission data of thin semiconducting films laid onto plane-parallel *opticallythick* dielectric substrates [46-50].Swanepoel [35] critically analyzed the T-formula of Manifacier et al. [45] and proposedinstead another T-formula for the total optical normal-incidence transmission of a thin conducting (or dielectric) film of a geometric thickness $d$laid on top of an opticallythick dielectric substrate of a geometric thickness d_{s}, with the film-substrate unit being immersed in air. It can be noted that the transmission formula of Swanepoel [35]takes into account both of the interference and absorption of the light plane waves executing multiple internal reflections inside the film, where *coherent*formulations are realized (with the condition $\Delta\lambda \ll (\lambda^2/2nd)$being fulfilled), in addition to the multiple internal reflections occurring in the finitely thick dielectric substrate, in which thesereflections were treated *incoherently*- that is, when $\Delta\lambda \gg (\lambda^2/2n_{\mathrm{s}}d_{\mathrm{s}})$.

In the Swanepoel's paper[35], the normal-incidence optical transmittanceformulae for a weakly-absorbing film laid onto a finitely thick dielectric substrate- that is, type-A optical system, is*just cited*, in terms of n, n_{s}, α or $x \equiv \exp(-\alpha\, d)$ and $\phi \equiv 4\pi nd/\lambda$, as described below

$$T(\lambda) \equiv \frac{16n^2 n_{\mathrm{s}} x}{(n+1)^3(n+n_{\mathrm{s}}^2) + x^2(n-1)^3(n-n_{\mathrm{s}}^2) - 2x(n^2-1)(n^2-n_{\mathrm{s}}^2)\cos\varphi} \qquad (145)$$

In the present article, full derivation of the formulae that describe the optical response of an ideal type-A optical whose thin film may be dielectric or conducting were conducted to arrive at the above-cited Swanepoel's $T(\lambda)$-formula, in addition to the corresponding $R(\lambda)$-formula, which has been quoted, using different notations, definitions, model approximations and formulations, in a number of literature papers[41, 51, 52, 54, 60, 61]. Thorough derivations have been carried out to accomplishthe proper formulations of the spectral$T(\lambda)$ and$R(\lambda)$of such an "ideal" type-A four-layered optical structure, upon which TE light plane wavesof a pseudo-singlewavelength $\lambda$are *normally incident*at the *air-film* interface. The resulting final forms of the normal-incidence $T(\lambda)$- and$R(\lambda)$-formulae for such an optical system are, respectively, summarized in Equation (106) or Equation (145) (Swanepoel's $T(\lambda)$-formula) and Equation (112).

Fifth, in the view of the model approximations$R_{\mathrm{vfs}} \cong R_{\mathrm{sfv}} \cong 1 - T_{\mathrm{vfs}}$, Equations (91) and (92), which determine the total normal-incidence specular reflectance R_{vfsv} and transmittance T_{vfsv} of an "ideal"{*air/thin film/thick dielectric substrate/air*}-structure, can be shown to have the forms

$$R_{\mathrm{vfsv}} = R_{\mathrm{vfs}} + \frac{R_{\mathrm{sv}} T_{\mathrm{vfs}}^2}{1 - R_{\mathrm{sv}} R_{\mathrm{sfv}}} = \frac{(1 - T_{\mathrm{vfs}}) + R_{\mathrm{sv}}(2\,T_{\mathrm{vfs}} - 1)}{1 - R_{\mathrm{sv}}(1 - T_{\mathrm{vfs}})} \tag{146}$$

$$T_{\mathrm{vfsv}} = \frac{T_{\mathrm{sv}} T_{\mathrm{vfs}}}{1 - R_{\mathrm{sv}} R_{\mathrm{sfv}}} = \frac{T_{\mathrm{sv}} T_{\mathrm{vfs}}}{1 - R_{\mathrm{sv}}(1 - T_{\mathrm{vfs}})} \tag{147}$$

Inserting the explicit values of T_{vfs}given in Equation (79)and of R_{sv} and T_{sv}given in Equations(89) and (90), respectively, and putting $\kappa = \kappa_{\mathrm{s}} = 0but\alpha \neq 0$, into Equation (147), onecan arrive at a rather imprecise expression for the total normal-incidence spectral transmittance $T(\lambda)$ of an {*air/thin film/thick dielectric substrate/air*}-structure as given below

$$T(\lambda) = \frac{16 n_{\mathrm{s}} n^2 x}{\{(n+1)^2(n+n_{\mathrm{s}})^2\} - 2x\{2n^2(n_{\mathrm{s}}-1)^2 - (n^2-1)(n^2-n_{\mathrm{s}}^2)\cos\varphi\} + \{(n-1)^2(n-n_{\mathrm{s}})^2\}x^2} \tag{148}$$

In view of the above-specifiedmodel approximations, the corresponding formula that describes the total normal-incidence specular reflectance $R(\lambda)$ of this structure is a little involved. Nevertheless, substituting the value of R_{sv} given in Equation (89) and T_{vfs}given in Equation(79) into Equation (146), one can get an $R(\lambda)$-expression of the form

$$R(\lambda) = \frac{4\mathrm{n_s} + (\mathrm{n_s^2} - 6\mathrm{n_s} + 1)T_{\mathrm{vfs}}}{4\mathrm{n_s} + (\mathrm{n_s} - 1)^2 T_{\mathrm{vfs}}}$$

$$= \frac{\{(n+1)^2(n+n_{\mathrm{s}})^2\} + \{(n-1)^2(n-n_{\mathrm{s}})^2\}x^2 - 2x\{2n^2(n_{\mathrm{s}}^2 - 6n_{\mathrm{s}} + 1) - (n^2-1)(n^2-n_{\mathrm{s}}^2)\cos\phi\}}{\{(n+1)^2(n+n_{\mathrm{s}})^2\} - 2x\{2n^2(n_{\mathrm{s}}-1)^2 - (n^2-1)(n^2-n_{\mathrm{s}}^2)\cos\phi\} + \{(n-1)^2(n-n_{\mathrm{s}})^2\}x^2} \tag{149}$$

It should be stressed again that the $T(\lambda)$-expression described in Equation (148) has been derived in the present work using the assumptions$R_{\mathrm{vfs}} \cong R_{\mathrm{sfv}} \cong 1 - T_{\mathrm{vfs}}$, a model approximation that can only be true when one set the factor exp $(-2\alpha d) \sim 1$ in only the numerators of the original R_{vfs}- and R_{sfv}- formulae given in Equations (78) and (80). These strict assumptions were adopted by Šantić and Scholz [58] who derived a $T(\lambda)$-expression (Equation (4) in their paper [58]), which is exactlysimilarto the above-acquiredEquation (148) for the total normal-incidence transmittance $T(\lambda)$ (= T_{vfsv}) of an "ideal" type-A optical structure. The transmission formula of Šantić and Scholz [58] and the transmission formula described in Equation (148) are both*dissimilar* to the well-known Swanepoel's formula already derived in Equation (145). It is worth noting here that Šantić and Scholz [58] have re-examined and criticized Swanepoel's $T(\lambda)$-expression [35]given in Equation (145), which they also claimedthat they couldn't derive it.

In principle, Equation (148) or the identical $T(\lambda)$-expression of Šantić and Scholz [58] coincides with the exactSwanepoel's formula given in Equation (145) only and only under the conditions $\alpha \cong 0$, $x \cong 1$, a situation that is considered to be true when the thin film of the above-specified four-layered structure is a perfectlytransparent dielectric. However, when optical absorption in the filmis significant, Swanepoel's $T(\lambda)$-formula is exact and realistic, so that it can magnificently be employed, as it is the case since few decades, for describing the total normal-incidence spectral transmittance $T(\lambda)$of numerous "ideal" type-A optical systems. Indeed, Swanepoel's $T(\lambda)$-formula given in Equation (145)is widely used for analyzing the measured normal-incidence $T(\lambda)$- λdata of such layered structures on the basis of the so-called *envelope method*, developed by Manifacier et al. [45] and became well known after Swanepoel [35].

In the present author's opinion, the crude approximation made by Šantić and Scholz [58] and here to arrive at Equation (148) is indeed debatable; thus,these formulas will not lead to the correct analysis of the normal-incidence transmission data except under the restricted hypothetical conditions described previously. In order to obtain a more accurate, though a little involved, description of both the total normal-incidence specular reflectance R_{vfsv} and transmittance T_{vfsv} of the type-A optical structure, one had to go back to their original expressions described by Equations (106) and (112), in which one should insert the unlike functional forms of R_{vfs} and R_{sfv} given in Equations (78) and (80) for the *vfs*- and *sfv*- *directions* of light propagation through the film. The values of the corresponding transmissivities T_{vfs} and T_{sfv} are identically equal when medium 1 is vacuum and the substrate is non-absorbing (dielectric); thus, pose *no* further complications in the procedure of calculating the total optical response of the above-described four-layered structure.

Sixth, in the present article, the complete formulae that describe both of the total normal-incidence *interference-free* optical transmission $T_{\alpha}$and reflection $R_{\alpha}$of an optically*thick absorbing film* laid on an optically*thicktransparentsubstrate*, the free surfaces of which are bounded by air (type-B four-layered structure), werealso derived in the framework of *incoherent* description of the internal multiple specular reflections taking place inside both of the film and substrate. The achieved normal-incidenceR_{α}- and T_{α}-formulae of this type-B four-layered structureare fully described in Equations (139) and (140), with the T_{α}-formula beingremarkably alike to that reportedin literature [28, 82-88].In the optical and infrared regions of the spectrum, this T_{α}-formula may beemployed to determine, within a somewhat large uncertainty, the absorption coefficient $\alpha(\lambda)$ of thick films from their interference-free transmission curves, but is not convenient in case of thin or thick films whose transmission spectra display interference fringes [28, 82-88].

Seventh, one must recall that the index of refraction $n(\lambda)$ and extinction coefficient $\kappa(\lambda)$ of the film (slab) material, or its optical absorption coefficient $\alpha(\lambda)(\equiv 4\pi\kappa(\lambda)/\lambda)$, are generally varying functions in the light wavelength λ- that is, these optical parameters are dispersive quantities. The theoretical or empirical expressions (dispersion relations) that describe the wavelength- dependence of optical constants normally contain twoor moreunknown constants. Thus,to perform computational curve-fitting of the measured reflectance $R(\lambda)$- and transmittance $T(\lambda)$- data ofeven a simple {*air*/*thin film*/*air*}-structure to the theoretical Equations (78) and (79), in which $n_{\text{s}} = 1$, $\kappa_{\text{s}} = 0$,$[\kappa(\lambda)]^2 \ll [n(\lambda)]^2$, $x(\lambda) \equiv \exp(-\alpha(\lambda)d)$, and $\cos\Delta_1 = \cos\Delta_1' = \cos(4\pi\, n(\lambda)d/\lambda) \cong \cos\Delta_2$ are considered, one must insert the proper dispersion relations of the optical parameters into the resulting $R(\lambda)$- and $T(\lambda)$- formulae to determine these unknown constants, which are often taken as the adjustable fitting parameters. Curve-fitting of optical data of structures having higher number of layers to their respective complex theoretical $R(\lambda)$- and $T(\lambda)$- formulaewill be more elaborate, as their optical constants are also complicatedly dependent on the energy of incident light photons. Further, one has to bear in mind that there are a variety of theoretical (empirical)dispersion formulations of several unknown constants that are valid in different regions of the spectrum, depending on the layer's material. This implies that the foundvalues of the adjustable fitting parameters would be controversial and sometimes doubtful, since there is no unique theoretical (empirical) model for describing their actual dispersion.

Besides, the traditional non-linear curve-fitting procedures often yield multi-solutions with different values for the adjustable fitting parameters, with their computed values may also be unrealistic and have no physical meaning. Thus, if one wishes to determine accurate and realistic values of theadjustable parameters, and hence the optical constants of a slab from their measured optical spectra, a powerful non-linear curve-fitting program that gives global solution for the function minimized must be employed. In the simplest multi-layered optical structures like, for example, an air-supported (free-standing in air) weakly-absorbing *thick* film (or slab) of known geometrical thickness, however, no need for a detailed non-linear curve-fitting procedures and one can readily solve analytically, under the model approximation $[\kappa(\lambda)]^2 \ll [n(\lambda)]^2$, the theoretical $R(\lambda)$- and $T(\lambda)$- formulae obtained from Equations (39) and (40)to calculate the values of $n(\lambda)$ and $\kappa(\lambda)$, at each spectral wavelength λ, in terms of the measured $R(\lambda)$- and $T(\lambda)$- data [80]. In "ideal" type-A four-layered structures, one can also employ analytical techniques that are based on the so-called envelope method [35] to calculate the various optical parameters of its film, but this optical analytical approach is limited to films whose $R(\lambda)$and $T(\lambda)$ spectra display a number of interference fringes. More details on the numerical and analytical solutions of the formulations describing the optical spectra of multi-layered spectra will not be given here further.

Appendix A: Format of the obliquely-incidence Fresnel's complex amplitude reflection and transmission coefficients at the interface of two dissimilar conducting media

The standardsets of boundary conditions imposed by Maxwell's equations on the components of electric and magnetic fields that are tangential and normal (perpendicular) to the interface (boundary) of two unlikeintimately adjacent conducting or non-conducting (dielectric)media are well established[9, 14-17, 20-24, 74-76]. In the frame of theE^{+}– E^{-} matrix method [20-23, 29-33, 76], however, it is appropriate to re-write these setsof boundary conditions in a unique compact matrix layout which will be formally similar for both TE and TM electromagnetic (EM) plane waves at any arbitrary angle of incidence. To realize such a reformulation, the boundary conditions will be algebraicallymanipulated so that the information about angles of incidence and refraction of alinearly-polarized light plane wave hitting an interface and the state of wave polarization are all embodied into an *effective indexofrefraction*. This helps one to rephrase the diverse equations of the ratios of the real EMfields of the plane waves reflected and transmitted from a layered structure to the respective real EM-field of the incident plane wave in same matrix form, whether the EM plane wave is a TE or TM plane wave- that is, *s*- or *p*- linearly polarized wave.

To see how this notation is generated, the differently-formatted expressions of the familiar Fresnel's complex amplitude reflection and transmission coefficients for the *s*-(TE) and *p*- (TM)light plane waves *obliquely incident* at the interface of two dissimilar adjacent media have to be reduced to a couple of algebraic expressions of similar arrangement for the two states of wave polarization [9, 21, 27, 29, 32]. Such identical mathematical layouts for both the *obliquely-incident*reflection and transmission coefficients of the *s*- and *p*- plane waves can be realizedby the use of an appropriate expression for the layer's effective index of refraction, which differs for the two states of wave polarization since the reflection and transmission of the components of the respective EM fields are governed by unlike laws [9-11, 13-17, 20-24, 74-76].

The required TE- or TM- formulas of the *obliquely-incident*reflection and transmission coefficients will beformally similar to the *normal-incidence*reflection and transmission coefficients that are already described in the text byEquations (8) and (9), but with the *normal complex indices of refraction* of layersbeing replaced by their respective *effectiverefractive indices*.In the present discussion, the solid layers (whether dielectric or conducting) will bepresumedto be linear, isotropic, homogeneous and nonmagnetic. Further, the incoming electromagnetic radiation will be supposed to be pseudo-monochromatic of a practically single angular frequency ω, that is- of a virtually well-defined spectral wavelength λ ($\equiv 2\pi c/\omega$, where c is the speed of light in free space).

Now, consider a monochromatic TE lightplane wave travelling in the medium $l$that hits the interface of two dissimilar stratified adjacent media l and m of different normal complex indices of refraction $\hat{n}_l$and $\hat{n}_m$. Further, let the complex propagation vector $\hat{\mathbf{k}}_l$of the wave in the l-medium to be inclined at an angle of incidence $\hat{\theta}_l$, with the real unit-vector $\mathbf{n}$normal to the boundary being pointing from the first into the second medium (here, taken to be in the direction of $\mathbf{e}_z$, a real unit-vector along the *positivez*-axis, the direction of stratification of the layers). Sucha TE light plane wave will in general be partly reflected specularly back into medium l with a complex propagation vector $\hat{\mathbf{k}}'_l$for the reflected TE light plane wave inclined to $\mathbf{n}$ at an angle of reflection $\hat{\theta}'_l$ and to be partly refracted into medium m with a complex propagation vector $\hat{\mathbf{k}}_m$for the refracted TE light plane wave making an angle of refraction $\hat{\theta}_m$ with the normal to the$l$-$m$boundary.

It is well known thatfor the case of dielectric stratified media, the indices of refraction n_l and n_m as well as the propagation vectors $\mathbf{k}_l$, $\mathbf{k}'_l$, and $\mathbf{k}_m$of, respectively, the incident, reflected and transmitted light plane waves at the$l$-$m$boundary and the angles of incidence, reflection, and transmission θ_l, θ'_l, and θ_m are *all real quantities*; thus, such angles have geometrical meaning since they can be drawn on a diagram as depicted in Figure A1 for an obliquely-incidence TE light plane wave.However, if media$l$and $m$are conductingwith complex indices of refraction $\hat{n}_l$and $\hat{n}_m$, the complex angles of incidence and refraction *cannot* be drawn geometrically,yetthey are still connected by the *modified* Snell's law $\hat{n}_l \sin\hat{\theta}_l = \hat{n}_m \sin\hat{\theta}_m$. The derivation of algebraic formulations incorporating complex quantities, such as the Fresnel's complex reflection and transmission coefficients and of related optical quantities, *do not appeal to* the geometry of the figure showing the reflection and refraction of a light plane wave incident at the interface between two contactingdielectric and/or conducting media and *remain formally correct*.

It is not difficult to show that the four vectors $\hat{\mathbf{k}}_l$, $\hat{\mathbf{k}}'_l$, $\hat{\mathbf{k}}_m$, and $\mathbf{n}$ are all *coplanar*lying in plane called the *plane of incidence* (taken here to be the Cartesian *xz*-plane), with the direction of the *normal*to this planeis determined from the direction of the *cross-product* of $\mathbf{n}$ ($= \mathbf{e}_z$) and any of these complex wave propagation vectors in the sense that$\mathbf{n} \times \hat{\mathbf{k}}_l$,$\mathbf{n} \times \hat{\mathbf{k}}'_l$ and $\mathbf{n} \times \hat{\mathbf{k}}_m$or in the direction of a real unit-vector $\mathbf{e}_y \equiv (\mathbf{n} \times \hat{\mathbf{k}}_l)/|\hat{\mathrm{k}}_l \sin\hat{\theta}_l|$[9, 76].The complex angles $\hat{\theta}_l$, $\hat{\theta}'_l$ and $\hat{\theta}_m$should also satisfy the relations $\hat{\theta}'_l \equiv \hat{\theta}_l$ (the law of reflection) and $\hat{\mathrm{k}}_l \sin\hat{\theta}_l = \hat{\mathrm{k}}_m \sin\hat{\theta}_m$ or, since $\hat{k} \equiv \hat{n}\,\omega/c$, $\hat{n}_l \sin\hat{\theta}_l = \hat{n}_m \sin\hat{\theta}_m$, the accredited*modified*Snell's law of refraction [9, 76].

As commonly known, none of the above-statedoptical consequences depends on the boundary conditions on theelectromagnetic plane-wave electric and magnetic fields derived from the fundamental Maxwell's Equations. The complex electric-field amplitude vectors $\hat{\mathbf{E}}_{ls}$, $\hat{\mathbf{E}}'_{ls}$and $\hat{\mathbf{E}}_{ms}$ associated with themonochromatic TE (*s*-polarized) light plane waves incident, reflected, and transmitted at the prescribed interface of the two different$l$ and m media are *all perpendicular* to the plane of incidence. In other words, they are all *perpendicular* to the complex propagation vectors $\hat{\mathbf{k}}_l$, $\hat{\mathbf{k}}'_l$, and $\hat{\mathbf{k}}_m$, as well as to the real unit-vector **n**that is normal to the$l$-m boundary(here, the Cartesian *xy*-plane). This implies that the *x*- and *z*-components of the electric-field amplitude vectors of the incident, reflected and transmitted monochromatic TE plane waves are all vanishing in the plane of incidence (here is the Cartesian*xz*-plane); thus, $\hat{\mathbf{E}}_{ls} = \mathbf{s}_l\hat{E}_{ls} = \mathbf{e}_y\hat{E}_{ls}$, $\hat{\mathbf{E}}'_{ls} = \mathbf{s}'_l\hat{E}'_{ls} = \mathbf{e}_y\hat{E}'_{ls}$and $\hat{\mathbf{E}}_{ms} = \mathbf{s}_m\hat{E}_{ms} = \mathbf{e}_y\hat{E}_{ms}$. The corresponding magnetic-field vector amplitudes of these TE plane waves can be found from the relation $\hat{\mathbf{B}} = (\hat{\mathbf{k}} \times \hat{\mathbf{E}})/\omega$, with the real parts of their vector amplitudes of the respective wave are certainly perpendicular to each other.

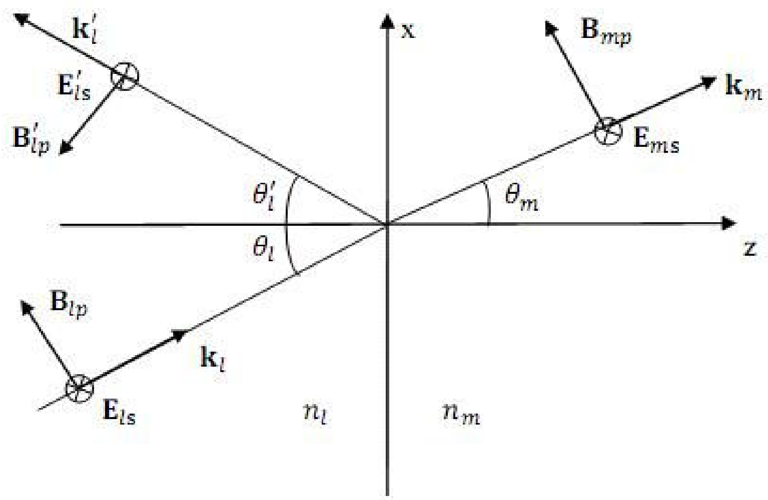

Figure A1.A sketch of the *obliquely-incidence* reflection and refraction of a monochromatic *s*-polarized(TE) electromagnetic plane wave at theinterface (the *xy*-plane) of two unlikedielectric media $l$and m. The *xz*-plane is the plane of incidence and the electric-field vectors$\mathbf{E}_{ls}$, $\mathbf{E}'_{ls}$and $\mathbf{E}_{ms}$ are directed into page, with the vectors **k**, **E**, **B** (in that order) form a *right-handed orthogonal set*.

Apply the customary boundary conditions [9-17, 74-76] imposed on the various components of both the electric-and magnetic-field vector amplitudes that are tangential and perpendicular to the interface of the media l and m (renamed here as$i$and $i+1$, respectively) to find the format of the Fresnel's complex electric-field amplitude reflection and transmission coefficients $(\hat{r}_{lm})_s \equiv (\hat{E}'_{ls}/\hat{E}_{ls})$ and $(\hat{t}_{lm})_s \equiv (\hat{E}_{ms}/\hat{E}_{ls})$ for a TE light plane wave *obliquely incident*at the prescribed interface. In general, the components of any electric-field and magnetic-field vector amplitudes $\hat{\mathbf{E}}$ and $\hat{\mathbf{B}}$ in a medium which are tangential to its boundary with another adjoining medium are given by the vector quantities:$\{-\mathbf{n} \times (\mathbf{n} \times \hat{\mathbf{E}})\}$and $\{-\mathbf{n} \times (\mathbf{n} \times \hat{\mathbf{B}})\}$, while their vector components normal to such a boundary are determined from the vector quantities $\mathbf{n}(\mathbf{n}.\hat{\mathbf{E}})$ and $\mathbf{n}(\mathbf{n}.\hat{\mathbf{B}})$ [9, 74-76].

For a TE (*s*-polarized) electromagnetic plane wave hitting the interface of two adjacent dissimilar media at oblique incidence, *neither* the incident plane TE wave *nor* the associated TE plane waves that are reflected from and transmitted through this interface have electric-field components *normal* to this boundary (or lying in the plane of incidence). Hence, the continuity equation on the electric-field components *normal to the boundary is of no use*.

On the other hand, the continuity condition on the tangential components of the electric-field vector amplitudes $\hat{\mathbf{E}}_{ls} = \mathbf{e}_y\hat{E}_{ls}$, $\hat{\mathbf{E}}'_{ls} = \mathbf{e}_y\hat{E}'_{ls}$and $\hat{\mathbf{E}}_{ms} = \mathbf{e}_y\hat{E}_{ms}$ of the incident, reflected and transmitted TE plane waves at such interface (the *xy*-plane) turns out to be satisfied automatically. Taking $\mathbf{n} = +\mathbf{e}_z$ and assuming that the transmitted wave propagating throughout the m-medium is not reflected back to its boundary (interface) with the l-medium, then the *tangential*(to the boundary) components of the *electric-field vector amplitudes* of these TE plane waves are mutually connected to each otherby the simple relationship

$$\hat{E}_{ls} + \hat{E}'_{ls} = \hat{E}_{ms} \quad \text{(A1)}$$

Since the conducting media under consideration is presumed to be linear, isotropic, homogeneous and nonmagnetic, the continuity of the *tangential*(to the boundary) components $\{-\mathbf{n}\times(\mathbf{n}\times\hat{\mathbf{B}})\}$ of the *magnetic-field vector amplitudes* of these TE light plane waves on either side of the l- m boundaryyields, with the *z*-component of $\hat{\mathbf{k}}'_l$being pointingalong the direction of the *negativez*-axis, another independent expression,recalling that $\hat{\mathbf{B}} = (\hat{\mathbf{k}}\times\hat{\mathbf{E}})/\omega$, with $\hat{k} \equiv \hat{n}\,\omega/c$, relating the corresponding TEwave electric-field vector amplitudes,of the form

$$\hat{n}_l \cos\hat{\theta}_l\,(\hat{E}_{ls} - \hat{E}'_{ls}) = \hat{n}_m \cos\hat{\theta}_m\,\hat{E}_{ms} \quad \text{(A2)}$$

Solve Equations (A1) and (A2) to find the Fresnel's reflection and transmission coefficients $(\hat{r}_{lm})_s \equiv (\hat{E}'_{ls}/\hat{E}_{ls})$ and $(\hat{t}_{lm})_s \equiv (\hat{E}_{ms}/\hat{E}_{ls})$for a TE light plane wave that is obliquely incident at the interface of the two media $l$and m. The result can be expressed in terms of a *complexeffectiveindex of refraction*$\hat{p}_j$ of the medium j, defined as$\hat{p}_j \equiv \hat{n}_j \cos\hat{\theta}_j$, by the formulae [9, 21]

$$\hat{r}_i = (\hat{r}_{lm})_s = \frac{\hat{n}_l\cos\hat{\theta}_l - \hat{n}_m\cos\hat{\theta}_m}{\hat{n}_l\cos\hat{\theta}_l + \hat{n}_m\cos\hat{\theta}_m} = \frac{\hat{p}_i - \hat{p}_{i+1}}{\hat{p}_i + \hat{p}_{i+1}} \quad \text{(A3)}$$

$$\hat{t}_i = (\hat{t}_{lm})_s = \frac{2\,\hat{n}_l\cos\hat{\theta}_l}{\hat{n}_l\cos\hat{\theta}_l + \hat{n}_m\cos\hat{\theta}_m} = \frac{2\,\hat{p}_i}{\hat{p}_i + \hat{p}_{i+1}} \quad \text{(A4)}$$

The corresponding Fresnel's complex electric-field amplitude reflection and transmission coefficients for the *oppositedirection* of *propagation* of the TE light plane wave through the interface of these adjoining l- and m-media can be shownto satisfy the expressions

$$\hat{r}'_i = (\hat{r}_{ml})_s = -\,\hat{r}_i = \frac{\hat{n}_m\cos\hat{\theta}_m - \hat{n}_l\cos\hat{\theta}_l}{\hat{n}_l\cos\hat{\theta}_l + \hat{n}_m\cos\hat{\theta}_m} = \frac{\hat{p}_{i+1} - \hat{p}_i}{\hat{p}_i + \hat{p}_{i+1}} \quad \text{(A5)}$$

$$\hat{t}'_i = (\hat{t}_{ml})_s = \frac{2\,\hat{n}_m\cos\hat{\theta}_m}{\hat{n}_l\cos\hat{\theta}_l + \hat{n}_m\cos\hat{\theta}_m} = \frac{2\,\hat{p}_{i+1}}{\hat{p}_i + \hat{p}_{i+1}} \quad \text{(A6)}$$

For a linearly *p*-polarized electromagnetic plane wave hitting an interface of two dissimilar media l and m at oblique incidence, the electric-field vector amplitudes $\hat{\mathbf{E}}_{lp}$, $\hat{\mathbf{E}}'_{lp}$and $\hat{\mathbf{E}}_{mp}$ of the incident, reflected and transmitted *p*-polarized plane waves are *allparallel* to (or lie in) the plane of incidence, which contains their complex wave propagation vectors $\hat{\mathbf{k}}_l$, $\hat{\mathbf{k}}'_l$, $\hat{\mathbf{k}}_m$and $\mathbf{n}$. *Conventionally*, the Fresnel's complex amplitude coefficients of reflection and transmission at the interface of two different media are expressed in terms of the ratios of amplitudes of the electric (*not magnetic*) vector fields of the reflected and transmitted plane waves to the amplitude of the electric vector field of the incident plane wave. Specifically, the Fresnel's coefficients for reflection and transmission of p-polarizedelectromagneticplane waves at such$l$-$m$interface are often expressed by the relations:$(\hat{r}_{lm})_{\mathrm{p}} \equiv (\hat{E}'_{l\mathrm{p}}/\hat{E}_{l\mathrm{p}})$ and $(\hat{t}_{lm})_{\mathrm{p}} \equiv (\hat{E}_{m\mathrm{p}}/\hat{E}_{l\mathrm{p}})$ [9-17, 20-24, 74-76].

It is known that the fields $\hat{\mathbf{E}}(\mathbf{r},t)$ and $\hat{\mathbf{B}}(\mathbf{r},t)$ of a time-harmonic monochromaticelectromagnetic plane wave of an angular frequency ω travelling in a linear and non-magnetic conducting medium of a complex index of refraction $\hat{n}$are both perpendicular (if $\hat{\epsilon}_{\mathrm{r}} \neq 0$), to its propagation vector $\hat{\mathbf{k}}$ ($\equiv \mathbf{k}_{\mathrm{r}} - j\mathbf{k}_{\mathrm{i}}$, where $\mathbf{k}_{\mathrm{r}}$ and $\mathbf{k}_{\mathrm{i}}$ are its real and imaginary vector parts). That is, the wave is transverse ($\hat{\mathbf{k}}.\hat{\mathbf{E}} = 0 = \hat{\mathbf{k}}.\hat{\mathbf{B}}$), where $\hat{\mathbf{E}}$ and $\hat{\mathbf{B}}$ arethe complex vector amplitudes of $\hat{\mathbf{E}}(\mathbf{r},t)$ and $\hat{\mathbf{E}}(\mathbf{r},t)$ and which are linked by $\hat{\mathbf{B}} = (\hat{\mathbf{k}} \times \hat{\mathbf{E}})/\omega$or $\hat{\mathbf{E}} = -[\omega/(\hat{\mathbf{k}}.\hat{\mathbf{k}})](\hat{\mathbf{k}} \times \hat{\mathbf{B}}) = -(c/\omega\,\hat{n}^2)(\hat{\mathbf{k}} \times \hat{\mathbf{B}})$.But, $\hat{\mathbf{E}}(\mathbf{r},t)$ and $\hat{\mathbf{B}}(\mathbf{r},t)$ and their real partsare *perpendicularto each other* if their *plane wave is only linearly polarized*even if the directions of $\mathbf{k}_{\mathrm{r}}$ and $\mathbf{k}_{\mathrm{i}}$are different.

Thus, when a linearly p-polarized monochromatic electromagneticplane waveis*obliquelyor normallyincident*atthe interface of two dissimilar linear, homogeneous and nonmagnetic conducting$l$- and$m$-media,its magnetic-field vector amplitude and those of the correspondingreflected and transmitted linearly p-polarized plane waves will be *perpendicular* to the plane of incidence, which contains, in addition to $\mathbf{n}$, their propagation wave vectors $\hat{\mathbf{k}}_l$, $\hat{\mathbf{k}}'_l$, and $\hat{\mathbf{k}}_m$. This is why a linearly p-polarized plane electromagnetic wave is often called a *transverse magnetic (*TM*) wave*. So, the magnetic-field vector amplitudes $\hat{\mathbf{B}}_{ls}$, $\hat{\mathbf{B}}'_{ls}$and $\hat{\mathbf{B}}_{ms}$ of the incident, reflected and transmitted TM plane waves are all *normal* to the plane of incidence (the *xz*-plane) and can readily be expressed by the relationships $\hat{\mathbf{B}}_{ls} = \mathbf{e}_{\mathrm{y}}\hat{B}_{ls}$, $\hat{\mathbf{B}}'_{ls} = \mathbf{e}_{\mathrm{y}}\hat{B}'_{ls}$and $\hat{\mathbf{B}}_{ms} = \mathbf{e}_{\mathrm{y}}\hat{B}_{ms}$.

The commonly adopted derivation of $(\hat{r}_{lm})_{\mathrm{p}}$ and $(\hat{t}_{lm})_{\mathrm{p}}$ coefficients for a monochromatic TMelectromagnetic plane wave hitting the interface of two media l and m is based on imposing the boundary conditions on the components of its respective fields tangential and normal to the interface to obtain the formulae relating the magnetic-field vector amplitudes $\hat{\mathbf{B}}_{ls}$, $\hat{\mathbf{B}}'_{ls}$and $\hat{\mathbf{B}}_{ms}$. The traditional way is then to write the Fresnel's coefficients $(\hat{r}_{lm})_{\mathrm{p}}$ and $(\hat{t}_{lm})_{\mathrm{p}}$ as explicit ratios of the magnitudes of the $\hat{\mathbf{E}}$-vectors of the TM plane waves as$(\hat{r}_{lm})_{\mathrm{p}} \equiv (\hat{E}'_{l\mathrm{p}}/\hat{E}_{l\mathrm{p}}) \equiv (\hat{B}'_{ls}/\hat{B}_{ls})$and $(\hat{t}_{lm})_{\mathrm{p}} \equiv (\hat{E}_{m\mathrm{p}}/\hat{B}_{l\mathrm{p}}) \equiv (\hat{n}_l/\hat{n}_m)(\hat{B}_{ms}/\hat{B}_{ls})$[9]. To generalize the E^{+}– E^{-}transfer matrix method so the general formulations describing the optical response of a multi-layered structure will be formally identical for both the TE and TM plane waves, $(\hat{r}_{lm})_{\mathrm{p}}$ and $(\hat{t}_{lm})_{\mathrm{p}}$shouldbe expressed in terms of*effective indices of refraction*that are different to those adopted in Equations (A3) – (A6) for the TE plane wave case, where an effective index of refraction for the medium $j$of the form$\hat{p}_j \equiv \hat{n}_j \cos\hat{\theta}_j$isused. This is simply attained by defining the Fresnel's complex amplitude coefficients of reflection and transmission of a linearly p-polarized (TM) plane wave obliquely incident at the l-$m$interface as explicit ratios of the *magnetic-field* (*not electric-field*) vector amplitudes $\hat{B}'_{ls}$ and $\hat{B}_{ms}$ of the reflected and transmitted TM plane waves to the magnetic-field vector $\hat{B}_{ls}$ of the incident TM plane wave, viz.:$(\hat{r}_{lm})_{\mathrm{p}} \equiv (\hat{B}'_{ls}/\hat{B}_{ls})$ and $(\hat{t}_{lm})_{\mathrm{p}} \equiv (\hat{B}_{ms}/$ *Bls*[9].

Impose the boundary conditions on the field components of the TM plane waves incident, reflected and transmitted at the interface of the two non-magnetic$l$- and m-media of complex indices of refraction $\hat{n}_l$ and$\hat{n}_m$that are tangential and normal to their interfaceto obtain a couple of independent relations connecting their$\hat{\mathbf{B}}_{ls}$, $\hat{\mathbf{B}}'_{ls}$and $\hat{\mathbf{B}}_{ms}$. Recalling that the vectors $\hat{\mathbf{k}}_l$, $\hat{\mathbf{k}}'_l$, $\hat{\mathbf{k}}_m$, and $\mathbf{n}$ are *all lying* in the plane of incidence, this can berealized by using the relation $\hat{\mathbf{E}} = -[\omega/(\hat{\mathbf{k}}.\hat{\mathbf{k}})](\hat{\mathbf{k}} \times \hat{\mathbf{B}}) = -(c/\omega\,\hat{n}^2)(\hat{\mathbf{k}} \times \hat{\mathbf{B}})$, in conjunction with the familiar BAC-CAB rule; so, $\hat{\mathbf{k}}_l.\hat{\mathbf{B}}_{ls} = \hat{\mathbf{k}}'_l.\hat{\mathbf{B}}'_{ls} = \hat{\mathbf{k}}_m.\hat{\mathbf{B}}_{ms} = 0$. The results can be expressed by the following formulae

$$\frac{\cos\hat{\theta}_l}{\hat{n}_l}\left(\hat{\boldsymbol{B}}_{ls}-\hat{\boldsymbol{B}}'_{ls}\right)=\frac{\cos\hat{\theta}_m}{\hat{n}_m}\hat{\boldsymbol{E}}_{ms} \quad \text{(A7)}$$

$$\hat{\boldsymbol{B}}_{ls}+\hat{\boldsymbol{B}}'_{ls}=\hat{\boldsymbol{B}}_{ms} \quad \text{(A8)}$$

To formulate the Fresnel's reflection and transmission coefficients of a TMelectromagnetic plane wave hitting the interface of two different media at oblique incidence, I shall refer to the *effectiveindexofrefraction* of a medium $j$as$\hat{q}_j=\cos\hat{\theta}_j/\hat{n}_j$[9]and not$\hat{q}_j=\hat{n}_j/\cos\hat{\theta}_j$(quoted in reference [21]), which is not suitable for describing allTM-wave Fresnel's reflection and transmission coefficients. Solving Equations (A7) and (A8) with$\hat{q}_j=\cos\hat{\theta}_j/\hat{n}_j$, one can express theseobliquely incident TM-wave Fresnel's coefficients at the interface of the media l ($\Rightarrow i$) and m ($\Rightarrow i+1$), for the opposite directions of wave propagation through such$l$-$m$interface, by [9]

$$\hat{r}_i=(\hat{r}_{lm})_{\mathrm{p}}=\frac{\frac{\cos\hat{\theta}_l}{\hat{n}_l}-\frac{\cos\hat{\theta}_m}{\hat{n}_m}}{\frac{\cos\hat{\theta}_l}{\hat{n}_l}+\frac{\cos\hat{\theta}_m}{\hat{n}_m}}=\frac{\hat{q}_i-\hat{q}_{i+1}}{\hat{q}_i+\hat{q}_{i+1}} \quad \text{(A9)}$$

$$\hat{r}'_i=(\hat{r}_{ml})_{\mathrm{p}}=-\,\hat{r}_i=\frac{\frac{\cos\hat{\theta}_m}{\hat{n}_m}-\frac{\cos\hat{\theta}_l}{\hat{n}_l}}{\frac{\cos\hat{\theta}_l}{\hat{n}_l}+\frac{\cos\hat{\theta}_m}{\hat{n}_m}}=\frac{\hat{q}_{i+1}-\hat{q}_i}{\hat{q}_i+\hat{q}_{i+1}} \quad \text{(A10)}$$

$$\hat{t}_i=(\hat{t}_{lm})_{\mathrm{p}}=\frac{2\,\frac{\cos\hat{\theta}_l}{\hat{n}_l}}{\frac{\cos\hat{\theta}_l}{\hat{n}_l}+\frac{\cos\hat{\theta}_m}{\hat{n}_m}}=\frac{2\,\hat{q}_i}{\hat{q}_i+\hat{q}_{i+1}} \quad \text{(A11)}$$

$$\hat{t}'_i=(\hat{t}_{ml})_{\mathrm{p}}=\frac{2\,\frac{\cos\hat{\theta}_m}{\hat{n}_m}}{\frac{\cos\hat{\theta}_l}{\hat{n}_l}+\frac{\cos\hat{\theta}_m}{\hat{n}_m}}=\frac{2\,\hat{q}_{i+1}}{\hat{q}_i+\hat{q}_{i+1}} \quad \text{(A12)}$$

Note that if the numerator of the middle term of Equation (A11) is multipliedby the quotient $(\hat{n}_l/\hat{n}_m)$, onecan getthe traditional expression cited for the Fresnel's complex amplitude transmission coefficient at the interface of the two dissimilar media l and m when $(\hat{t}_{lm})_{\mathrm{p}}$is defined relative to the TMelectric-field vector amplitudes as$(\hat{t}_{lm})_{\mathrm{p}}\equiv\left(\hat{E}_{m\mathrm{p}}/\hat{E}_{l\mathrm{p}}\right)$[9, 20-24, 74-76].

Appendix B: The $E^{+}-E^{-}$transfer matrix for transmission and specular reflection coefficients of multi-layered structures

Writing the Fresnel's complex amplitude reflection and transmission coefficients of bothTE and TM monochromatic electromagnetic plane waves that are obliquely-incident at the interface of two unlike media in identical formatsincluding only theirrespective effective indices of refraction will allow the use of unlimited number of interfaces of adjoining stratified layers of a multi-layered structure. This common notation is valuable in the development of the $E^{+}-E^{-}$transfer matrix method [20-23, 29-33, 76]andother methods[9-11]. These Fresnel's complex reflection and transmission coefficients will bedefined asthe ratios of the amplitudes of electric-field vectors and of the amplitudes of magnetic-field vectors in the TE and TM cases, respectively[9].

Now, consider a pile of j-layered structure with $(j-1)$ interfaces separating j different layers with normal complex indices of refraction $\hat{n}_i$ (i = 1, 2, …..j – 1,and$j$), with $\hat{n}_1$being the index of refraction of the *first*layer in which a light plane wave is propagating toward its interface to layer 2 of anindex of refraction $\hat{n}_2$etc. Multiple internal specular reflections may occur inside all layers and if one layer isoptically thin, interference effects between these reflections are significant. In a pile of j successive different layers, the second layer to the nextith-*layer* has two interfaces which are formally identical for all in-between layers of the entire structure. Thus,one needs only to configure the problem for a generalized interface and then repeat the calculation $(j-1)$ times to include the effect of the $(j-1)$ interfaces of the entire stack of j layers, as shown in Figure B1.

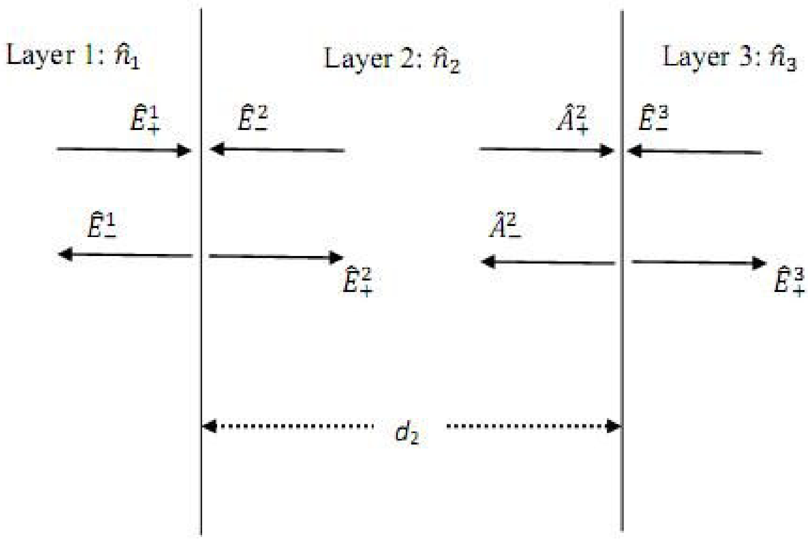

Figure B1.A geometrical sketch for light plane waves propagating in homogeneous different layers, in which light waves execute multiple internal reflections and/or absorption

Recall here that the state of plane-wave polarization and both the angles of incidence and refraction are hidden in the new common format, in which the Fresnel's complex amplitude reflection and transmission coefficients are defined in terms of the effective indices of refraction of the media, as described in Equations (A3) – (A6) and Equations (A9) – (A12). Then, all reflected and transmitted monochromatic TE (or TM) lightplane waves can be represented by their respective fields normally incident at each side of the interface of two adjacent media, as depicted in Figure B1 for the case of TE light waves.Impose, the boundary conditions on the electric fields reflected and transmitted through a generalized interface ofthe two media i and $i+1$of Figure B1to obtain the formulations relating the electric fields of the incident, reflected, transmitted TE plane waves at such interface in terms of $\hat{p}_i \equiv \hat{n}_i \cos\hat{\theta}_i$ and $\hat{p}_{i+1} \equiv \hat{n}_{i+1} \cos\hat{\theta}_{i+1}$as[21]

$$\hat{p}_i\left(\hat{E}^{\mathrm{i}}_{+} - \hat{E}^{\mathrm{i}}_{-}\right) = \hat{p}_{i+1}(\hat{E}^{\mathrm{i+1}}_{+} - \hat{E}^{\mathrm{i+1}}_{-}) \tag{B1}$$

$$\hat{E}^{\mathrm{i}}_{\mathrm{s}+} + \hat{E}^{\mathrm{i}}_{\mathrm{s}-} = \hat{E}^{\mathrm{i+1}}_{\mathrm{s}+} + \hat{E}^{\mathrm{i+1}}_{\mathrm{s}-} \tag{B2}$$

Solution of Equations (B1) and (B2) yields a couple of relations for the electric-field components of the obliquely-incidence monochromatic*s*-polarized (TE) light plane wave of the forms

$$\hat{E}_{s+}^{i} = \left(\frac{\hat{p}_i + \hat{p}_{i+1}}{2\,\hat{p}_i}\right)\hat{E}_{s+}^{i+1} + \left(\frac{\hat{p}_i - \hat{p}_{i+1}}{2\,\hat{p}_i}\right)\hat{E}_{s-}^{i+1} \tag{B3}$$

$$\hat{E}_{s-}^{i} = \left(\frac{\hat{p}_i - \hat{p}_{i+1}}{2\,\hat{p}_i}\right)\hat{E}_{s+}^{i+1} + \left(\frac{\hat{p}_i + \hat{p}_{i+1}}{2\,\hat{p}_i}\right)\hat{E}_{s-}^{i+1} \tag{B4}$$

Using the formats ofthe Fresnel's complex electric-field amplitude reflection and transmission coefficients described in Equations (A3) and (A6) for TE light plane waves, one can simplify the notationsof Equations (B3) and (B4) to give two general independent equations of the form[21]

$$\hat{E}_{+}^{i} = \frac{\hat{E}_{+}^{i+1} + \hat{r}_i\hat{E}_{-}^{i+1}}{\hat{t}_i} \tag{B5}$$

$$\hat{E}_{-}^{i} = \frac{\hat{r}_i\hat{E}_{+}^{i+1} + \hat{E}_{-}^{i+1}}{\hat{t}_i} \tag{B6}$$

Equations (B5) and (B6) can be combined into a *uniquematrixequation* for the interface separating two successive dissimilar optical media designated by the letters $i$and $i+1$ of the form

$$\begin{pmatrix}\hat{E}_{+}^{i}\\ \hat{E}_{-}^{i}\end{pmatrix} = \begin{pmatrix}\frac{1}{\hat{t}_i} & \frac{\hat{r}_i}{\hat{t}_i}\\ \frac{\hat{r}_i}{\hat{t}_i} & \frac{1}{\hat{t}_i}\end{pmatrix}\begin{pmatrix}\hat{E}_{+}^{i+1}\\ \hat{E}_{-}^{i+1}\end{pmatrix} \tag{B7}$$

Normally, Equation (B7) is written in a more compact matrix formulation as below

$$\hat{\boldsymbol{E}}^{i} \equiv \hat{\boldsymbol{I}}_i\hat{\boldsymbol{E}}^{i+1} \tag{B8}$$

The operator $\hat{\boldsymbol{I}}_i$in Equation (B8) represents the *ith-interface* Fresnel's coefficients matrix, which joins the Fresnel's complex reflection and transmission coefficients $\hat{r}_i$ and $\hat{t}_i$ at the ith-interface of the $i$and $i+1$ layers with $\hat{r}_i = \hat{r}_{i,\ i+1}$ and $\hat{t}_i = \hat{t}_{i,\ i+1}$ in the manner given below

$$\hat{\boldsymbol{I}}_i = \begin{pmatrix}\frac{1}{\hat{t}_i} & \frac{\hat{r}_i}{\hat{t}_i}\\ \frac{\hat{r}_i}{\hat{t}_i} & \frac{1}{\hat{t}_i}\end{pmatrix} \tag{B9}$$

When the electromagnetic planewave crossesan interface tothe neighboring layer, it could be partially absorbed and/or interfere with the back and forth reflected plane waves traversing the layer. Both of these phenomena are often incorporated in terms of a complex phase change $\hat{\delta}$of the electric field (or magnetic field) of the light plane wave traversing the layer. In general, for a lightplane wave of anangular frequency ω (wavelength λ_0) impinging at an angle of incidence $\hat{\theta}$at an interface of a layer ofthickness d and a complex index of refraction $\hat{n} = n - j\kappa$, $\hat{\delta}$ produced upon a *single traversal* of the wave in the layer is $\hat{\delta} = d(\omega/c)\hat{n}\cos\hat{\theta} = (2\,\pi d/\lambda_0) * (u - jv)$, where u and v are real quantities which are, respectively, equal to

n and κ at normal incidence [9, 76]. This helps, as discussed shortly,to find the wave field quantities $\hat{A}_+$and $\hat{A}_-$, which are clearlydepicted as $\hat{A}_+^2$ and $\hat{A}_-^2$for layer2 of Figure B1.

The electromagnetic wave fields $\hat{A}_+^2$ and $\hat{E}_-^2$ in a particular conducting (or dielectric) layer (layer 2 in Figure B1) must thus be modified by the phase shift (change) $\hat{\delta}_2$ they experience after traversing *once* through layer 2that has a finite geometrical thickness d_2 and a normal complex index of refraction $\hat{n}_2$. For the monochromatic TE electromagnetic plane waves that are incident from the semi-infinite media 1 (Layer 1 in Figure B1) atits interface to layer 2, these wave-field quantities can then be expressed by the following set of relations

$$\hat{A}_+^2 \equiv e^{j\hat{\delta}_2}\hat{E}_+^2\hat{E}_-^2 \equiv e^{j\hat{\delta}_2}\hat{A}_-^2 \quad \text{or} \quad \hat{A}_-^2 \equiv e^{-j\hat{\delta}_2}\hat{E}_-^2 \tag{B10}$$

Equation (B10) can easily be manipulated in terms of a simple 2 x 2 *matrix*of the form

$$\begin{pmatrix} \hat{A}_+^2 \\ \hat{A}_-^2 \end{pmatrix} = \begin{pmatrix} e^{j\hat{\delta}_2} & 0 \\ 0 & e^{-j\hat{\delta}_2} \end{pmatrix} \begin{pmatrix} \hat{E}_+^2 \\ \hat{E}_-^2 \end{pmatrix} \tag{B11}$$

For the layer 2 in question, Equation (B11) can be written in a more compact matrix form as

$$\hat{\boldsymbol{A}}^2 \equiv \hat{\boldsymbol{T}}_2\hat{\boldsymbol{E}}^2 \tag{B12}$$

One can generalize Equation (B12) for the ith-layeras $\hat{\boldsymbol{A}}^{\mathrm{i}} \equiv \hat{\boldsymbol{T}}_{\mathrm{i}}\hat{\boldsymbol{E}}^{\mathrm{i}}$, with the so-called *transmission* matrix $\hat{\boldsymbol{T}}_{\mathrm{i}}$being defined by the following matrix expression

$$\hat{\boldsymbol{T}}_{\mathrm{i}} = \begin{pmatrix} e^{j\hat{\delta}_{\mathrm{i}}} & 0 \\ 0 & e^{-j\hat{\delta}_{\mathrm{i}}} \end{pmatrix} \tag{B13}$$

The final result for a stack of $j$successive stratified layers with$j-1$intimately contacted smoothand homogeneous interfaces, with the layers index $j = 2, 3, 4, \dots ....$, can be formulated in a single general matrix equation as given by

$$\begin{pmatrix} \hat{E}_+^1 \\ \hat{E}_-^1 \end{pmatrix} = \boldsymbol{M} \begin{pmatrix} \hat{E}_+^f \\ \hat{E}_-^f \end{pmatrix} \tag{B14}$$

The fields $\hat{E}_+^1$ and $\hat{E}_-^1$are, respectively, the electric field components travelling through and reflected from the first (incident) layer, while the fields $\hat{E}_+^f$ and $\hat{E}_-^f$are, respectively, the electric field components transmitted into and reflected from the final (last) layer of the jth-layered stack.

Let us now find the expression that represents the matrix $\boldsymbol{M}$ for the $(j-1)$ interfaces separating the j dissimilar dielectric and/or conducting stratified layers of this jth-layered stack, including the (first) layer from which the light wave is being incident onto the second layer of the structure. Thisgeneral matrix $\boldsymbol{M}$ (usually referred to as the *characteristic* matrix of the j-layered structure) can be evaluated from theoverall product of the*ith-layerFresnel'scoefficient* matrix $\hat{\boldsymbol{I}}_{\mathrm{i}}$ and transmission matrix $\hat{\boldsymbol{T}}_{\mathrm{i}}$as given below

$$\boldsymbol{M} \equiv \hat{\boldsymbol{I}}_1 \circ \hat{\boldsymbol{T}}_2 \circ \hat{\boldsymbol{I}}_2 \circ \hat{\boldsymbol{T}}_3 \circ \hat{\boldsymbol{I}}_3 \circ \hat{\boldsymbol{T}}_4 \circ \hat{\boldsymbol{I}}_4 \circ \dots\dots\dots\dots\dots\dots \circ \hat{\boldsymbol{I}}_{j-2} \circ \hat{\boldsymbol{T}}_{j-1} \circ \hat{\boldsymbol{I}}_{j-1} \tag{B15}$$

To find the expressions describing the total transmittance and specular reflectance of an "ideal"j-layered stack, itis more convenient to evaluate separately a*characteristic* matrix $\boldsymbol{C} \equiv \boldsymbol{T} * \boldsymbol{I}$for each of the structure $(j-1)$ layers following the *first* (*incident*) layer, whichis normally a semi-infinite, linear, homogeneous, nonmagnetic, and dielectric mediawith a constant refractive index. To be more precise, one should bear in mind that in the present treatment I have taken the index $i \geq 2$ and the corresponding Fresnel's complex amplitude reflection and transmission coefficients $\hat{r}_{\mathrm{i}}$ and $\hat{t}_{\mathrm{i}}$ represent, respectively, $\hat{r}_{\mathrm{i,\ i+1}}$ and $\hat{t}_{\mathrm{i,\ i+1}}$. Each individual *characteristicmatrix*$\boldsymbol{C}_{\mathrm{i}} \equiv \boldsymbol{T}_{\mathrm{i}}\boldsymbol{I}_{\mathrm{i}}$ of the *i*th-layer of this m-layered stack can be expressed in a simple manner as described below

$$\boldsymbol{C}_{\mathrm{i}} \equiv \boldsymbol{T}_{\mathrm{i}}\boldsymbol{I}_{\mathrm{i}} = \begin{pmatrix} e^{j\hat{\delta}_{\mathrm{i}}} & 0 \\ 0 & e^{-j\hat{\delta}_{\mathrm{i}}} \end{pmatrix} \begin{pmatrix} \dfrac{1}{\hat{t}_{\mathrm{i}}} & \dfrac{\hat{r}_{\mathrm{i}}}{\hat{t}_{\mathrm{i}}} \\ \dfrac{\hat{r}_{\mathrm{i}}}{\hat{t}_{\mathrm{i}}} & \dfrac{1}{\hat{t}_{\mathrm{i}}} \end{pmatrix} = \begin{pmatrix} \dfrac{e^{j\hat{\delta}_{\mathrm{i}}}}{\hat{t}_{\mathrm{i}}} & \dfrac{\hat{r}_{\mathrm{i}}e^{j\hat{\delta}_{\mathrm{i}}}}{\hat{t}_{\mathrm{i}}} \\ \dfrac{\hat{r}_{\mathrm{i}}e^{-j\hat{\delta}_{\mathrm{i}}}}{\hat{t}_{\mathrm{i}}} & \dfrac{e^{-j\hat{\delta}_{\mathrm{i}}}}{\hat{t}_{\mathrm{i}}} \end{pmatrix} = \frac{1}{\hat{t}_{\mathrm{i}}} \begin{pmatrix} e^{j\hat{\delta}_{\mathrm{i}}} & \hat{r}_{\mathrm{i}}e^{j\hat{\delta}_{\mathrm{i}}} \\ \hat{r}_{\mathrm{i}}e^{-j\hat{\delta}_{\mathrm{i}}} & e^{-j\hat{\delta}_{\mathrm{i}}} \end{pmatrix} \tag{B16}$$

The characteristic matrix$\boldsymbol{C}_{\mathrm{i}}$ of each *i*th-layer ($i \geq 2$) of the j-layered structure involves its own *complex* phase-change angle $\hat{\delta}_{\mathrm{i}}(\equiv \operatorname{Re}\hat{\delta}_{\mathrm{i}} - j \operatorname{Im}\hat{\delta}_{\mathrm{i}})$, with the *imaginary* part $\operatorname{Im}\hat{\delta}_{\mathrm{i}}$being integrated in the *opticalabsorption* terms of the formulas describing the total intensity of the transmitted and specularly reflected light signals produced by the optical system in question. In terms of the characteristic matrix of the individual in-between layers of an j-layered structure, the overall characteristic matrix $\boldsymbol{M}$ of the structure can now be given by the compact expression

$$\boldsymbol{M} \equiv \hat{\boldsymbol{I}}_1 \circ \prod_{i=2}^{i=j-1} \boldsymbol{C}_i \tag{B17}$$

The overall (net) electric-field amplitude specular reflection coefficient $\hat{r}_{\mathrm{net}}$ and transmission coefficient $\hat{t}_{\mathrm{net}}$of the whole multi-layered stack, relative to the amplitude of the electric-field amplitude of the monochromatic TE electromagnetic plane wave incident on the structure, can then be evaluated from the following relations

$$\hat{r}_{\mathrm{net}} \equiv \frac{\hat{E}_-^1}{\hat{E}_+^1} \tag{B18}$$

$$\hat{t}_{\mathrm{net}} \equiv \frac{\hat{E}_+^f}{\hat{E}_+^1} \tag{B19}$$

In a strictly analogous way one can treat the Fresnel's reflection and transmission of monochromatic *p*-polarized (TM) plane electromagnetic waves obliquely incident onto a j-layered structure. This can be implemented by replacing the respective electric-field amplitudes $\hat{E}_\pm^i$ of Figure B1 by the corresponding magnetic-field amplitudes $\hat{B}_\pm^i$ and evaluating all the expressions required to arrive at the final expression describing the total TM reflection and transmission coefficients $\hat{r}_{\mathrm{net}} \equiv \hat{B}_-^1/\hat{B}_+^1$ and $\hat{t}_{\mathrm{net}} \equiv \hat{B}_+^f/\hat{B}_+^1$of the entire structure. However, one should evaluate the characteristic matrix $\boldsymbol{C}_{\mathrm{i}} \equiv \boldsymbol{T}_{\mathrm{i}}\boldsymbol{I}_{\mathrm{i}}$ of each *ith-layer* of the structure by making use of the Fresnel's reflection and transmission coefficients $\hat{r}_{\mathrm{i}}$ and $\hat{t}_{\mathrm{i}}$ corresponding to the TM case that are instead described in Equations (A9) and (A12).

Appendix C: Expressions of the normal-incidencevalues of the cosine termscos Δ_1,cos Δ_2,andcosΔ'_1forthe {air/film/substrate}-structure

Now, let us find the normal-incidence expressions for the cosine terms of the angles Δ_1, Δ_2 and Δ'_1 defined in the formulas of the total transmittance and reflectance of an {air/film/substrate}-stack, that is, $\cos\Delta_1 = \cos[(\phi_{\text{fs}} - \phi_{\text{vf}}) - \varphi_2]$, $\cos\Delta'_1 = \cos[(\phi_{\text{fv}} - \phi_{\text{sf}}) - \varphi_2]$, and $\cos\Delta_2 = \cos[(\phi_{\text{sf}} + \phi_{\text{fv}}) - \varphi_2]$. Usetrigonometric identities $\sin(\vartheta_1 \pm \vartheta_2) = \sin\vartheta_1\cos\vartheta_2 \pm \cos\vartheta_1\sin\vartheta_2$ and $\cos(\vartheta_1 \pm \vartheta_2) = \cos\vartheta_1\cos\vartheta_2 \mp \sin\vartheta_1\sin\vartheta_2$to simplify these cosine terms as

$$\begin{aligned}\cos\Delta_1 &= \cos(\phi_{\text{fs}} - \phi_{\text{vf}})\cos\varphi_2 + \sin(\phi_{\text{fs}} - \phi_{\text{vf}})\sin\varphi_2\\ &= [\cos\phi_{\text{fs}}\cos\phi_{\text{vf}} + \sin\phi_{\text{fs}}\sin\phi_{\text{vf}}]\cos\varphi_2\\ &\quad + [\sin\phi_{\text{fs}}\cos\phi_{\text{vf}} - \cos\phi_{\text{fs}}\sin\phi_{\text{vf}}]\sin\varphi_2 \end{aligned} \tag{C1}$$

$$\begin{aligned}\cos\Delta'_1 &= \cos(\phi_{\text{fv}} - \phi_{\text{sf}})\cos\varphi_2 + \sin(\phi_{\text{fv}} - \phi_{\text{sf}})\sin\varphi_2\\ &= [\cos\phi_{\text{fv}}\cos\phi_{\text{sf}} + \sin\phi_{\text{fv}}\sin\phi_{\text{sf}}]\cos\varphi_2\\ &\quad + [\sin\phi_{\text{fv}}\cos\phi_{\text{sf}} - \cos\phi_{\text{fv}}\sin\phi_{\text{sf}}]\sin\varphi_2 \end{aligned} \tag{C2}$$

$$\begin{aligned}\cos\Delta_2 &= \cos(\phi_{\text{fs}} + \phi_{\text{vf}})\cos\varphi_2 + \sin(\phi_{\text{fs}} + \phi_{\text{vf}})\sin\varphi_2\\ &= [\cos\phi_{\text{fs}}\cos\phi_{\text{vf}} - \sin\phi_{\text{fs}}\sin\phi_{\text{vf}}]\cos\varphi_2\\ &\quad + [\sin\phi_{\text{fv}}\cos\phi_{\text{sf}} + \cos\phi_{\text{fv}}\sin\phi_{\text{sf}}]\sin\varphi_2 \end{aligned} \tag{C3}$$

Assume the substrate of this structure to be a transparent nonmagnetic dielectric layer with $\kappa_{\text{s}} = 0$ and an index of refraction n_{s} and the film (layer 2) to be partially absorbing with an index of refraction n and extinction coefficient κ. Then, substitute the values of the trigonometric sines and cosines of the phase angles ϕ_{vf} and ϕ_{fs} obtained from the expressions of $\tan\phi_{\text{vf}}$ $(= \tan\phi_{\text{fv}})$ and $\tan\phi_{\text{fs}}$ $(= \tan\phi_{\text{sf}})$ described in the text by Equations (92) and (93)into the above-cited Equations (C1), (C2), and (C3)toget

$$\begin{aligned}\cos\Delta_1 =& \left[\frac{(n_2^2 + \kappa_2^2 - 1) * (n_2^2 + \kappa_2^2 - n_{\text{s}}^2) + 4\, n_{\text{s}}\kappa_2^2}{C_{\text{com}}}\right]\cos\varphi_2\\ &+ 2\kappa_2\left[\frac{(n_2^2 + \kappa_2^2 - n_{\text{s}}^2) - n_{\text{s}}(n_2^2 + \kappa_2^2 - 1)}{C_{\text{com}}}\right]\sin\varphi_2\end{aligned} \tag{C4}$$

$$\begin{aligned}\cos\Delta'_1 =& \left[\frac{(n_2^2 + \kappa_2^2 - 1) * (n_2^2 + \kappa_2^2 - n_{\text{s}}^2) + 4\, n_{\text{s}}\kappa_2^2}{C_{\text{com}}}\right]\cos\varphi_2\\ &+ 2\kappa_2\left[\frac{n_{\text{s}}(n_2^2 + \kappa_2^2 - 1) - (n_2^2 + \kappa_2^2 - n_{\text{s}}^2)}{C_{\text{com}}}\right]\sin\varphi_2\end{aligned} \tag{C5}$$

$$\begin{aligned}\cos\Delta_2 =& \left[\frac{(n_2^2 + \kappa_2^2 - 1) * (n_2^2 + \kappa_2^2 - n_{\text{s}}^2) - 4\, n_{\text{s}}\kappa_2^2}{C_{\text{com}}}\right]\cos\varphi_2\\ &- 2\kappa_2\left[\frac{n_{\text{s}}(n_2^2 + \kappa_2^2 - 1) + (n_2^2 + \kappa_2^2 - n_{\text{s}}^2)}{C_{\text{com}}}\right]\sin\varphi_2\end{aligned} \tag{C6}$$

Where the parameter coefficientC_{com} is given by the expression

$$C_{\mathrm{com}} = \sqrt{[(n_2^2+\kappa_2^2-1)^2+4\kappa_2^2]*[(n_2^2+\kappa_2^2-n_s^2)^2+4\,n_s^2\kappa_2^2]} \tag{C7}$$

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