Criteria of True and Its Applications

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Abstract
Some criteria and examples of its applications are considered. A new concept of "probability of probability" for checking the true of density function is issued by a criterion of true: "A true must be the same result for repeating any times under same conditions". The convergence of series issued by repeating probability of probability in probability Banach space can be proved by g-contractive mapping theorem. And its fixed point (i.e., a type of exponential density function) is found. Finally, a conclusion is made.

Key words: criteria of true, probability, probability of probability, g-contraction mapping, fixed point.

1. Introduction:

The criterion of true is a widely concerned topic in many fields. Here we just list some of examples of its applications. The focus of this paper is the application of criterion of true on theory of probability.

On theory of probability

Outcome of an uncertain event has no rule and thus it can not be predictable. However, a lot of people believe that rule exists and try to find the rule. The concept of probability and theory of probability set up for the attempt of finding "rules in the sense of statistics" using mathematical tools. Although theory of probability has been successfully used in many areas, however, a fatal weak of the theory is: it can not guaranteed an event with "very small probability" never happened. For example, it has frequently been reported that the winner won a great lottery prize just bough few ticks. It shows that an event occurs or not is not connected with probability. Nevertheless, no other better choice, the theory of probability is still widely used.

It is of great importance in all statistical applications that samples with smaller size be properly chosen with "representative" from population. Usually, such a sampling is drawn randomly from a (given) probability distribution (Irving W. Burr, 1974).

A question is arisen: "how to choose a probability distribution?" Or "how to judge a chosen probability distribution is true (better or proper)?"

The goal of this paper is to study the criterion of judging a properly chosen of a probability distribution based on the principle of "a true must be the same result for repeating any times under same conditions". The concept of "probability of probability" is therefore introduced. And the convergence of series issued from repeating probability of probability in Banach space had been proved by g-contractive mapping theorem.

In section 2, The criteria of judging true or false without experiment are listed And it is applied to judging the correctness of chosen probability density function by the convergence of series obtained by repeating "probability of probability".

Finally, a conclusion is made.

2. Criteria of true

The discussion on "the criterion of true" ("The social practice is the only criterion of truth",1978) played an important role for political power struggle in China in 1978. However, there are many cases, in which no practice, or experiment can be done, or it costs very expensive. What criterion can be used for judging true or false? Herein, some criteria of judging true or false are listed:

(1). Axiom, general acknowledged truth, self-evident truth. Such as: "**Conservation Law of Energy"**, which suits for things on earth in anywhere at any time.

Example 1.

The author and his student had used the "**Reciprocity Theorem of Work"** based on "**Conservation Law of Energy"** as a criterion to check the accuracy of displacement calculation of an in-plan hinged-jointed rigid sloping piles embedded in an elastic half-space by line-loaded integral equation method (yun & Li, 1995). In which, no experiment could be done, and no previous work on similar case can be used for comparing. If no criterion for judging the accuracy of the calculation can be found, then, the method, even it is nice, would not be recognized.

(2). The principle of "**:no contradiction with the logic proof**" for judging the true or false of a "proposition", a "statement", etc.

Example 2.

To judge true or false of the statement:

"For freely falling objects A and B, with weights $w_A > w_B$, then, their falling velocity $v_A > v_B$.". (2-1)

Proof:

Suppose that (2-1) is true. If we tie A and B together to form an object C, then we have

$$w_c = w_A + w_B > w_A > w_B, \quad (2\text{-}2)$$

Then, by the statement, we have

$$v_c > v_A > v_B, \quad (2\text{-}3)$$

On the other hands, since $v_A > v_B$, A tied with B, its velocity must be lessen than v_A, i.e., $v_c = v_A - \Delta$,

$(\Delta > 0)$, (2-4)

B tied with A, its velocity must be greater than v_B, i.e.,

$v_c = v_B + \delta$, $(\delta > 0)$, (2-5)

(2-3) is contradicted with (2-4). So the statement is false. (The freely falling object experiment is not necessary for proof).

Example 3, checking a chosen probability density function is true (proper) or not.

A true (e.g., a rule) must be the same result for repeating any times under the same conditions. However, what is repeating for probability density function? A concept of "probability of probability" is therefore introduced via continuously density function of random variable x_i.

Probability

$$P_i = P_i(x_i > x_{i0}) = \int_{x_{i0}}^{\infty} f_i(x_i)dx_i = F_i(x_{i0}, f_i)\,, \tag{2-6}$$

is a function of x_{i0} (a real) and f_i, where f_i is a continuous density function of random variable x_i, with properties f_i >0, and $\int_{-\infty}^{\infty} f_i(x_i)dx_i = 1$, $F_i(x_{i0}, f_i)$ is the probability distribution. The subscript i denotes the i-th iteration. Let

$$x_{i+1} = F_i(x_{i0}, f_1) = P_i(x_i > x_{i0})\,, \tag{2-7}$$

be the function of random variable x_i, i.e.,

$$x_{i+1} = F_i(x_i) = F_i(x_{i0}, f_i(x_i))\,, \tag{2-8}$$

Eq. (2-8) can be viewed as a mapping $T_i : X \to X$ of random variable set into itself, or

$$x_{i+1} = T_i x_i\,, \quad (x_{i+1}, x_i \in X)\,, \tag{2-9}$$

Repeating the process, we have

$$x_{i+2} = T_{i+1}x_{i+1} = T_{i+1} \circ T_i x_i = T_{i+1} \circ T_i \circ \ldots \circ T_1 x_1\,, \tag{2-10}$$

Where $F \circ G$ represents the composition of mapping F and mapping G.
If the series obtained by repeating process converges, then, we have

$$\lim_{i\to\infty} \|x_{i+1} - x_i\| = 0\,, \tag{2-11}$$

Where $\|x\|$ is the norm of x in X. X is a probability Banach space, i.e., a non-empty complete metric space satisfied the properties of probability.
By (2-11), we have

$$x_{i+1} = T_i x_i = x_i = x^*, \qquad (2\text{-}12)$$

Where x^* is the fixed point of the sequential mapping. The convergence of series similar to (2-10) in Banach space had been proved by g-contraction mapping theorem (Yun, 2001) for any initial choice of x_1 .In (Yun, 2011), the required condition is the geometric mean contraction ratio less than a constant less than 1, which is looser than the Banach contraction mapping theorem, the later requires each mapping must less than a constant less than 1. More information on g-contraction mapping and its application can be found in Refs (Yun, 2011, 2013).

(2-12) shows no difference of x^* between x^* and Tx^*, i.e., repeating any times x^* keeps the same. If the first chosen $x_1 = x^*$, for example, the exponential distribution, then, we have $T_1 x^* = x^*$. So, the choice of probability density function is true (proper).

Conclusion

If the first chosen $x_1 = x^*$, then, it is proper. However, for a convergent series issued by repeating probability of probability, it does not matted the first choice of x_1, x^* will be reached after mappings.

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