Plane Symmetric Cosmological Model with Interacting Dark Matter and Holographic Dark Energy Using Exponential Volumetric Expansion

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Abstract
In this paper, we have studied the plane symmetric cosmological model filled with interacting Dark matter and Holographic Dark energy. The solutions of the field equations have been obtained under the assumption of constant deceleration parameter (using exponential volumetric expansion). The physical and geometrical aspects of the model are also discussed.

Keywords: Plane symmetric cosmological space-time, Interacting dark fluids, deceleration parameter, Statefinder parameters, Coincidence problem.

1. Introduction:

Wilkinson Microwave Anisotropy Probe (WMAP) proved that dark energy occupies about 73% of the energy of our universe and dark matter occupies about 23% whereas the baryon matter occupies only about 4% of the total energy of the universe.

A special class of interacting models in which holographic Dark Energy is allowed to interact with Dark Matter (Gong, 2004; Gong and Zhang, 2005; Wang *et al*. 2006; Nojiri and Odintsov, 2006 ; Guo, *et al*. 2007 ; Banerjee and Pavon, 2007; Zimdahl and Pavon, 2007; Zimdahl, 2008) have been studied by many authors. In addition, Guo *et al.* 2007 have shown that the proposal of interacting dark energy is compatible with the current observations of the SNIa and CMB data.

Recently, Sarkar (2014a, 2014b, 2014c) has studied non-interacting holographic dark energy with linearly varying deceleration parameter for Bianchi type-I and V universe and interacting holographic dark energy in Bianchi type-II respectively. Motivated by this, in this paper we have considered the plane symmetric cosmological model filled with interacting Dark matter and Holographic Dark energy. The solutions of the field equations have been obtained under the assumption of constant deceleration parameter (using exponential volumetric expansion). The physical and geometrical aspects of the model are also discussed.

2. Metric and Field Equations:

In view of the importance of the plane symmetry, we consider the line element in plane symmetric form [Zhang & Noh 2009, Setare & Momeni 2010, Shen & Zhao 2012] as

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2 dz^2 , \tag{2.1}$$

where A and B are the scale factors and functions of the cosmic time t only .

The Einstein's field equations are ($8\pi G = 1$ and $c = 1$)

$$R_{ij} - \frac{1}{2} g_{ij} R = -{}^{m}T_{ij} + {}^{\Lambda}T_{ij} \quad , \tag{2.2}$$

where ${}^{m}T_{ij} = \rho_m\, u_i\, u_j$ and ${}^{\Lambda}T_{ij} = (\rho_\Lambda + p_\Lambda) u_i u_j + g_{ij} p_\Lambda$ (2.3)

are matter tensors for dark matter (pressureless i.e. $w_m = 0$) and holographic dark energy respectively. Here ρ_m is the energy density of dark matter and ρ_Λ and p_Λ are the energy density and pressure of holographic dark energy.

The Einstein's field equations (2.2) for metric (2.1) with the help of equations (2.3) can be written as

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} = \rho_m + \rho_\Lambda \ , \tag{2.4}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = -p_\Lambda , \tag{2.5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -p_\Lambda , \tag{2.6}$$

where overhead dot (˙) represents derivative with respect to time *t*.

We have assumed that both components do not conserve separately but interact with each other in such a manner that the balance equations take the form

$$\dot{\rho}_m + \left(\frac{\dot{V}}{V}\right)\rho_m = Q \tag{2.7}$$

$$\dot{\rho}_\Lambda + \left(\frac{\dot{V}}{V}\right)(1 + w_\Lambda)\rho_\Lambda = -Q \ , \tag{2.8}$$

where $w_\Lambda = p_\Lambda / \rho_\Lambda$ is the equation of state parameter for holographic dark energy and $Q > 0$ measures the strength of the interaction.

A vanishing Q implies that dark matter and dark energy remain separately conserved. In view of continuity equations, the interaction between dark energy and dark matter must be a function of the energy density multiplied by a quantity with units of inverse of time, which can be chosen as the Hubble factor *H*. There is freedom to choose the form of the energy density, which can be any combination of dark energy and dark matter. Thus, the interaction between dark energy and dark matter could be expressed phenomenologically in the form (Guo *et al.,* 2007; Amendola *et al.,* 2007)

$$Q = 3b^2 H\rho_m = b^2 \frac{\dot{V}}{V}\rho_m , \tag{2.9}$$

where b^2 is coupling constant.

From equations (2.13) and (2.11), we get the energy density of dark matter as

$$\rho_m = \rho_0 V^{(b^2-1)} \quad , \tag{2.10}$$

where $\rho_0 > 0$ is a real constant of integration.

Using equations (2.9) and (2.10), we get the interacting term as

$$Q = 3\,\rho_0 b^2 H V^{(b^2-1)} \quad . \tag{2.11}$$

3. Cosmological Solutions for Constant Deceleration Parameter:

In order to obtain exact solutions of the field equations (2.4)-(2.6), we impose a law of variation for the Hubble parameter which yields the constant value of deceleration parameter. This law was first introduced by Berman, 1983. According to this law the variation of the mean Hubble parameter for plane symmetric space-time is given by

$$H = k(A^2B)^{-m/3}, \tag{3.1}$$

where $k > 0$ and $m \geq 0$ are constants.

The volume V in terms of scale factors is given by

$$V = a^3 = A^2B\,. \tag{3.2}$$

The directional Hubble parameters in the directions of x, y and z axes respectively are

$$H_x = H_y == \frac{\dot{A}}{A}, \quad H_z = \frac{\dot{B}}{B}. \tag{3.3}$$

The mean Hubble parameter H is given by

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right). \tag{3.4}$$

The volumetric deceleration parameter q is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2}\,. \tag{3.5}$$

Equating equation (3.1) with (3.4) and integrating we get

$$V = A^2B = c_1 e^{3kt} \quad , \qquad \text{for} \qquad m = 0\ , \tag{3.6}$$

and

$$V = A^2B = \left(mkt + c_2\right)^{\frac{3}{m}} \quad , \qquad \text{for } m \neq 0\ , \tag{3.7}$$

where c_1 and c_2 are positive constant of integration.

Using (3.1) with (3.6) for $m = 0$ and with (3.7) for $m \neq 0$, the mean Hubble parameters are

$$H = k \qquad , \qquad \text{for } m = 0\ , \tag{3.8}$$

and

$$H = k\left(mkt + c_2\right)^{-1} \quad , \qquad \text{for } m \neq 0\ . \tag{3.9}$$

Using (3.2), (3.6) and (3.7) in (3.5), we get constant values for the deceleration parameter for mean scale factor as

$$q = -1 \qquad , \qquad \text{for } m = 0\ , \tag{3.10}$$

and

$$q = m - 1 \quad , \qquad \text{for } m \neq 0 . \tag{3.11}$$

The sign of q indicates whether the model accelerates or not. The positive sign if $q(m > 1)$ corresponds to decelerating models where as the negative sign $-1 \leq q < 0$ for $0 \leq m < 1$ indicates acceleration and $q = 0$ for $m = 1$ corresponds to expansion with constant velocity.

Model for $m = 0$ **[*Exponential Volumetric Expansion Model*]:**

Subtracting equation (2.6) from equation (2.5) and using equation (3.2), we get

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\frac{\dot{V}}{V} = 0 \ . \tag{3.12}$$

On integration of equation (3.12) and considering equation (3.6), we obtain

$$\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = c_3 e^{-3kt} , \tag{3.13}$$

where c_3 is constant of integration.

On integration of (3.13) and using (3.2), we get exact values of the scale factors as:

$$A = (c_1 c_4)^{\frac{1}{3}} \ e^{\left(kt - \frac{c_3 e^{-3kt}}{9k}\right)} \quad , \tag{3.14}$$

$$B = (c_1 c_4^{-2})^{\frac{1}{3}} \ e^{\left(kt + \frac{2 c_3 e^{-3kt}}{9k}\right)} \quad , \tag{3.15}$$

where c_4 is constant of integration.

Using equations (3.14) and (3.15) in equation (3.2), the volume V of the universe is given by

$$V = c_1 e^{3kt} . \tag{3.16}$$

Using equations (3.14) and equation (3.15) in equation (3.4) and in equation (3.5), we get the mean Hubble parameter and deceleration parameter as

$$H = k \tag{3.17}$$

$$q = -1 \tag{3.18}$$

The mean anisotropy parameter of expansion is defined as $\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2$ and given by

$$\Delta = \frac{2c_3^2}{9k^2} e^{-6kt} \quad . \tag{3.19}$$

Using equation (3.16) in equations (2.10) and (2.11), we get

$$\rho_m = \rho_0 (c_1)^{(b^2-1)} e^{3k(b^2-1)t} , \tag{3.20}$$

$$Q = 3kb^2 \rho_0 (c_1)^{(b^2-1)} e^{3k(b^2-1)t} \quad . \tag{3.21}$$

Using equations (3.14) and (3.15) and (3.20) in the equation (2.4), we obtain the energy density of holographic dark energy as

$$\rho_\Lambda = 3k^2 - \frac{1}{3}c_3^2 e^{-6kt} - \rho_0 (c_1)^{(b^2-1)} e^{3k(b^2-1)t} \ . \tag{3.22}$$

Using equations (3.14) and (3.15) in the equation (2.5), we obtain the pressure of holographic dark energy as

$$p_\Lambda = -3k^2 - \frac{c_3^2}{3} e^{-6kt} \, . \tag{3.23}$$

The EoS parameter of holographic dark energy is given by

$$w_\Lambda = \frac{-\left(3k^2 + \frac{c_3^2}{3} e^{-6kt}\right)}{3k^2 - \frac{1}{3}c_3^2 e^{-6kt} - \rho_0 (c_1)^{(b^2-1)} e^{3k(b^2-1)t}} \, . \tag{3.24}$$

The coincidence parameter $\bar{r} = \rho_m / \rho_\Lambda$ i.e. the ratio of dark matter energy density to the dark energy density is given by

$$\bar{r} = \frac{\rho_0 (c_1)^{(b^2-1)} e^{3k(b^2-1)t}}{3k^2 - \frac{1}{3}c_3^2 e^{-6kt} - \rho_0 (c_1)^{(b^2-1)} e^{3k(b^2-1)t}} \, . \tag{3.25}$$

4. Statefinder Diagnostic:

In order to differentiate between all competing cosmological scenarios involving DE, a sensitive and robust diagnostic for DE models is mostly required. For this purpose Sahni *et al. ,2003* introduced a diagnostic proposal that makes use of parameter pair $\{r, s\}$, called as "statefinder". The statefinder probes the expansion dynamics of the universe through higher derivatives of the expansion factor $\ddot{a}$ and is a natural companion to the deceleration parameter which depends upon $\ddot{a}$.

The statefinder pair $\{r, s\}$ is defined as follows

$$r = \frac{\dddot{a}}{aH^3} \qquad \text{and} \qquad s = \frac{r-1}{3(q-1/2)} \, .$$

The spatially flat Λ CDM scenario corresponds to a fixed point in the diagram $\{s, r\}\big|_{\Lambda CDM} = \{0, 1\}$. Departure of a given dark energy model from this fixed point provides a good way of establishing the 'distance' of this model from Λ CDM (Alam *et al.*, 2003). The statefinder can successfully differentiate between a wide variety of dark energy models including the cosmological constant, quintessence, the Chaplygin gas, braneworld models and interacting dark energy models (Sahni, 2003; Alam *et al.*, 2003; Gorini *et al.*, 2003; Zimdahl and Pavon, 2004).

The statefinder parameters r and s for exponential volumetric expansion model (i.e model for $m = 0$) are given by

$$r = 21 \qquad \text{and} \qquad s = -4.44 \, .$$

5. Discussion:

The physical and geometrical behavior of the model :

i) The deceleration parameter **(q):** For this model with the deceleration parameter q is negative (= -1) for $m = 0$ as shown in Figure-1, This result is consistent with the observations made by Perlmutter *et al.* (1998, 1999) and Riess *et al.* (1998) showing that the present day universe is undergoing accelerated expansion.

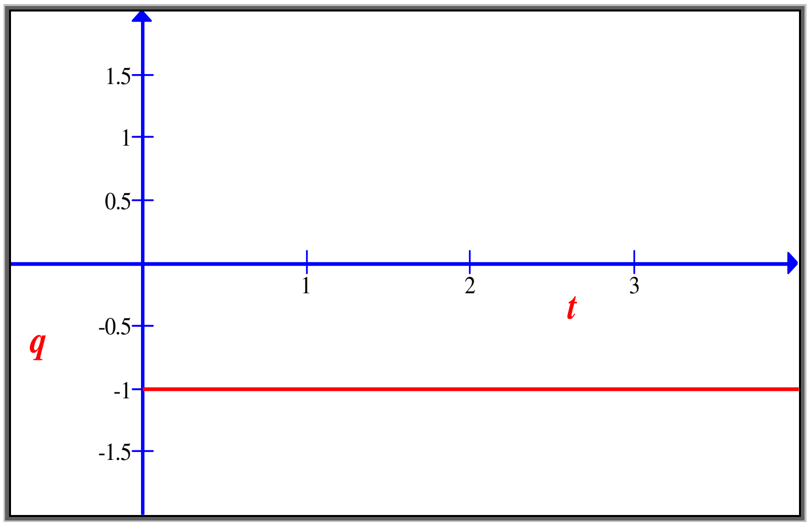

Figure-1: Evolution of deceleration parameter *q* vs. t.

ii) The anisotropy parameter of expansion **(Δ) :** In Figure-2, we have plotted anisotropy parameter of expansion Δ against cosmic time t for exponential volumetric expansion model. It is observed that anisotropy decreases to zero quickly. Hence, the model reaches to isotropy after some finite time which matches with the recent observations as the universe is isotropic at large scale.

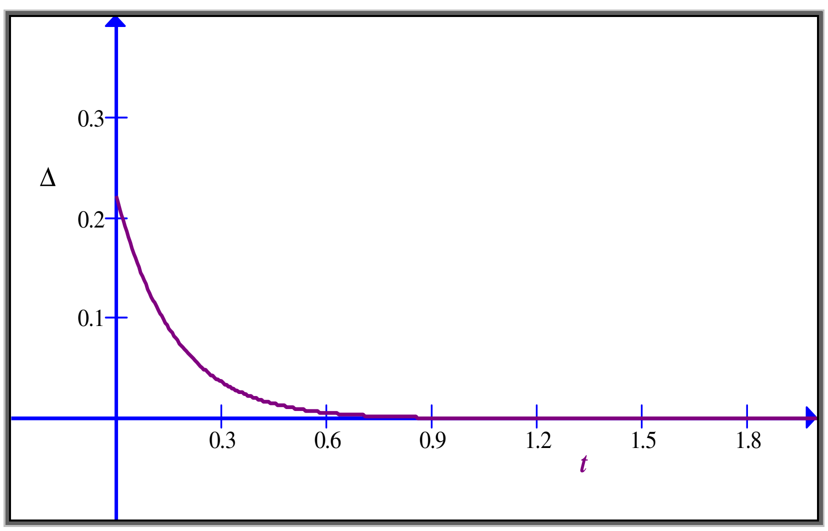

Figure-2: Evolution of anisotropy parameter of expansion Δ vs. t for $k = 1$.

iii) The equation of state parameter **(w_Λ):**

In the Figure-3 , we have shown the variation of EoS parameter (w_Λ) with cosmic time t for exponential expansion model. In exponential expansion model w_Λ starts phantom region $(w_\Lambda < -1)$ and attains the value $w_\Lambda = -1$ after some finite t .

i.e. the model approaches to $\wedge$CDM model after some finite *t*.
Wang *et al.,* 2005, 2006 proved that the interacting holographic dark energy model can accommodate the transition of the dark energy equation of state w_Λ from $w_\Lambda > -1$ to $w_\Lambda < -1$.

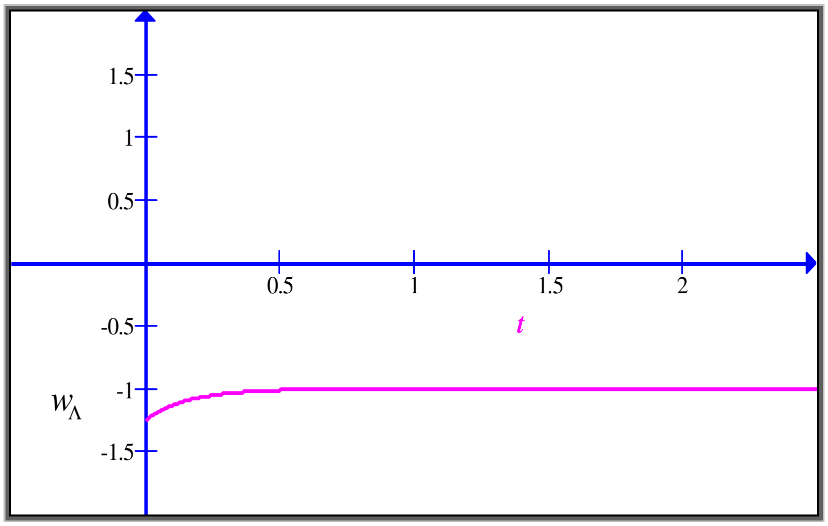

Figure-3: Evolution of anisotropy parameter of expansion w_Λ vs. t for $k=1$.

6. Conclusion:

In this paper, we have studied the plane symmetric cosmological model filled with interacting Dark matter and Holographic Dark energy. For $Q=0$ (non-interacting case) our model reduces to the particular model of Sarkar, 2014 (the model when $k=0$ in Sarkar, 2014). Also, in this model, the anisotropy of expansion dies out very quickly and attains isotropy after some finite time i.e. we can say that the plane symmetric space-time reduces to flat FRW soon after inflation. The Statefinder diagnostic is applied to the model in order to distinguish between our dark energy model with other existing dark energy models. The evolving trajectory in the *s-r* plane for the obtained model is quite different from those of other dark energy models.

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