Validity domain extension of multiaxial high cycle fatigue criteria

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Abstract:
Some multiaxial criteria for high-cycle fatigue analysis are generally limited to some materials, depending on the endurance strength ratios. The extension of the validity domain of three multiaxial high cycle fatigue criteria is proposed. This extension of the validity domain was successfully validated on fatigue tests from the literature. Results show that the criteria are in good accordance with the experimental data and that the author's criterion is very efficient.

Key Words: high cycle fatigue, validity domain, fatigue limit, non-proportional loading.

1 Introduction

It is well known that mechanical components such as crankshafts, propeller shafts, vehicle axles, shafts of flywheels, railroad wheels, turbine blades and structures such as submarine hulls and pressure vessels, undergo multiaxial complex fatigue loading resulting in biaxial and triaxial stress. To determine the stress level allowable for the safety of the component, many multiaxial fatigue endurance criteria have been proposed in the literature for metals [1, 3, 7, 8, 12, 14, 17]. Several multiaxial fatigue models have also been reviewed or compared; some reviews and comparisons of existing multiaxial fatigue models can be found in [2, 4, 5, 6, 10, 11, 15, 16].

Most of the current multiaxial fatigue criteria are limited to specific materials or loading conditions. Another disadvantage associated with these criteria is the fact that they require quite lengthy and complicated calculations or are too time consuming , requiring the user to be a specialist in fatigue [2, 6, 10]. As design tools, criteria can be used quite successfully as long as the user is aware of each criterion's limitations.

In recent decades, extensive research has been performed for multiaxial fatigue, and a limitation of fatigue models that have been studied extensively is the account of the detrimental effect of non-proportional loading on fatigue endurance, as reported by experimental observations [9]. Non proportional loading is the more general case that permits arbitrary stress histories, with principal directions and/or the ratios of the principal stresses varying with time during the loading cycle. Another limitation of criteria in high cycle fatigue is their range of validity as reported in [3, 7, 8, 10, 17]. The range of validity of fatigue criteria results from constraints imposed on parameters of the criteria and lead to their inapplicability to some materials for which they could have rendered good predictions.

Thus a key aspect of all criteria is that their formulation imposes to them to be valid not for all metals, but to a certain category of metals as a result of the constraints that are imposed on their constants. Various models available in the literature are limited to specific materials depending on the ratio of the fatigue limit in reversed torsion and in reversed bending (or tension), t_{-1}/f_{-1}, see Table 1 for the definition ranges and the materials categorized.

This ratio of material properties is of essential importance in multiaxial fatigue, as indicated in [3, 7, 8], and determine the nature of the materials. In consequence, the objective of the present work is to develop a method enabling the extension of the domain of validity of some criteria and to predict for materials out of the validity domain, the fatigue strength limit of components subjected to cyclic loading.

We first review some stress-based multiaxial fatigue endurance criteria for metals based on the linear combination of two stress parameters as the governing variables in the crack initiation process. A measure of the shear solicitation to fatigue and the hydrostatic stress as far as their domain of validity (Section 2). We propose adapted criteria that enable the extension of the validity domain (Section 3). The method is then validated by analyzing by means of the present adapted criteria, relevant experimental tests available in the literature [9, 13, 14, 17]; to ensure that the adapted criteria coincide with the original formulation while predicted fatigue strength limit of components showed good agreement with experimental data out of the domain of validity (Section 4), followed by discussion (Section 5). Conclusions are drown in section 6.

2 Presentation of some multiaxial fatigue criteria

2.1 Authors criterion

Many stress-based criteria for high-cycle fatigue analysis have already been proposed to assist in the design of components experiencing multiaxial stresses. However the implementation of these criteria is not in general an easy task. The authors proposed in a recent work [12] an equivalent stress approach implemented in the following fatigue function of the criterion

$$E_A = \frac{\sqrt{J_{2a} + \alpha_A \cdot sign(\sigma_{H,\max})(\sigma_{H,\max})^2}}{\beta_A}; \tag{1}$$

$$\beta_A = t_{-1}, \qquad \alpha_A = 3\left(3\left(\frac{t_{-1}}{f_{-1}}\right)^2 - 1\right). \tag{2}$$

f_{-1}, t_{-1} are respectively the fatigue limits in fully reversed bending and torsion. And $\sigma_{H,\max}$ is the maximum hydrostatic stress; J_{2a} is the amplitude of the second invariant of stress deviator tensor.

The condition $\alpha_A > 0$ determines the range of validity of the criterion, expressed as:

$$\frac{t_{-1}}{f_{-1}} > \frac{1}{\sqrt{3}}. \tag{3}$$

This model is limited to materials, in which $\frac{t_{-1}}{f_{-1}} > 0.58$.

2.2 Crossland criterion

As reported in [10, 11] Crossland proposed to reduce a complex multiaxial loading to an equivalent uniaxial loading and to compare it with the value of β_C – characteristic parameter of each material. The criterion is expressed as:

$$E_C = \frac{\sqrt{J_{2a}} + \alpha_C \sigma_{H,\max}}{\beta_C} \;; \tag{4}$$

$$\beta_C = t_{-1} \;,\; \alpha_C = 3\left(\frac{t_{-1}}{f_{-1}} - \frac{1}{\sqrt{3}}\right). \tag{5}$$

The quantity $\sqrt{J_{2a}}$ is the amplitude of the square root of the second invariant of the alternating deviator stress tensor.

The condition $\alpha_C > 0$ determines the range of validity of the criterion, expressed as:

$$\frac{t_{-1}}{f_{-1}} > \frac{1}{\sqrt{3}} \quad . \tag{6}$$

This model is limited to materials, in which $\frac{t_{-1}}{f_{-1}} > 0.58$.

2.3 Papadopoulos criterion [1997]

Papadopoulos et al. [10] proposed a fatigue criterion based on the average stress approach that relies on the argument that the accumulated plastic deformations at mesoscopic level, at each slip plane, are proportional to the resolved shear stress amplitude T_a. An average of the stress components within an elementary volume involving the critical point is given as $\sqrt{\langle T_a \rangle}$. The expression of the criterion is

$$E_{P_1} = \frac{\sqrt{\langle T_a \rangle} + \alpha_{P_1} \sigma_{H,\max}}{\beta_{P_1}} \quad . \tag{7}$$

$$\beta_{P_1} = t_{-1} \;,\; \alpha_{P_1} = 3\left(\frac{t_{-1}}{f_{-1}} - \frac{1}{\sqrt{3}}\right) . \tag{8}$$

The condition $\alpha_{P_1} > 0$ determines the range of validity of the criterion, expressed as:

$$\frac{t_{-1}}{f_{-1}} > \frac{1}{\sqrt{3}} \quad . \tag{9}$$

2.4 Papadopoulos criterion [1993]

Taking advantage of the work of Dang Van published in 1973, many researchers have attempted to propose criteria based on an interpretation at a mesoscopic level of the material fatigue behavior. Papadopoulos proposed in 1993 a criterion reported in [5] that depends on nature of the material.

- For soft metals $0.5 < \frac{t_{-1}}{f_{-1}} < 0.6$:

The fatigue function of the criterion is written as:

$$E_{P_2} = \frac{\underset{\varphi,\gamma}{Max}\left[T_\sigma(\varphi,\gamma)\right] + \alpha_{P_2} \sigma_{H,\max}}{\beta_{P_2}} \;; \tag{10}$$

$$T_\sigma(\varphi,\gamma)=\sqrt{\int_{\psi=0}^{\psi=2\pi}\tau_a^2(\varphi,\gamma,\psi)d\psi}\ ;$$
$$\beta_{P_2}=\sqrt{\pi}\,t_{-1},\ \alpha_{P_2}=3\sqrt{\pi}\,\frac{2t_{-1}-f_{-1}}{2f_{-1}}. \tag{11}$$

$\tau_a(\varphi,\gamma,\psi)$ is obtained by projecting the amplitude of the shear stress along a line oriented by ψ in the plane of the facet, which itself is parameterized by its normal and angles φ and γ in the spherical reference [5].

- For hard metals $0.6<\frac{t_{-1}}{f_{-1}}<0.8$:

The fatigue function of the criterion is written as:

$$E_{P_2}=\frac{M_\sigma+\alpha'_{P_2}\sigma_{H,\max}}{\beta'_{P_2}} \tag{12}$$

$$M_\sigma(\varphi,\gamma)=\sqrt{\int_{\varphi=0}^{\varphi=2\pi}\int_{\gamma=0}^{\gamma=\pi}T_\sigma^2(\varphi,\gamma)\sin\gamma d\varphi d\gamma}\ ;$$
$$\beta'_{P_2}=\pi\sqrt{\frac{8}{5}}t_{-1},\ \alpha'_{P_2}=\pi\sqrt{\frac{8}{5}}\,\frac{3t_{-1}-\sqrt{3}f_{-1}}{f_{-1}}. \tag{13}$$

The domain of validity of the second Papadopoulos criterion is given by the interval for soft and hard metals, and will therefore not be considered in the procedure of extending the limit of validity of multiaxial fatigue criteria.

3 Validity domain extension

The multiaxial high cycle fatigue criteria reviewed above , and that aim at reducing a given multiaxial stress state to an equivalent uniaxial stress, which is then compared to the uniaxial fatigue strength at a specified number of loading cycles are in general valid depending on the materials types; ductile (or low ductile) materials, hard materials, brittle materials.

The ratio between the fatigue strength at a reference number of loading cycles N under fully reversed torsion and that under fully reversed bending (or tension), are generally used to define the materials, see Table 1.

To give the possibility for a criterion in his formulation not to be limited to specific materials, we use a method based on the formulation of an adapted criterion; the extended validity domain comes later just as a result of the reformulation.

3.1 Methodology

The shear stress is considered in many multiaxial high-cycle fatigue models as one of the driving forces of the fatigue process – since plasticity plays an important role in crack. The first step in the extension of the validity domain of stress-based criterion generally expressed in terms of the equivalent shear stress and hydrostatic stress dependent parameters is to formulate its equivalent. That is we re-express the criterion by introducing parameters such that the condition of validity will now also depend on values given to the introduced parameters.

Mathematically , for a multiaxial high cycle criteria based on the linear combination of a quantity related to the shear stress and hydrostatic stress, i.e. (τ_{eq} and σ_H) as in equations (1, 4, 7); τ_{eq} in the expression of the criterion is replaced by its equivalent, in the form:

$$\tau'_{eq}=\tau_{eq}\left(1+\lambda\left(\frac{\sigma_H}{\tau_{eq}}-\xi\right)\right). \tag{14}$$

The introduction of τ'_{eq} yields the adapted criterion and will later on enables the validity domain extension.

To simplify the expression of the adapted criterion, the value of parameter ξ is taken such that in Eq. (14), $\tau_{eq} = \tau'_{eq}$ in fully reversed (or repeated) bending tests. Thus ξ is obtained from fatigue limits in fully reversed (or repeated) bending as

$$\xi = \frac{\sigma_H}{\tau_{eq}}. \tag{15}$$

The new validity domain of criteria, obtained from their adapted formulations are now depending on λ, the validity domain extension parameter; a material independent real parameter.

3.2 Extension of the validity domain of Crossland criterion

Crossland criterion is based on the linear combination of the amplitude of the square root of the second invariant of the alternating deviator, $\sqrt{J_{2a}}$ and the maximum hydrostatic stress, i.e. ($\sqrt{J_{2a}}$ and $\sigma_{H,\max}$). Based on the considerations developed along Sections 3.1, $\xi = \sqrt{3}/3$. Crossland equivalent criterion is expressed as:

$$E'_C = \frac{\sqrt{J_{2a}}\left(1 + \lambda_C\left(\frac{\sigma_{H,\max}}{\sqrt{J_{2a}}} - \frac{\sqrt{3}}{3}\right)\right) + \alpha'_C \sigma_{H,\max}}{\beta'_C} \tag{16}$$

Where α'_C and β'_C are the new parameters of the adapted criterion. These constants can be determined by uniaxial and torsional fatigue limits.

For fully reversed torsion, one has $\sqrt{J_{2a}} = t_{-1}$, $\sigma_{H,\max} = 0$. Application of the criterion, Eq. (16) provides the material parameter β'_C as:

$$\beta'_C = t_{-1}\left(1 - \frac{\lambda_C}{\sqrt{3}}\right). \tag{17}$$

For fully reversed bending, $\sqrt{J_{2a}} = \frac{f_{-1}}{\sqrt{3}}$ *and* $\sigma_{H,\max} = \frac{f_{-1}}{3}$. One finds that the expression of parameter α'_C as:

$$\alpha'_C = 3\left[\left(1 - \frac{\lambda_C}{\sqrt{3}}\right)\frac{t_{-1}}{f_{-1}} - \frac{1}{\sqrt{3}}\right] \tag{18}$$

The physical meaning of α'_C in Eq. (18) as of α_C in Eq. (4) is the contribution of damage caused by the hydrostatic stress amplitude [4]. To preserve the observed detrimental influence of a tensile mean stress the parameter α'_C should be non-negative; thus,

$$\begin{cases} \alpha'_C > 0 \\ \beta'_C > 0 \end{cases}. \tag{19}$$

Equation (19) gives the new range of validity for Crossland adapted criterion, now expressed in terms of parameter λ as:

$$\lambda_C < \sqrt{3}\left(1 - \frac{f_{-1}}{\sqrt{3}t_{-1}}\right). \tag{20}$$

3.3 Extension of the validity domain of Papadopoulos criterion [1997]

The criterion formulated by Papadopoulos is based on the mesoscopic scale approach. Applied in out-of-phase bending and torsion, the average measure of $\sqrt{\langle T_a \rangle}$ is:

$$\sqrt{\langle T_a \rangle} = \sqrt{\frac{5}{8\pi^2}\int_{\varphi}^{2\pi}\int_{\theta}^{\pi}\int_{\chi}^{2\pi} T_a(\varphi,\theta,\chi)d\chi \sin(\theta)d\theta d\varphi}\,; \\ = \sqrt{\frac{\sigma_{xa}^2}{3}+\tau_{xya}^2}\ . \quad (21)$$

Where the bending stress amplitude is σ_{xa} ; τ_{xya} is the torsion stress amplitude. With $\xi = \sqrt{3}/3$, Papadopoulos adapted criterion is proposed in the form:

$$E'_{P_1} = \frac{\sqrt{\langle T_a \rangle}\left(1+\lambda_{P_1}\left(\frac{\sigma_{H,\max}}{\sqrt{\langle T_a \rangle}}-\frac{\sqrt{3}}{3}\right)\right)+\alpha'_{P_1}\sigma_{H,\max}}{\beta'_{P_1}} \quad (22)$$

In fully reversed torsion, from Eq. (21) we have $\sqrt{\langle T_a \rangle} = t_{-1}$, *and* $\sigma_{H,\max} = 0$. The material constants β'_{P_1} is given as:

$$\beta'_{P_1} = t_{-1}\left(1-\frac{\lambda_{P_1}}{\sqrt{3}}\right). \quad (23)$$

Under fully-reversed bending (tension), $\sigma_{H,\max} = \frac{f_{-1}}{3}$ *and* $\sqrt{\langle T_a \rangle} = \frac{f_{-1}}{\sqrt{3}}$; the material constant α'_{P_1}, is

$$\alpha'_{P_1} = 3\left[\left(1-\frac{\lambda_{P_1}}{\sqrt{3}}\right)\frac{t_{-1}}{f_{-1}}-\frac{1}{\sqrt{3}}\right]. \quad (24)$$

The following restrictions,

$$\begin{cases}\alpha'_{P_1} > 0\\ \beta'_{P_1} > 0\end{cases}, \quad (25)$$

give the new range of validity of the criterion, which is now by construction applicable to all metals, when the real λ that satisfies

$$\lambda_{P_1} < \sqrt{3}\left(1-\frac{f_{-1}}{\sqrt{3}t_{-1}}\right)\ . \quad (26)$$

3.4 Extension of the domain of validity of the author's criterion

The authors' criterion was re-expressed, and the new expression of the fatigue function is

$$E'_A = \frac{\sqrt{J_{2a}\left(1+\lambda_A\left(\frac{(\sigma_{H,\max})^2}{J_{2a}}-\frac{1}{3}\right)\right) + \alpha'_A\, sign(\sigma_{H,\max})(\sigma_{H,\max})^2}}{\beta'_A}. \quad (27)$$

Where $\xi = 1/3$; observing that under fully-reversed torsional fatigue loading the relevant stress quantities are equal to: $\sigma_{H,\max} = 0 \; and \; J_{2a} = (t_{-1})^2$; and under fully reversed uniaxial fatigue loading to: $\sigma_{H,\max} = \frac{f_{-1}}{3} \; and \; J_{2a} = \left(\frac{t_{-1}}{\sqrt{3}}\right)^2$; constants β'_A, and α'_A turn out to be:

$$\beta'_A = t_{-1}\sqrt{1-\frac{\lambda_A}{3}} \; ; \tag{28}$$

$$\alpha'_A = 9\left[\left(1-\frac{\lambda_A}{3}\right)\left(\frac{t_{-1}}{f_{-1}}\right)^2 - \frac{1}{3}\right]. \tag{29}$$

The range of validity of the criterion is given under the conditions

$$\begin{cases} \alpha'_A > 0 \\ \beta'_A > 0 \end{cases}. \tag{30}$$

The criterion is now valid for all metals, when the value of the extension parameter λ_A satisfies Eq. (31).

$$\lambda < 3\left(1-\left(\frac{f_{-1}}{\sqrt{3}t_{-1}}\right)^2\right) \tag{31}$$

4 Results

To show the accuracy of the adapted criteria to effectively extend the validity domain as proposed, results from a systematic bibliographical investigation on un-notched samples reported in [13, 17] are used. Table 2 summarizes the fatigue properties of the materials and the values of material parameters obtained from the original and adapted criteria. The values of the criteria validity domain extension parameters, are $\lambda_C = \sqrt{3} - 2$; $\lambda_{P_1} = \sqrt{3} - 2$; $\lambda_A = -1$, respectively for the Crossland, Papadopoulos and the Authors criteria. It needs to be pointed out that material parameters ($\alpha'_C, \alpha'_{P_1}, \alpha'_A$) in the expressions of adapted criteria should be non-negative for the extension methodology to be valid.

The accuracy of results of the application of the proposed models to experimental data when predicting fatigue limits is determined by comparing the proximity of E, the predicted total damage, to unity. Therefore, the error index is,

$$I = \frac{E-1}{1} \times 100(\%). \tag{32}$$

The predicted fatigue damage indicator I measures the relative difference between the estimation of the criterion and the experimental data. A negative value of I means that, the criterion predicts a greater fatigue limit than experimental one; resulting in a non-conservative prediction. Conversely, a positive value of I corresponds to a conservative prediction. If the error index I is close to zero, it means that the agreement is good between prediction and experimental results.

The minimum circumscribed ellipse proposed in [8] is used to compute the effective shear stress amplitude in Crossland criterion. The equivalent stress approach proposed in [12] was used in the computation of the Authors criterion.

The applicability of the present models are validated through an error index I Eq. (32); firstly with a hard steel. Secondly “out of their validity domain” with two extra-ductile carbon steel labelled as C1007, C20; and by two ductile carbon steels C1035, C1010.

Tables 3- 7 list the results for these materials. Where, the error index I_C, refers to the results of Crossland criterion, I'_C to the adapted Crossland criterion; I_{P_1} to the Papadopoulos criterion, I'_{P_1} to the adapted Papadopoulos criterion. And finally I_A and I'_A are respectively the Authors and adapted Authors criteria.

5 Discussion

One can clearly see from Table 2 that we have negative values for parameters α_C, α_{P_1} and α_A respectively in Crossland, Papadopoulos, and the Authors criteria. The negative values indicates that the materials are out the validity domain. These negative values are now positive with the adapted criteria, as a consequence of the extension of the validity domain.

In spite of the small number of experimental data, Table 3, the proposed adapted multiaxial fatigue criteria enables the extension of the validity domain of stress based criteria without modifying their prediction capacity. Predictions coincides with the original formulations since results between the adapted criteria and their original formulation are in perfect agreement as shown in Table 3.

From the two extra-ductile materials (Table 4-7) we were able to predict with a good degree of confidence the fatigue strength limit. These results evidences the applicability of the studied criteria out of their natural validity domain. Although the Papadopoulos' criterion is restricted to hard metals [10], the results shows that when the validity domain of the criterion is extended, the criterion is equivalent to Crossland criterion implemented with the MCE approach.

The comparisons of the predictions, shows that the non-proportional to proportional stresses transformation when used in the Authors criterion gives much improved predictions (lowest average absolute value of the error index) than the MCE approach. The equivalent stress approach is an efficient and easy-of-use procedure that considers the non-proportional loading effects when implemented in the criterion proposed by the authors.

6 Conclusions

A new fatigue stress transformation for adapted criteria and validity domain extension is developed in this paper. The transformation is applied on some multiaxial fatigue criteria so as to extend their domain of validity to extra-ductile and ductile materials.

Initially, the range of validity of the Crossland, Papadopoulos and Authors criteria were given by the condition $t_{-1}/f_{-1} > 1/\sqrt{3}$. The criterion was not valid for ductile metals, and extremely ductile metals. Through the proposed adapted formulations, the new equivalent criteria can now be used safely to predict fatigue limit.

Based on the proposed modifications the adapted criteria from their new formulations are now applicable for all materials: hard, brittle, ductile, and extremely ductile materials. Thus the proposed models for adapted criteria ensure that these criteria are no more limited to some specific materials.

At present, the validity domain extension are formulated from criteria based on the stress invariants approach. The method must be generalized to the critical plane and integral approaches criteria to better assess the fairness of the proposed transformations. These topics are in-progress.

Table 1 : Definition of classes for metals, in the high-cycle fatigue context, by the ratio between torsion fatigue limit t_{-1}, and alternating bending limit f_{-1} [3, 7, 8, 10, 17].

Metal class	Range for t_{-1}/f_{-1}
Extra-ductile	$t_{-1}/f_{-1} < 0.58$
Ductile	$t_{-1}/f_{-1} = 0.58$
Hard	$0.58 < t_{-1}/f_{-1} \le 0.80$
Brittle	$0.8 \le t_{-1}/f_{-1} < 1.0$
Extremely brittle	$t_{-1}/f_{-1} \ge 1$

Table 2 : Example of material from [13, 17] and criteria material parameters computation.

Metal class	Material	t_{-1}	f_{-1}	t_{-1}/f_{-1}	Sign of material parameters					
					α_c	α'_c	α_{P1}	α'_{P1}	α_A	α'_A
Extra-ductile	NiCrMo steel (75-80 tons)	342.7	660.7	0.52	-0.18	0.06	-0.18	0.06	-0.58	0.23
	NiCr steel (Hollow samples)	339.6	653.2	0.52	-0.17	0.07	-0.17	0.07	-0.57	0.24
	NiCr steel (Solid samples)	369.7	666.7	0.55	-0.07	0.19	-0.07	0.19	-0.23	0.69
	C1007 steel	147	264	0.56	-0.06	0.20	-0.06	0.20	-0.21	0.72
	NiCrMo steel S81	331.9	589.7	0.56	-0.04	0.22	-0.04	0.22	-0.15	0.80
	XC18	186	332	0.56	-0.05	0.21	-0.05	0.21	-0.18	0.77
	NiCrMo steel (60-70 tons)	337.7	602.1	O.56	-0.05	0.21	-0.05	0.21	-0.17	0.77
	0.1% C steel (normalised)	151.3	268.6	0.56	-0.04	0.22	-0.04	0.22	-0.14	0.81
	0.4% C steel (spheroidized)	155.9	274.8	0.57	-0.03	0.23	-0.03	0.23	-0.10	0.86
	St35	130	230	0.57	-0.04	0.23	-0.04	0.23	-0.12	0.83
Ductile	High strength steel	364	630	0.58	0.00	0.27	0.00	0.27	0.00	1.01
	High strength steel	172	298	0.58	-0.00	0.27	-0.00	0.27	-0.00	1.00
	High strength steel	184.7	320	0.58	-0.00	0.27	-0.00	0.27	-0.00	1.00
	R7T (Axial)	297	514.4	0.58	0.00	0.27	0.00	0.27	0.00	1.00
	R7T (Circum.)	310	537	0.58	-0.00	0.27	-0.00	0.27	-0.00	1.00
	0.34% C steel	218	378	0.58	-0.00	0.27	-0.00	0.27	-0.01	0.99
	CrMo steel	366.6	628.3	0.58	0.02	0.29	0.02	0.29	0.06	1.09
	Mild Steel	137.3	235.4	0.58	0.02	0.29	0.02	0.29	0.06	1.08
	C1035	138	239	0.58	0.00	0.27	0.00	0.27	0.00	1.00
	C1010	137	235	0.58	0.02	0.29	0.02	0.29	0.06	1.08

Table 3: Experimental fatigue limits and model errors for hard steel (f_{-1}=313.9MPa, t_{-1}=196.2MPa) [14].

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\sigma_{xy,a}$	$\sigma_{xy,m}$	$\beta(°)$	$I_{C_{MCC}}$	$I'_{C_{MCC}}$	I_{P_1}	I'_{P_1}	I_A	I'_A
1	138.1	0	167.1	0	0	-2.28	-2.28	-2.28	-2.28	-4.14	-4.14
2	140.4	0	169.9	0	30	-2.55	-2.55	-0.64	-0.64	-1.78	-1.78
3	145.7	0	176.3	0	60	-3.61	-3.61	3.10	3.10	1.94	1.94
4	150.2	0	181.7	0	90	-3.74	-3.74	6.27	6.27	4.29	4.29
5	245.3	0	122.6	0	0	1.44	1.44	1.44	1.44	0.06	0.06
6	249.7	0	124.8	0	30	0.02	0.02	3.26	3.26	2.24	2.24
7	252.4	0	126.2	0	60	-8.35	-8.35	4.39	4.39	3.38	3.38
8	258.0	0	129.0	0	90	-17.81	-17.81	6.70	6.70	5.28	5.28
9	299.1	0	62.8	0	0	0.92	0.92	0.92	0.92	0.52	0.52
10	304.5	0	63.9	0	90	-2.99	-2.99	2.74	2.74	2.33	2.33
Mean value of errors index						-3.89	-3.89	2.59	2.59	1.41	-1.70
Mean absolute value of the error index						4.37	4.37	3.17	3.17	2.60	3.94

Table 4 : Experimental fatigue limits and model errors, for extra-ductile C1007 (f_1=264MPa, t_{-1}=147MPa) [17].

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\sigma_{xy,a}$	$\sigma_{xy,m}$	$\beta(°)$	$I'_{C_{MCE}}$	I'_A	I'_{P_1}
1	264	0	0	0	0	0	0	0
2	109	0	131	0	0	-2.66	-1.78	-2.66
3	194	0	97.1	0	0	-1.87	-1.19	-1.87
4	254	0	52.6	0	0	2.44	2.65	2.44
5	0	0	147	0	0	0	0	0
Mean value of errors index						-0.42	-0.07	-0.42
Mean absolute value of the error index						1.39	1.13	1.40

Table 5: Experimental fatigue limits and model errors, as cited in [14], for extra-ductile C20 (f_1=332 MPa, t_{-1}=186 MPa).

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\sigma_{xy,a}$	$\sigma_{xy,m}$	$\beta(°)$	$I'_{C_{MCE}}$	I'_A	I'_{P_1}
1	246	0	138	0	0	4.21	4.86	4.21
2	246	0	138	0	45	4.21	5.42	4.21
3	264	0	148	0	90	11.79	12.52	11.79
Mean value of errors index						6.73	7.60	6.73
Mean absolute value of the error index						6.73	7.60	6.73

Table 6: Experimental fatigue limits and model errors, as cited in [17], for ductile C1035 (f_1=239 MPa, t_{-1}=138 MPa).

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\sigma_{xy,a}$	$\sigma_{xy,m}$	$\beta(°)$	$I'_{C_{MCE}}$	I'_A	I'_{P_1}
1	239	0	0	0	0	-0.00	-0.00	-0.00
2	69.6	0	130	0	0	-1.40	-1.40	-1.40
3	131	0	113	0	0	-1.46	-1.46	-1.46
4	180	0	89.9	0	0	-0.42	-0.42	-0.42
5	214	0	61.7	0	0	0.08	0.08	0.08
6	236	0	31.6	0	0	1.37	1.37	1.37
7	0	0	138	0	0	0	0	0
Mean value of errors index						-0.26	-0.26	-0.26
Mean absolute value of the error index						0.68	0.68	0.68

Table 7: Experimental fatigue limits and model errors, as cited in [17], for ductile C1010 (f_1=235 MPa, t_{-1}=137 MPa).

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\sigma_{xy,a}$	$\sigma_{xy,m}$	$\beta(°)$	$I'_{C_{MCE}}$	I'_A	I'_{P_1}
1	235.0	0	0.0	0	0	0	0	0
2	97.6	0	118.0	0	0	-4.15	-4.38	-4.15
3	101.0	0	122.0	0	0	-0.88	-1.12	-0.88
4	101.0	0	122.0	0	60	-0.88	-0.15	-0.88
5	109.0	0	131.0	0	90	6.53	6.33	6.53
6	180.0	0	90.1	0	0	1.14	0.96	1.14
7	187.0	0	93.6	0	60	5.07	5.42	5.07
8	201.0	0	101.0	0	90	13.12	12.96	13.12
9	223.0	0	46.2	0	0	0.76	0.71	0.76
10	227.0	0	46.9	0	90	2.54	2.49	2.54
Mean value of errors index						2.11	2.11	2.11
Mean absolute value of the error index						3.19	3.14	3.19

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