PLANE SYMMETRIC VACUUM COSMOLOGICAL MODEL IN LAU & PROKHOVNIK THEORY OF GRAVITATION

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Abstract

The vacuum plane symmetric model has been obtained in scalar tensor relativistic theory formulated by Lau and Prokhonvik (1986) under the assumption of a certain relationship between the cosmological term Λ and scalar field ψ. The dynamical behavior of the model have been studied.

1. Introduction

Lau and Prokhovnik (1986) formulated a scalar tensor theory in terms of an action principle. This theory is a generalization of Einstein's general relativity and Lau's theory (1985).This developed theory is applied to a cosmological model compatible with large number of hypothesis and ψ is then deduced. Further, Maharaj and Beesham (1988) pointed out an error in field equations obtained by Lau and Prokhovnik (1986). They have obtained a vacuum solution to the generalized field equations given by Lau and Prokhovnik (1986) for the Robertson-Walker space-time. This is in contrast to the earlier theory of Lau (1985) in which the vacuum solutions turned out to be identical to the corresponding general relativistic solutions with constant λ. Der Sarkissian (1985), Beesham (1987) discussed various cases of Lau's theory. Alfonso-Faus(1986) has studied variable G-cosmology based on Einstein's field equations.

In this paper we have studied the plane symmetric cosmological model in vacuum. Its dynamical behavior has been discussed.

2. Metric &Field equations:

Lau and Prokhovnik in 1986 have started with the variation of action given by

$$I = \int \sqrt{-g}\left(R - 2\lambda + g^{ij}\psi_{,i}\psi_{,j} - 16\pi G L_m\right)d^4x \quad , \tag{2.1}$$

Where R is Ricci scalar, L_m is mass Lagrangian density including all non-gravitational fields, ψ is scalar field , λ and G depend upon time t .

The variation of action given by equation (2.1) with respect to g_{ij} gives required field equations as

$$R_{ij} - \frac{1}{2}g_{ij}R + \Lambda g_{ij} = -8\pi G T_{ij} , \tag{2.2}$$

where T_{ij} is energy momentum tensor given by

$$T_{ij} = \frac{2}{\sqrt{-g}} \frac{\partial}{\partial g^{ij}} \left(\sqrt{-g} L_m \right) \quad (2.3)$$

R_{ij} is Ricci tensor and Λ is a new cosmological term depends upon the spatial coordinates and *t*.

For spatially homogeneous space time (as g^{44} is unity), Λ is given by

$$\Lambda = \lambda - \frac{1}{2}\dot{\psi}^2 \quad (2.4)$$

In case of vacuum $L_m = 0$, $T_{ij} = 0$.

Consider the plane symmetric metric given by

$$ds^2 = dt^2 - A^2\left(dx^2 + dy^2\right) - B^2 dz^2 , \quad (2.5)$$

where A and B are functions of *t* only.

In this case, the vacuum field equations (2.2) for metric (2.5) reduce to

$$\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \Lambda \quad (2.6)$$

$$\frac{2\dot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 = \Lambda \quad (2.7)$$

$$\left(\frac{\dot{A}}{A}\right)^2 + \frac{2\dot{A}\dot{B}}{AB} - \psi^2 = \Lambda \quad (2.8)$$

We have three equations in four unknowns A, B, Λ and ψ .

By subtracting equation (2.7) from equation (2.6), we get

$$\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\ddot{A}}{A} - \left(\frac{\dot{A}}{A}\right)^2 = 0 \quad (2.9)$$

To solve the above field equations, let we consider

$$A = B^n$$

Differentiating, we get

$$\dot{A} = nB^{n-1}\dot{B}$$

&

$$\ddot{A} = nB^{n-1}\ddot{B} + n(n-1)B^{n-2}\dot{B}^2$$

Using these values of $A, \dot{A}, \ddot{A}$ in equation (2.9), we obtain

$$\frac{\ddot{B}}{B} - \frac{2n^2}{1-n}\left(\frac{\dot{B}}{B}\right)^2 = 0 \quad (2.10)$$

Multiplying by B^2 we get,

$$B\ddot{B} - \frac{2n^2}{1-n}\left(\dot{B}\right)^2 = 0 \quad (2.11)$$

Let $\dot{B} = f(B)$

$\ddot{B} = ff'$, where $f' = \dfrac{df}{dB}$

The equation (2.11) gives

$$Bff' - \frac{2n^2}{1-n}(f)^2 = 0$$

Dividing by *B*, we get

$$ff' - \frac{2n^2}{1-n}\left(\frac{f^2}{B}\right) = 0$$

Multiply by 2 we get

$$2ff' - \frac{4n^2}{1-n}\left(\frac{f^2}{B}\right) = 0$$

Further calculation and simplifications and integration give us

$B^{\frac{-4n^2}{1-n}} f^2 = K$, where K is constant of integration.

$$f^2 = KB^{\frac{4n^2}{1-n}} \tag{2.12}$$

But $f = \dot{B} = \dfrac{dB}{dt} \Rightarrow f^2 = \left(\dfrac{dB}{dt}\right)^2$

$$\left(\frac{dB}{dt}\right)^2 = KB^{\frac{4n^2}{1-n}}$$

$$B^{\frac{-4n^2}{1-n}}(dB)^2 = K(dt)^2$$

Taking square root of both sides, we get

$$\left[B^{\frac{-4n^2}{1-n}}\right]^{\frac{1}{2}} dB = \sqrt{K}\,dt$$

$$B^{\frac{-2n^2}{1-n}} dB = \sqrt{K}\,dt$$

Integrating, we get

$$B = K_3(K_1 t + K_2)^{\frac{1}{N}}, \text{ where } K_3 = (N)^{\frac{1}{N}} \tag{2.13}$$

But, we have $A = B^n$

Therefore

$$A = K_4(K_1 t + K_2)^{\frac{n}{N}}, \text{ where } K_4 = (K_3)^n \tag{2.14}$$

Now we consider, special volume as $V^3 = A^2 B$

Using values of *A* and *B* from equation (2.13) and equation (2.14), we get

$$V = K_5(K_1 t + K_2)^{\frac{2n+1}{3N}}, \text{ where } K_5 = \left(K_3 K_4{}^2\right)^{\frac{1}{3}} \tag{2.15}$$

3. Dynamical parameters:

The dynamical parameters calculated for the above metric and found to be as

Shear scalar σ^2 is given by

$$\sigma^2 = \frac{2K_1(n-1)}{3N(K_1t+K_2)^2} \tag{3.1}$$

And the scalar expansion θ is given by

$$\theta = 3K_6(K_1t+K_2)^{-1}, \text{ where } K_6 = K_1\frac{2n+1}{3N} \tag{3.2}$$

The Hubble parameter H is given by

$$H = K_6(K_1t+K_2)^{-1} \tag{3.3}$$

The deceleration parameter q is given by,

$$q = \frac{3N}{2n+1} - 1 \tag{3.4}$$

The cosmological term Λ is given by

$$\Lambda = K_7(K_1t+K_2)^{-2}, \text{ where } K_7 = K_1^2\frac{n}{N}\left(3\frac{n}{N}-1\right) \tag{3.5}$$

And $\psi = K_8 \log(K_1t+K_2)$, (3.6)

Where $K_8 = \frac{\left(-2K_1^2\frac{n}{N}\left(3\frac{n}{N}-1\right)\right)^{\frac{1}{2}}}{K_1}$

4. Conclusion:

We have studied the plane symmetric cosmological model in vacuum.

The solutions are consistent with physical nature of the field equations.

This universe has singularity at $t = -\frac{K_2}{K_1}$.

The parameters θ, σ, Λ and H decreases continuously and after long time shear scalar vanishes.

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