Bianchi Type – III Dark Energy Cosmological Model in *f(R)* Theory of Gravitation

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Abstract
In this paper we have used different f(R) action, coupled to two scalar fields in order to obtain a new Bianchi type – III cosmological model in f(R) theory of Gravity. We have derived the standard cosmological quantities and compared them with the respective cosmological quantities in General Relativity.

Keywords: Equation of State, *f(*R)-Gravity, Bianchi Type – III Model, Dynamical Parameters

1. Introduction

In modern cosmology, the concept of cosmic microwave background (CMB) is most important. Its theoretical aspects play an important role. Standard model of cosmology is based on the inflation theory as well as theoretical aspects of cosmic microwave background. These fundamental concepts are expressed in several recent text books. The standard model of cosmology is mathematically treated by the Bianchi type - III model. This model is consistent with the early and current state universe. It has been observed that the universe is homogeneous and isotropic in the large scale structure [7]. We find that some standard cosmological models are based on general relativity, which are unable to explain; like unisotropy or the accelerated expansion of the universe. Therefore, the standard model cosmology should be replaced by other types, based on alternative theories of gravity [9-22]. *f(R)*theory of gravity [1-6] is one of them. In this theory the common Einstein-Hilbert action

$$S_{E-H} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R-2\Lambda\right)+\int d^4x\sqrt{-g}L_m \tag{1.1}$$

is replaced by

$$S_{f(R)} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(f(R)-2\Lambda\right)+\int d^4x\sqrt{-g}L_m \tag{1.2}$$

Where *f*(R) is a function of space-time Ricci scalar curvature R and L_m is the matter Lagrangian. Now, by varying the action (1.2) with respect to the space-time metric $g_{\mu\nu}$, we get the corresponding field equations as

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu \nabla_\nu f'(R) + g_{\mu\nu} \Box f'(R) + g_{\mu\nu}\Lambda = 8\pi G T_{\mu\nu} \tag{1.3}$$

where $f'(R) = \frac{df(R)}{dR}$, $\Box \equiv \nabla^\mu \nabla_\mu$, ∇_μ is the covariant derivative, $T_{\mu\nu}$ is the standard matter energy-momentum tensor derived from the Lagrangian L_m. These are the fourth order partial differential equations in the metric tensor $g_{\mu\nu}$.

2. Bianchi type –III cosmological Model Coupled to Scalar Fields in Standard Cosmology

We introduce the following relation as a substitution for relation (1.2)

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[G(\varphi)R + F(\psi) - 2\Lambda\right] + \int d^4x\sqrt{-g}\,L_m \tag{2.1}$$

Here $\varphi \equiv \varphi(R)$ and $\psi \equiv \psi(R)$ the consequent field equations would be

$$\begin{aligned}&\left[G(\varphi) + RG'(\varphi) + F'(\psi)\right]R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\left[RG(\varphi) + F(\psi) - 2\Lambda\right]\\&-\nabla_\mu\nabla_\nu\left[G(\varphi) + RG'(\varphi) + F'(\psi)\right]\\&+ g_{\mu\nu}\Box\left[G(\varphi) + RG'(\varphi) + F'(\psi)\right] = 8\pi G T_{\mu\nu}\end{aligned} \tag{2.2}$$

where the prime stands for differentiation with respect to R.

The dynamics of standard cosmology come from the Bianchi type– III model and its geometric interpretations of space-time, using the line element

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2ax}dy^2 - C^2dz^2 \tag{2.3}$$

in which A, B and C are the functions of the cosmic time t only and 'a' is non-zero constant.

According to this model, we have a homogeneous isotropic distribution of matter, forming the energy-momentum tensor of the perfect fluid.

$$T_{\mu\nu} = (\rho + p)U \otimes U + g_{\mu\nu}p \tag{2.4}$$

here, ρ is the energy density and p is the pressure of the cosmic fluid. Also U_μ is the 4-velocity vector.

According to the field equations, we can derive

$$\begin{aligned}8\pi G T_{00} = 8\pi G\rho &= \left[G(\varphi) + RG'(\varphi)F'(\psi)\right]R_{00} - \frac{1}{2}g_{00}\left[RG(\varphi) + F(\psi) - 2\Lambda\right]\\&-\partial_0\partial_0\left[G(\varphi) + RG'(\varphi) + F'(\psi)\right]\\&+ g_{00}\left\{g^{00}\partial_0\partial_0\left[G(\varphi) + RG'(\varphi) + F'(\psi)\right] - \left(\Gamma^0_{11} + \Gamma^0_{22} + \Gamma^0_{33}\right)\partial_0\left[RG(\varphi) + F(\psi) - 2\Lambda\right]\right\}\end{aligned} \tag{2.5a}$$

and

$$8\pi GT_{11}=8\pi Gp=\left[G(\varphi)+RG'(\varphi)+F'(\psi)\right]R_{11}-\frac{1}{2}g_{11}\left[RG(\varphi)+F(\psi)-2\Lambda\right]$$
$$-\Gamma_{11}^{0}\partial_{0}\left[G(\varphi)+RG'(\varphi)+F'(\psi)\right]$$
$$+g_{11}\left\{g^{00}\partial_{0}\partial_{0}\left[G(\varphi)+RG'(\varphi)+F'(\psi)\right]-\left(\Gamma_{11}^{0}+\Gamma_{22}^{0}+\Gamma_{33}^{0}\right)\partial_{0}\left[RG(\varphi)+F(\psi)-2\Lambda\right]\right\}$$

(2.5b)

Using equation (2.3) in (2.5), we get

$$8\pi G\rho=\left[G+R\frac{dG}{d\varphi}\frac{d\varphi}{dR}+\frac{dF}{d\psi}\frac{d\psi}{dR}\right]R_{00}-\frac{1}{2}\left[RG+F-2\Lambda\right]$$
$$+\left(\Gamma_{11}^{0}+\Gamma_{22}^{0}+\Gamma_{33}^{0}\right)\left\{2\frac{dG}{d\varphi}\varphi'\dot{R}+R\left[\left(\frac{dG}{d\varphi}\right)^{\cdot}\varphi'+\frac{dG}{d\varphi}\dot{\varphi}'\right]+\left(\frac{dF}{d\psi}\right)^{\cdot}\psi'+\frac{dF}{d\psi}\dot{\psi}'\right\} \quad (2.6a)$$

$$8\pi Gp=\left[G+R\frac{dG}{d\varphi}\frac{d\varphi}{dR}+\frac{dF}{d\psi}\frac{d\psi}{dR}\right]R_{11}+\frac{1}{2}A^{2}\left[RG+F-2\Lambda\right]$$
$$+\left(-2A^{2}\right)\left[\left(\frac{dG}{d\varphi}\right)^{\cdot}\varphi'\dot{R}+\frac{dG}{d\varphi}\dot{\varphi}'\dot{R}+\frac{dG}{d\varphi}\ddot{R}\right]-A^{2}\left\{\dot{R}\left[\left(\frac{dG}{d\varphi}\right)^{\cdot}\varphi'+\frac{dG}{d\varphi}\dot{\varphi}'\right]+R\left[\left(\frac{dG}{d\varphi}\right)^{\cdot\cdot}\varphi'+2\left(\frac{dG}{d\varphi}\right)^{\cdot}\varphi'+\frac{dG}{d\varphi}\ddot{\varphi}'\right]\right\}$$
$$-A^{2}\left[\left(\frac{dF}{d\psi}\right)^{\cdot\cdot}+2\left(\frac{dF}{d\psi}\right)^{\cdot}\dot{\psi}'+\frac{dF}{d\psi}\ddot{\psi}'\right]+A^{2}\left(\Gamma_{11}^{0}+\Gamma_{22}^{0}+\Gamma_{33}^{0}\right)\left[2\frac{dG}{d\varphi}\varphi'\dot{R}+R\left(\left(\frac{dG}{d\varphi}\right)^{\cdot}\varphi'+\frac{dG}{d\varphi}\dot{\varphi}'\right)+\left(\frac{dF}{d\psi}\right)^{\cdot}\psi'+\frac{dF}{d\psi}\dot{\psi}'\right]$$

(2.6b)

The dot stands for differentiation with respect to cosmic time *t*. These values will be derived explicitly, when we consider

$$R_{00}=\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}$$
$$R_{11}=-\left(A\ddot{A}+\frac{A\dot{A}\dot{B}}{B}+\frac{A\dot{A}\dot{C}}{C}\right)+a^{2} \quad (2.7)$$
$$R=2\left(\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}-\frac{a^{2}}{A^{2}}\right)$$

And

$$\Gamma_{11}^{0}=A\dot{A}$$
$$\Gamma_{22}^{0}=B\dot{B}e^{-2ax}$$
$$\Gamma_{33}^{0}=C\dot{C}$$

3. Dynamical Properties for Definite Scalar Potentials in Bianchi Type-III

Now let us consider

$G(\varphi)\equiv G(R)\doteq R$

$$F(\psi) \equiv F\left(\frac{1}{R}\right) \doteq \frac{1}{R} \tag{3.1}$$

Using equation (3.1) in equation (2.6), while considering the values in equation (2.7), we obtain

$$8\pi G\rho = \left[4(\alpha) - \frac{1}{4}(\alpha)^{-2}\right]\left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}\right) - 2(\alpha)^2 - \frac{1}{4}(\alpha)^{-1} + \Lambda$$

$$-\left(A\dot{A} + B\dot{B}e^{-2ax} + C\dot{C}\right)\left[4(\beta) + \frac{1}{2}(\alpha)^{-3}(\beta)\right] \tag{3.2a}$$

and

$$8\pi Gp = \left[4(\alpha) - \frac{1}{4}(\alpha)^{-2}\right]\left(-A\ddot{A} - \frac{A\dot{A}\dot{B}}{B} - \frac{A\dot{A}\dot{C}}{C} + a^2\right) + A^2\left[2(\alpha)^2 + \frac{1}{4}(\alpha)^{-1} + \Lambda\right]$$

$$+ A^2\left\{2(\beta) + \frac{1}{2}(\alpha)^{-3}(\beta) - 4(\gamma) + \frac{3}{2}(\alpha)^{-4}(\beta)^2 - \frac{1}{2}(\alpha)^{-3}(\gamma) + \left(A\dot{A} + B\dot{B}e^{-2ax} + C\dot{C}\right)\left[4(\beta) + \frac{1}{2}(\alpha)^{-3}(\beta)\right]\right\} \tag{3.2b}$$

Where,

$$\alpha = \left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{a^2}{A^2}\right] \tag{3.3}$$

$$\beta = \frac{A^{(3)}}{A} - \frac{\dot{A}\ddot{A}}{A^2} + \frac{B^{(3)}}{B} - \frac{\dot{B}\ddot{B}}{B^2} + \frac{C^{(3)}}{C} - \frac{\dot{C}\ddot{C}}{C^2} + \left(\frac{\dot{A}\ddot{B}}{AB} + \frac{\ddot{A}\dot{B}}{AB} - \frac{\dot{A}\dot{B}^2}{AB^2} - \frac{\dot{A}^2\dot{B}}{A^2B}\right) + \left(\frac{\dot{B}\ddot{C}}{BC} + \frac{\ddot{B}\dot{C}}{BC} - \frac{\dot{B}\dot{C}^2}{BC^2} - \frac{\dot{B}^2\dot{C}}{B^2C}\right)$$

$$+\left(\frac{\dot{C}\ddot{A}}{CA} + \frac{\ddot{C}\dot{A}}{CA} - \frac{\dot{C}\dot{A}^2}{CA^2} - \frac{\dot{C}^2\dot{A}}{C^2A}\right) + 2a^2\frac{\dot{A}}{A^3}$$

(3.4)

and

$$\gamma = \frac{A^{(4)}}{A} - 2\frac{A^{(3)}\dot{A}}{A^2} - \frac{\ddot{A}^2}{A^2} + 2\frac{\dot{A}^2\ddot{A}}{A^3} + \frac{B^{(4)}}{B} - 2\frac{B^{(3)}\dot{B}}{B^2} - \frac{\ddot{B}^2}{B^2} + 2\frac{\dot{B}^2\ddot{B}}{B^3} + \frac{C^{(4)}}{C} - 2\frac{C^{(3)}\dot{C}}{C^2} - \frac{\ddot{C}^2}{C^2} + 2\frac{\dot{C}^2\ddot{C}}{C^3}$$

$$+\left(\frac{\dot{A}B^{(3)}}{AB} + \frac{\ddot{A}\ddot{B}}{AB} - \frac{\dot{A}\dot{B}\ddot{B}}{AB^2} - \frac{\dot{A}^2\ddot{B}}{A^2B}\right) + \left(\frac{\ddot{A}\ddot{B}}{AB} + \frac{A^{(3)}\dot{B}}{AB} - \frac{\ddot{A}\dot{B}^2}{AB^2} - \frac{\dot{A}\ddot{A}\dot{B}}{A^2B}\right) - \left(2\frac{\dot{A}\dot{B}\ddot{B}}{AB^2} + \frac{\ddot{A}\dot{B}^2}{AB^2} - 2\frac{\dot{A}\dot{B}^3}{AB^3} - \frac{\dot{A}^2\dot{B}^2}{A^2B^2}\right)$$

$$-\left(\frac{\dot{A}^2\ddot{B}}{A^2B} + 2\frac{\dot{A}\ddot{A}\dot{B}}{A^2B} - \frac{\dot{A}^2\dot{B}^2}{A^2B^2} - 2\frac{\dot{A}^3\dot{B}}{A^3B}\right) + \left(\frac{\dot{B}C^{(3)}}{BC} + \frac{\ddot{B}\ddot{C}}{BC} - \frac{\dot{B}\dot{C}\ddot{C}}{BC^2} - \frac{\dot{B}^2\ddot{C}}{B^2C}\right) + \left(\frac{\ddot{B}\ddot{C}}{BC} + \frac{B^{(3)}\dot{C}}{BC} - \frac{\ddot{B}\dot{C}^2}{BC^2} - \frac{\dot{B}\ddot{B}\dot{C}}{B^2C}\right)$$

$$-\left(2\frac{\dot{B}\dot{C}\ddot{C}}{BC^2} + \frac{\ddot{B}\dot{C}^2}{BC^2} - 2\frac{\dot{B}\dot{C}^3}{BC^3} - \frac{\dot{B}^2\dot{C}^2}{B^2C^2}\right) - \left(\frac{\dot{B}^2\ddot{C}}{B^2C} + 2\frac{\dot{B}\ddot{B}\dot{C}}{B^2C} - \frac{\dot{B}^2\dot{C}^2}{B^2C^2} - 2\frac{\dot{B}^3\dot{C}}{B^3C}\right)$$

$$+\left(\frac{\dot{C}A^{(3)}}{CA} + \frac{\ddot{C}\ddot{A}}{CA} - \frac{\dot{C}\dot{A}\ddot{A}}{CA^2} - \frac{\dot{C}^2\ddot{A}}{C^2A}\right) + \left(\frac{\ddot{C}\ddot{A}}{CA} + \frac{C^{(3)}\dot{A}}{CA} - \frac{\ddot{C}\dot{A}^2}{CA^2} - \frac{\dot{C}\ddot{C}\dot{A}}{C^2A}\right) - \left(2\frac{\dot{C}\dot{A}\ddot{A}}{CA^2} + \frac{\ddot{C}\dot{A}^2}{CA^2} - 2\frac{\dot{C}\dot{A}^3}{CA^3} - \frac{\dot{C}^2\dot{A}^2}{C^2A^2}\right)$$

$$-\left(\frac{\dot{C}^2\ddot{A}}{C^2A} + 2\frac{\dot{C}\ddot{C}\dot{A}}{C^2A} - \frac{\dot{C}^2\dot{A}^2}{C^2A^2} - 2\frac{\dot{C}^3\dot{A}}{C^3A}\right) + 2a^2\left(\frac{\ddot{A}}{A^3} - 3\frac{\dot{A}^2}{A^4}\right)$$

(3.5)

4. The Model to Determine EoS

According to general relativity, the energy deposit of universe could be derived from

$$R^i_j - \frac{1}{2}R\delta^i_j + \Lambda = 8\pi G T^i_j \tag{4.1}$$

For which the Bianchi-III model implies that

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{a^2}{A^2} = -\rho \tag{4.2}$$

Also the same procedure for the fluid pressure p , results in

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = p \tag{4.3}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = p \tag{4.4}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{a^2}{A^2} = p \tag{4.5}$$

And

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 \tag{4.6}$$

When$8\pi G = 1\ and \Lambda = 0$.

Above equation (4.6) implies

$$A = B \tag{4.7}$$

Using above equation the field equations (4.2) to (4.5) implies

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = p \tag{4.8}$$

$$2\frac{\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 - \frac{a^2}{A^2} = p \tag{4.9}$$

And

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{C}}{AC} - \frac{a^2}{A^2} = -\rho \tag{4.10}$$

The field equations (4.8) to (4.10) are a system of three non-linear differential equations with four unknowns A, C, ρ, p .

Hence in order to solve the system completely we assume physical condition that shear scalar σ is proportional to scalar expansion θ, which gives the following relation between metric function as

$$C = A^n \tag{4.11}$$

where $n \neq 1, n. > 1$, is an arbitrary constant.

Equating equation (4.8) and equation (4.9), we have

$$\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} + \left(\frac{\dot{A}}{A}\right)^2 - \frac{\dot{A}\dot{C}}{AC} - \frac{a^2}{A^2} = 0 \tag{4.12}$$

Using equation (4.11) in equation (4.12), we get

$$A\ddot{A} + (1+n)\dot{A}^2 = \frac{a^2}{(1-n)} \tag{4.13}$$

Let us consider

$\dot{A} = g(A)$,

$$\ddot{A} = gg^{*} \text{ where } g^{*} = \frac{dg}{dA} \tag{4.14}$$

With the help of equation (4.14), equation (4.13) reduces to

$$2gg^{*} + 2(n+1)\frac{g^2}{A} = 2\frac{a^2}{(1-n)A} \tag{4.15}$$

Solving equation (4.15) and on integration, we get

$$A^{(2n+2)}g^2 = 2\frac{a^2}{(1-n)}\frac{A^{(2n+2)}}{(2n+2)} + k_1 \tag{4.16}$$

Where k_1 is the constant of integration.

$$\text{But } g = \dot{A} \text{ and } g^2 = \dot{A}^2 \tag{4.17}$$

Using equation (4.17) inequation (4.16), we get

$$\frac{A^{(1+n)}dA}{\sqrt{\frac{a^2}{(1-n^2)}A^{(2n+2)} + k_1}} = dt \tag{4.18}$$

To get determinate solution, we take $k_1 = 0$

$$\left[\frac{a^2}{(1-n^2)}\right]^{-\frac{1}{2}} dA = dt \tag{4.19}$$

On integration,

$$A = (t + k_2)\left[\frac{a^2}{(1-n^2)}\right]^{\frac{1}{2}} \tag{4.20}$$

Where k_2 is the constant of integration.

Using equation (4.7) and equation (4.11), we obtain the scale factors A, B *and* C as

$$A = k_3(t + k_2) \tag{4.21}$$

$$B = k_3(t + k_2) \tag{4.22}$$

And

$$C = k_4(t + k_2)^n \tag{4.23}$$

Where,

$$k_3 = \left[\frac{a^2}{(1-n^2)}\right]^{\frac{1}{2}} \text{ and } k_4 = k_3^{\,n}$$

Using equations (4.21) and (4.23) in equation (4.10), we obtain the energy density (ρ) as

$$\rho = k_5(t + k_2)^{-2} \tag{4.24}$$

Using equations (4.21) and (4.23) in equation (4.9), we obtain the pressure (p) as

$$p = k_6(t + k_2)^{-2} \tag{4.25}$$

From equation (4.24) and equation (4.25), we can write

$$\rho = p \tag{4.26}$$

Also we know that the EoS could be derived from

$$\omega = \frac{p}{\rho} \tag{4.27}$$

Which implies $\omega = \textbf{\textit{constant}}$ (4.28)

5. Calibrating the energy density and the fluid pressure using *f(R)* equations

Using equation (4.7), equation (4.21) and equation (4.23) in equations (3.3), (3.4) and (3.5) we get

$$\alpha = N_1 (t + k_2)^{-2} \tag{5.1}$$

$$\beta = N_2 (t + k_2)^{-3} \tag{5.2}$$

$$\gamma = N_3 (t + k_2)^{-4} \tag{5.3}$$

Where $N_{1,} N_2 and N_3$ are constant terms.

Using equations (5.1), (5.2) and (5.3) in equations (3.2a),(3.2b) while considering $8\pi G = 1$ and $\Lambda = 0$, we obtain energy density and fluid pressure in terms of *f(R)* gravity

$$\rho = \frac{4n(n-1)N_1}{(t+k_2)^4} - \frac{n(n-1)}{4N_1^2 (t+k_2)^2} - \frac{2N_1^2}{(t+k_2)^4} - \frac{1}{4N_1 (t+k_2)} - \left[k_3^2 (t+k_2)(1+e^{-2ax}) + k_4^2 n (t+k_2)^{2n-1}\right]\left[\frac{4N_2}{(t+k_2)^3} + \frac{N_2}{2N_1^3 (t+k_2)^2}\right] \tag{5.4}$$

$$p = \frac{-4k_3^2(n+1)}{(t+k_2)^2} + \frac{4a^2 N_1}{(t+k_2)^2} + \frac{k_3^2(n+1)}{4N_1^2} - \frac{a^2}{4N_1^2} + \frac{2k_3^2 N_1^2}{(t+k_2)^2} + \frac{k_3^2(t+k_2)}{4N_1} + \frac{2k_3^2 N_2}{(t+k_2)} + \frac{k_3^2 N_2}{2N_1^3} - \frac{4k_3^2 N_3}{(t+k_2)^2} + \frac{3k_3^2 N_2^2}{2N_1^4 (t+k_2)^2} - \frac{k_3^2 N_3}{2N_1^3 (t+k_2)} + \left[k_3^2 (t+k_2)(1+e^{-2ax}) + k_4^2 n (t+k_2)^{2n-1}\right]\left[\frac{4k_3^2 N_2}{(t+k_2)} + \frac{k_3^2 N_2}{2N_1^3}\right] \tag{5.5}$$

Using equations (5.4) and (5.5), the EoS $\left(\omega = \frac{p}{\rho}\right)$ could be derived in *f*(R) Gravity

It's like

$$\omega = \frac{p}{\rho} = \frac{1}{t^2}\frac{t^4}{1} = t^2 \tag{5.6}$$

6. Physical behavior of the Model

The Hubble parameter H is defined by

$$H = \frac{\dot{V}}{V} \tag{6.1}$$

Where $V^3 = ABCe^{ax}$ and (6.2)

The deceleration parameter q is

$$q = -\frac{V\ddot{V}}{\dot{V}^2} \tag{6.3}$$

The scalar of expansion

$$\theta = 3\frac{\dot{V}}{V} \tag{6.4}$$

And the dynamical parameters are the shear σ defined by

$$\sigma^2 = \frac{1}{12}\left\{\left[\frac{g_{11,0}}{g_{11}} - \frac{g_{22,0}}{g_{22}}\right]^2 + \left[\frac{g_{22,0}}{g_{22}} - \frac{g_{33,0}}{g_{33}}\right]^2 + \left[\frac{g_{33,0}}{g_{33}} - \frac{g_{11,0}}{g_{11}}\right]^2\right\} \tag{6.5}$$

For our model, these parameters are

$$H = \frac{(2+n)}{3(t+k_2)}, \qquad q = \frac{(1-n)}{(2+n)} \tag{6.6}$$

$$\theta = \frac{(2+n)}{(t+k_2)} \text{ and } \sigma^2 = \frac{2(n^2 - 2n + 1)}{3(t+k_2)^2}$$

7. Conclusion:

We have considered Bianchi type –III cosmological model in *f*(R) theory and determined field equations. Some physical properties had been studied. Then we derived the EoS and we observed that in general relativity it remain constant. Having the resultant field equations in the standard cosmological model, we derived the energy density and the fluid pressure in our model, which were specified for definite interpretations of the scalar fields. We observed that, if cosmic time t goes on increasing continuously and after a very long time, energy density ρ and fluid pressure p will vanished.

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