PRIMER REGULATED AND ORIENTED MODELING AND PARAMETER TUNING FOR STIFFNESS MODELING IN ROBOTICS

Gangadhar T.G*[1]., Balkeshwar Singh [2], Ganesh Hegde [3], Shaik Abdul Kareem [4], Syed Mudassir [5]

[1,2,3,4,5] Mechanical & Industrial Engineering Section, Salalah College of Technology, Sultanate of Oman

*Corresponding Author Email: tggangsadi@gmail.com

Abstract

This paper on stiffness modeling and parameter tuning uses the methodology of primer regulated and oriented approach to provide a real time treatment for the solution of Robot end point deflection. The joint stiffness model has two components: 1. the normalized Cartesian stiffness depending on the geometry, material and configuration of manipulator links. 2. The active complimentary stiffness developed by the configuration of the manipulator and force applied at the end point. The stiffness model without the consideration of the later component is incomplete and its validity is true in cases where: 1. Dynamic situation governed by D'Alembert's principle is not significant and. 2. The Jacobian matrix does not change in the work volume as in case of a Cartesian Robot. The illustrative system assumed here is a two link articulated manipulator forming a base for stiffness formulation which is solved for joint forces end point deflection and joint rotations using Primer Regulated and Oriented Modeling and Parameter tuning (PROMPT) procedure which is developed on strong foundation of Hegde's primer value theorem. The experimental measurements and illustrative procedure adopted in literature as a solution to non-linearity sighted in the joint stiffness formulation is successfully avoided by this analytical procedure. The parameter tuning of joint stiffness matrix estimates the joint torques/forces leading to a supplement for the compliance and force control. This study provides a supplementary contribution to the previously reported works on an analytical basis leaving a scope to verify experimentally.

Keywords: robotics, stiffness modeling, robot manipulator, joint stiffness matrix

1. Introduction

For establishing static or dynamic equilibrium in various stages of Robotic manipulations, it is necessary to know about the magnitude and direction of the various forces and moments that are at work. The understanding and control of compliance necessitates the analysis and determination of forces and moments reflected at the end point of the manipulator. The compatibility of compliance requirements of the tasks is useful in choosing the right manipulator configuration. Further the manipulator configuration describes the end effectors stiffness.

The components of joint stiffness are the complimentary stiffness and the Cartesian stiffness as cited in [1]. For a Cartesian Robot Manipulator, the Joint Stiffness equates with the structural Cartesian stiffness as the Jacobian Matrix is a constant throughout its workspace. Even for the quasistatic configuration the complementary stiffness contributed by the Jacobian varying with link positions is zero with no loading.

The application of one of the forces, say F_Y at the end point of the manipulator results in the rotation of the frame located at the end effector due to the other forces developed by the contribution of the non-diagonal elements of the joint stiffness matrix. The appearance of the non-diagonal elements is attributed by the coupling among the rotational and translational movements of the links of the serially connected robot manipulator contributed by the inconsistent behaviour of the Cartesian space and the joint space. The joint stiffness accurately modeled would highly useful in compensating the coupling and posing errors caused by the external seen and unseen forces at the end tool connected to the manipulator. The determination of elements of complementary stiffness is governed by the D'Alembert's principle leading to the formation of the partial derivatives of the Jacobian with respect to change in angular position of the joint.

Gursel A.et al. [2] sighted the non linearity in determination of the deflection vector, ΔX and proposed a least square estimation algorithm to determine the unknown joint stiffness matrix, by minimizing the summed square of the error vector associated with the experimentally measured values, under a range of payloads.

Further the optimization problem is solved by interior-reflective Newton method or Levenburg-Marquardt algorithm. Even though the error is appreciably small after long iterations, the procedure is a approach perfection solution as compared to contained perfection of Primer Regulated and Oriented Modeling and Parameter Tuning (PROMPT) procedure.

PROMPT Procedure starts with bifurcation of the force equation in to a governing virtual pivot equation formed by column wise co-efficient summation and a constraint enzyme equation which goes with the known force component along one of the degrees of freedom. Further the Primer Adapter is calculated as a function of stiffness co-efficient and the given force, followed by the determination of the deflection vector and the force vector.

The Complementary stiffness computed with the Cartesian stiffness gives the angular rotation vector and the joint stiffness matrix. The attribution of virtuallity touch to the joint forces is eliminated by parameter tuning, giving tuned joint stiffness matrix, which satisfy the reality attached in the determination of the joint forces.

In section (II) the modeling of the stifnesses namely the Cartesian stiffness matrix, complementary stiffness matrix and the joint stiffness matrix, and expressions relating each other are derived based on the references [3] and [4].

In section (III) the primer regulated and oriented modeling and the parameter Tuning of the joint stiffness based on Hegde's Primer Value (HPV) Theorem is explained, which is author's new and original supplementary contribution. The parameter tuning of Joint stiffness to eliminate the induction of Virtuality is done in section (IV).The inability of author's accessibility to a practical robot model to experimentally verify the results provides a scope for further studies. The expressions for Jacobian Matrix of two link manipulator and the complementary stiffness derivations are presented in Appendix-I. Hegde's Primer Value (HPV) Theorem with proof, which is a strong base for PROMPT, is briefly explained in Appendix – II.

The advantages and the outcome of the PROMPT is briefly outlined in conclusion, as in section (V).The literature from references provides basis for author's thinking and development of the novel and original PROMPT procedure which needs a horizontal forward spread from the scientific and technical community in the form of experimental and applied support to derive its real time benefit.

2. Modeling Stiffness

The Force Vector F= [F_X, F_Y, F_Z] required to cause the manipulator end effectors to experience differential change of position ΔX= [δx, δy, δz]^T is given by

F= [K_{ij}] ΔX ---- (1)

$$[K_{ij}] = diag\left[K_{ij}\right]$$

$$= \left[\frac{AE}{L}, \frac{3EI_y}{L^3}, \frac{3EL_z}{L^3}\right]$$

The Angular change $\Delta\theta = \lfloor \delta\theta_{1,} \delta\theta_2, \delta\theta_3 \rfloor$ from the unloaded position of the manipulator end point, is resulted by the application of actuator torque/Force can be expressed by

$\varsigma = [K_\theta]\Delta\theta$ ----- (2)

Where τ is the (3x1) torque vector, $\varsigma = \lfloor \varsigma_{1,} \varsigma_{2,} \varsigma_3 \rfloor^T$ needed to balance the external force vector, F represented by the generalized transformation relation as

$\tau = J^T$ F ----- (3)

Where J is Jacobian matrix, the elements are derived in Appendix I, which is given by

$J=J_{ij}$ for i = j =1, 2, 3

Differentiating (3) with respect to θ yields the following relation

$K_\theta = K_c + \bar{K}_X$ ----- (4)

Where $K_c = (\frac{\partial J^T}{\partial \theta})$F, The complimentary stiffness matrix as addressed in [1] ,and $\bar{K}_X = J^T \lfloor K_j \rfloor$J, is the Normalized stiffness matrix. K_θ is the joint stiffness matrix.

3. PROMPT Procedure

3.1. Estimation of Force Vector:

In a practical situation of the robot operation it is the maximum payload the only force i.e., F_y is known among force vectors and other forces F_X, and F_Z the displacement vector being unknown solution to equation (1) looks impossible by the conventional treatment techniques. But the following procedure outlines the novel possibility by adapting the basis of Hegde's primer Value (HPV) Theorem briefly presented in the Appendix-II.

(i). The bifurcation of (1) results in two equations-

a) The Virtual Pivotal

b) Constrained Enzyme

The Virtual pivotal equation has the form

$[K_{11} K_{22} K_{33}]\, \Delta X = F_V$ ----- (5)

Where $F_v = F_x + F\, y + F\, z$

The Constraint Enzyme equation has the form

$[\ 0 \quad K_{22}\, 0]\, \Delta X = F_y$ ------ (6)

Where F_y is the vertical payload that is specified.

(ii). The estimation of Primer Adapter based on HPV Theorem is

$$h = \frac{F_y}{K_{22}\left(\frac{K_{11}}{K_{22}} - 1\right)} \quad \ldots.\ (7)$$

(iii) The displacement vectors are given by the substitution of Primer Adapter, h. $\Delta X = [\delta_{x,} \delta_y, \delta_z]^T$

$$= \left[-h, \left(\frac{K_{11}}{K_{22}} - 1 \right) h, \left[\frac{K_{22}}{K_{23}} + 1 \right] h \right]$$

(iv) The Force Vector are

$$F = [K_{11}\ \delta_{11}\ , F_Y\ , K_{33}\ \delta_Z]^T$$

Estimation of Joint Stiffness on substitution of the Force Vector in (4) gives the Joint Stiffness with the computation of complementary stiffness matrix as

$$K_C = \left[\frac{\partial J^T}{\partial \theta_1} F, \frac{\partial J^T}{\partial \theta_2} F, \frac{\partial J^T}{\partial \theta_3} F \right] \text{ ---- } (8)$$

(v) Estimation of Joint Torque

The Joint Torque is obtained by combining (2) and (3) which also provides the change in angular position about the co-ordinate frame located at the joint.

$J^T\ F = [K_\theta]\, \Delta\theta$ ---------------- (9)

Considering the First row of the equation (9), The Equation formed is

$[K_{\theta 1} K_{\theta 2} K_{\theta 3}]\, \Delta\theta$ - $[J_{11}\ J_{12}\ J_{13}]\ F = 0$ ---(10)

By solving (10) by HPV Theorem

$$\Delta\theta = [-h_\theta, \left(\frac{K_{\theta 1}}{K_{\theta 2}} - 1 \right) h\,\theta, \left(\frac{K_{\theta 2}}{K_{\theta 3}} + 1 \right) h_\theta]$$

Where the Primer Adapter is

$$h_\theta = \frac{J_{11} J\, x + J_{12} J_y + J_{13} J_z}{K\theta 3}$$

4. Stiffness Parameter Tuning

Because of Induction of Vituality in the Joint Forces Calculated it requires a parameter tinning to restore the reality in the Joint Stiffness Matrix, Which is given by

$$K_{\theta T} = \begin{matrix} K_{\theta 1}-\lambda 1 & K_{\theta 2} & K_{\theta 3} \\ K_{\theta 4} & K_{\theta 5}-\lambda 2 & K_{\theta 6} \\ K_{\theta 7} & K_{\theta 8} & K_{\theta 9}-\lambda 3 \end{matrix}$$

$K_{K\theta}$ is the Tunned Joint Stiffness Matrix.

Where

$$\lambda i = \frac{J_{ij} F - K_{\theta} \Delta_{\theta}}{\Delta \theta}$$

X is called as the Tunner.

The Joint Torques are obtained by Using the tunned joint stiffness Matrix, as

$$\varsigma = [K_{\theta T}] \Delta\theta \text{ ------------------------- } (11)$$

6. Conclusion

Primer Regulated and Oriented Modeling and Parameter Tuning (PROMPT) of stiffness of Robot Manipulator is outlined to find the end point deflections as a function of single primer Adapter obtained from the co-efficients of stiffness Matrix.

The Non –Linearity appearing due to the addition of active stiffness to the addition of active stiffness to the Cartesian stiffness in forming the Joint stiffness, has been eliminated analytically by simple arithmetical Operations used in HEGDE's Primer Value Theorem .The concept of Virtuality introduced in Joint Forces enhances the Speed of Computation as there is no evidence of inversion or Iterations.

APPENDIX -I

The (3*3) Jacobian Matrix for a two Link Manipulator with reference to (3) is given by

$$[J_{ij}]^T = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}$$

Where

$J_{11} = -l_1 \text{ Sin } \theta_1 - l_2 \text{ Sin } (\theta_1+\theta_2)$

$J_{12} = l_1 \text{ Cos } \theta_1 + l_2 \text{ Cos } (\theta_1+\theta_2)$

$J_{13} = 1$

$J_{21} = -l_2 \text{ Sin } (\theta_1+\theta_2)$

$J_{22} = l_2 \text{ Cos } (\theta_1+\theta_2)$

$J_{23} = 1$

$J_{31} = 0$

$J_{32} = 0$

$J_{33} = 1$

$[\frac{\partial J^T}{\partial \theta_1}] = [-l_1 \text{ Cos } \theta_1 - l_2 \text{ Cos } (\theta_1 + \theta_2),$

$-l_1 \text{ Sin } \theta_1 - l_2 \text{ Sin}(\theta_1 + \theta_2), 0,$

$-l_2 \text{ Cos } (\theta_1 + \theta_2), -l_2 \text{ Sin } (\theta_1 + \theta_2),$

$0, 0, 0, 0]$

$\frac{\partial J^T}{\partial \theta_3} = 0$

$K_c = [K_{c1}, K_{c2}, K_{c3}]^T$

$K = [\{-l_1 C\theta_1 - l_2 C(\theta_1+\theta_2)\} F_x, \{-l_2 C(\theta_1+\theta_2) - l_2 S(\theta_1+\theta_2)\} F_Y, 0]$

$K_{c2} = [\{-l_2 C(\theta_1+\theta_2) - l_2 S(\theta_1+\theta_2)\} F_x, \{-l_2 C(\theta_1+\theta_2) - l_2 S(\theta_1+\theta_2)\} F_Y, 0]$

$K_{c3} = [0,0,0]$

The Normalized Cartesian Stiffness Components

$\bar{K}_{11} = \frac{AE}{l_1} [\{l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2)\}^2] + \frac{3EI_Y}{l^3_1} [\{l_2 \sin(\theta_1 + \theta_2)\}^2] \quad \bar{K}_{12} = \frac{AE}{l_1}$

$[\{-l_1 \sin\theta_1 - l_2 \sin(\theta_1 + \theta_2)\}\{l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)\}] + \; + \frac{3EI_Y}{l^3_1} [\{-l_2{}^2 \sin(\theta_1 + \theta_2)\} \cos(\theta_1 + \theta_2)]$

$\bar{K}_{13} = \frac{AE}{l_1} \left[\{-l_1 \sin\theta_1 - l_2 \sin(\theta_1 + \theta_2)\} + \frac{3EI_Y}{l^3_1} [\{-l_2 \sin(\theta_1 + \theta_2)\}] \right] \bar{K}_{21} = \bar{K}_{12}, \bar{K}_{22} = \frac{AE}{l_1}$ x

$[\{l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2)\}^2] + \frac{3EI_Y}{l^3_1} [\{l_2 \cos(\theta_1 + \theta_2)\}^2]$

$\bar{K}_{23} =$

$\frac{AE}{l_1} [\{l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2)\}] + \frac{3EI_Y}{l^3_1} [\{l_2 \cos(\theta_1 + \theta_2)\}]$

$\bar{K}_{31} = \bar{K}_{13}, \; \bar{K}_{32} = \bar{K}_{23}$

$\bar{K}_{33} = \frac{AE}{l_1^3} + \frac{3EI_Y}{l^3_1} + \frac{3EI_Z}{l^3_1}$

$K_{\theta 1} = \{-l_1 C\theta_1 - l_2 C(\theta_1 + \theta_2)\} F_x + \bar{K}_{11}$

$K_{\theta 2} = \{-l_2C(\theta_1+\theta_2) - l_2S(\theta_1+\theta_2)\}F_y + \bar{K}_{12}$

$K_{\theta 3} = \bar{K}_{13}$

$K_{\theta 4} = \{-l_2C(\theta_1+\theta_2)\}F_x + \bar{K}_{21}$

$K_{\theta 5}$ $\{-l_2C(\theta_1+\theta_2) - l_2\sin(\theta_1+\theta_2)\}F_y + \overline{K}_{22}$ $K_{\theta 6} = \bar{K}_{23}$

$K_{\theta 7} = \bar{K}_{31}$

$K_{\theta 8} = \bar{K}_{32}$

$K_{\theta 9} = \bar{K}_{33}$

APPENDIX-II

Hegde's Primer value theorem for two algebraic Equations
$\sum a_i X_i = b$ And $\sum c_i X_i = d$ where b is unknown and d is known. The Solution to X_i are
$X_1 = -h$

$X_{i+1} = \left(\frac{a_i}{a_{i+1}} - 1\right)h$ for i =1,2,------n-1

$X_n = \left(\frac{a_{n-1}}{a_n} + 1\right)h$

Where the Primer Adapter, h

$$h = \frac{d}{-C_1 + \sum C_{i+1}\left(\frac{a_i}{a_{i+1}} + 1\right) + C_n\left(\frac{a_{n-1}}{a_n} + 1\right)}$$

Proof:
The Virtual Pivotal equation $\sum a_i X_i = b$ can be written as

$a_1x_1 + V_1 = 0$

$(x_1, V_1) = (-h, a_1h)$

Further

$a_2x_2 + V_2 = a_1h$

$(x_2, V_2) = \left[\left(\frac{a_1}{a_2} - 1\right)h, a_2h\right]$

Hence

$(x_{i+i}, V_{i+1}) = \left[\left(\frac{a_1}{a_2} - 1\right)h, a_2h\right]$

So on $a_nx_n - b = 0$

(x n ,b) = $\left[\left(\frac{a_{n-1}}{a_n}+1\right)h, a_n h\right]$

By substituting X$_i$ in $\sum C_i X_i = d$

$$h = \frac{d}{-C_1 + \sum C_{i+1}\left(\frac{a_i}{a_{i+1}} - 1\right) + C_n\left(\frac{a_{n-1}}{a_n} + 1\right)}$$

When b is known

$$h = \left[\frac{b}{a_n}\right]$$

Acknowledgement

We are thankful to Mr. N. Sellappan, HOS of Mechanical Engineering, Dr. Said Mohammed Al-Mashikhi, Head, Department of Engineering, and Dr. Maryam AlAwadi, Dean, Salalah College of Technology for their valuable inspiration, encouragement and providing necessary facilities for the study.

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