Blasius and Sakiadis Problems in Nanofluids using Buongiorno Model and Thermophysical Properties of Nanoliquids

Nur Syazana Anuar [1] and Norfifah Bachok* [2]

[1,2] Department of Mathematics and Institute for Mathematical Research, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor, Malaysia

*Corresponding Author E-mails: norfifah@upm.edu.my; norfifah78@yahoo.com

Abstract

This study aims to investigate the classical problems of boundary layer flow and heat transfer characteristics past a semi-infinite static flat plate (Blasius problem) and past a moving semi-infinite flat plate (Sakiadis problem) in a water-based nanofluid with Prandtl number $\mathrm{Pr} = 6.2$ *which containing three different types of nanoparticles, namely Copper* (Cu), *Alumina* $(\mathrm{Al_2O_3})$, *and Titania* $(\mathrm{TiO_2})$. *The model used for the nanofluid incorporates the effects of Brownian motion* Nb, *thermophoresis* Nt *and solid volume fraction* φ *parameters. The governing partial differential equations are transformed into a system nonlinear ordinary differential equations using a similarity transformation which is then solved numerically. Numerical results are presented in tables or graphs for the skin friction coefficients and local Nusselt number which represents the heat transfer rate at the surface as well as velocity, temperature and nanoparticle volume fraction profiles and the physical aspects are discussed in details. It is found that the Brownian motion, thermophoresis and solid volume fraction affects the fluid flow and heat transfer characteristics.*

Key words: Boundary layer, Heat Transfer, Nanofluid, Brownian motion, Thermophoresis.

1. Introduction

In a past few decades, the thermal conductivity of nanofluids has been investigated by many researchers. Many of the publications on nanofluids are about understanding of their behaviors so that they can be utilized where straight heat transfer enhancement is paramount as in many industrial application, nuclear reactors, transportation, aerodynamic extrusion of plastic sheets, hot rolling, glass fiber, the cooling and drying of paper, electronics as well as biomedicine and food. A good list of applications is available in Das et al.[1]. Choi [2] was the first person who introduces the word "nanofluid"; he defined nanofluid as a liquid containing dispersed submicronic solid particles (nanoparticles). Fluids such as water, oil and ethylene glycol mixture are poor heat transfer fluids. Therefore, nanofluid is expected to have an effective thermal conductivity in order to enhance heat transfer because nanofluids contains nanoparticle that have extremely large surface areas. Thus, it has a great potential for application in heat transfer.

Blasius [3] were among the first who investigated the problems of boundary layer flow past a static semi-infinite flat plate, without considering the heat transfer aspect. Different from Blasius [3], Sakiadis [4] investigated the boundary layer flow over a continuous solid surface moving with constant velocity. Since then, many researchers has been considered the problem of viscous boundary layer flow on a moving or fixed flat plate. Cortell [5] extended the research that had been made by Sakiadis [4] by considering on heat transfer aspect. There are also several numerical and studies on moving flat plate that can be found in the review papers by Ishak et al. [6], Bachok and Ishak [7] and Ishak et al.[8]. However, Cortell [9] discussed the problem in Blasius and Sakiadis by considering the radiation effects with a convective surface boundary conditions.

However, the above problems have not dealt with the nanofluid. There are two types of model that have been found in this field such as Buongiorno [10] and Tiwari and Das [11]. Tiwari and Das [11] model analyses the behaviour of nanofluids taking into account the solid volume fraction. It is worth mentioning that the nanofluid model proposed by Tiwari and Das [11] was frequently used by Rohni et al. [12], Ahmad et al. [13], Bachok et al. [14], Bachok et al. [15], Elbashbeshy et al. [16] and Subashini and Sumathi [17] in their papers. Unlike Tiwari and Das [11] model, less attention has been focused on Buongiorno [10] model for this field. In Buongiorno [10] model, it considers a problem of nanofluids where this model focuses on two types of mechanisms which are Brownian motion Nb and thermophoresis Nt. To the best of our knowledge, there are only the papers by Bachok et al. [18], which are dealing with boundary layer flow over a moving surface in a flowing fluid and Rosca and Pop [19], for the problem of unsteady boundary layer flow past a moving surface in an external uniform free stream.

Different from the previous approach, the aim of the present paper is to consider a problem using a combination of Buongiorno [10] and Tiwari and Das [11] models. The problem with combination of both nanofluid models has been considered by Pop et al. [20] and Noor et al. [21], which are dealing with boundary layer flow past a moving surface. However, we are going to extend the problem of Ahmad et al. [13] by considering the effect of Brownian motion, thermophoresis and nanoparticle volume fraction towards fluid flow and heat transfer characteristics. The governing equations in the form of partial differential equations are transformed into nonlinear ordinary differential equations and then solved numerically using a shooting method built in Maple software. Three different types of nanofluids made of Cu, $\mathrm{Al_2O_3}$ and $\mathrm{TiO_2}$ are tested with Prandtl number, $\Pr = 6.2$.

2. Mathematical formulation

Consider a steady two-dimensional boundary layer flow past a fixed (Blasius [3]) or past a moving (Sakiadis [4]) semi-infinite flat plate. It is assumed that the nanofluid is incompressible, laminar flow, and the viscous dissipation and radiation effects are neglected. Let u and v be the velocity components along x and y directions. The flow takes place at $y \geq 0$ where y is the coordinate measure normal to the surface. Further, we assume that the uniform temperature and the uniform nanofluid volume fraction at the surface of the plate are T_w and C_w, while the uniform temperature and the uniform nanofluid volume fraction far from the surface of the plate are T_∞ and C_∞, respectively. Under these assumptions and following the model equations of a nanofluid proposed by Buongiorno [10] and Tiwari and Das [11], the boundary layer equations of mass, momentum, and thermal energy can be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right], \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2}. \quad (4)$$

Subject to the boundary conditions:

i) Blasius problem

$v=0$, $u=0$ at $y=0$,

$u=U$ as $y\to\infty$ (5a)

ii) Sakiadis problem

$v=0$, $u=U$ at $y=0$,

$u=0$ as $y\to\infty$ (5b)

where u and v be the velocity components along the axes x and y, and U is the constant velocity of the free stream (inviscid flow) or that of a moving flat plate. The boundary conditions for the equations (3) and (4) are

$T=T_w$ at $y=0$, $T=T_\infty$ as $y\to\infty$ (6)

$C=C_w$ at $y=0$, $C=C_\infty$ as $y\to\infty$ (7)

It is assumed that ρ is the pressure of nanofluid, μ_{nf} is the dynamic viscosity of the nanofluid, ρ_{nf} is the density of the nanofluid, and α_{nf} is thermal diffusivity of the nanofluid, which is given by Oztop and Abu-Nada [22]:

$$\mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}}, \quad \rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s, \quad \alpha_{nf}=\frac{k_{nf}}{(\rho C_\rho)_{nf}},$$

$$\frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\varphi(k_f-k_s)}{k_s+2k_f+\varphi(k_f-k_s)}, \quad (\rho C_\rho)_{nf}=(1-\varphi)(\rho C_\rho)_f+\varphi(\rho C_\rho)_s. \quad (8)$$

Where φ is the solid volume fraction parameter of the nanofluid, k_{nf} is the thermal conductivity of nanofluid, k_f is the thermal conductivity of the fluid fraction, k_s is the thermal conductivity of the solid volume fraction, ρ_f is the reference density of the fluid fraction, ρ_s is the reference density of solid fraction, μ_f is the viscosity of the fluid fraction, and $(\rho C_p)_{nf}$ is the heat capacitance of the nanofluids, where C_p is the specific heat at constant pressure. The viscosity μ_{nf} of the nanofluid given by Brinkman can be approximated as viscosity of the base fluid μ_f containing dilute suspension of fine spherical particles.

The governing Eqs. (1) - (4) subjected to the boundary conditions (5) – (7) can be expressed in a simpler form by introducing the following similarity transformation:

$$\psi=(U\nu_f x)^{\frac{1}{2}}f(\eta), \quad \eta=\left(\frac{U}{\nu_f x}\right)^{\frac{1}{2}}y,,$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \quad \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}. \quad (9)$$

Where ν_f is the kinematic viscosity of the fluid fraction and ψ is the stream function that is defined as

$u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, which satisfy Eq. (1). By applying the Eqs. (8) and (9), Eqs. (2) - (4) can be reduce to the following ordinary differential equations:

$$\frac{1}{(1-\varphi)^{2.5}\left(1-\varphi+\varphi\rho_s/\rho_f\right)} f''' + \frac{1}{2} ff'' = 0 , \qquad (10)$$

$$\frac{1}{\Pr}\left[\frac{k_{nf}/k_f}{1-\varphi+\varphi(\rho C_p)_s/(\rho C_p)_f}\right]\theta'' + \frac{1}{2} f\theta' + Nb\phi'\theta' + Nt\theta'^2 = 0 , \qquad (11)$$

$$\phi'' + \frac{1}{2} Le\phi' f + \frac{Nt}{Nb}\theta'' = 0 . \qquad (12)$$

subjected to the boundary conditions (5a) and (5b) which become

i) Blasius problem

$$f(0) = 0, \quad f'(0) = 0, \quad f'(\infty) = 1 . \qquad (13a)$$

ii) Sakiadis problem

$$f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) = 0 . \qquad (13b)$$

And for Eqs. (6) and (7), we have

$$\theta(0) = 1, \quad \theta(\infty) = 0, \quad \phi(0) = 1, \quad \phi(\infty) = 0 \qquad (14)$$

where primes denoted as differentiation with respect to η and the four parameters are defined by

$$\Pr = \frac{\nu_f}{\alpha_f} , \qquad Le = \frac{\nu_f}{D_B} ,$$

$$Nb = \frac{\tau D_B (C_w - C_\infty)}{\nu_f} , \qquad Nt = \frac{\tau D_T (T_w - T_\infty)}{T_\infty \nu_f} . \qquad (15)$$

Here $\Pr$, Le, Nb, and Nt denote the Prandtl number, Lewis number, the Brownian motion parameter and the thermophoresis parameter. The physical quantities of interest in this study are the skin friction coefficients C_f, local Nusselt number Nu and local Sherwood number Sh_x, which are defined as:

$$C_f = \frac{\tau_w}{\rho_f U^2}, \quad Nu = \frac{xq_w}{k_f (T_w - T_\infty)}, \quad Sh_x = \frac{xq_m}{D_B (C_w - C_\infty)} . \qquad (16)$$

where τ_w is the skin friction or the shear stress, q_w is the heat flux from the plate and q_m is the mass flux at the surface which given by:

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0} , \quad q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} , \quad q_m = -D_B\left(\frac{\partial C}{\partial y}\right)_{y=0} \qquad (17)$$

substituting Eqs. (9) into (16) and (17), we will obtain

$$(\mathrm{Re}_x)^{\frac{1}{2}} C_f = \frac{1}{(1-\varphi)^{2.5}} f''(0), \quad (\mathrm{Re}_x)^{-\frac{1}{2}} Nu = -\frac{k_{nf}}{k_f}\theta'(0), \quad (\mathrm{Re}_x)^{-\frac{1}{2}} Sh_x = -\phi'(0) \qquad (18)$$

where $\mathrm{Re}_x = Ux/\nu_f$ is the local Reynolds number.

Table 1 Thermophysical properties of fluid and nanoparticles (Oztop and Abu-Nada [22]).

Physical properties	Fluid phase (water)	Cu	Al_2O_3	TiO_2
C_p (J/kg K)	4179	385	765	686.2
ρ (kg/m^3)	997.1	8933	3970	4250
k (W/mK)	0.613	400	40	8.9538

3. Method of solution

The nonlinear ordinary differential Eqs. (10) - (12) subject to the boundary conditions (13) and (14) has been solved numerically using the shooting method. It is an iterative algorithm technique implemented in MAPLE program which attempts to identify the appropriate initial conditions for a related initial value problem (IVP) that provides the solution to the original boundary value problem (BVP). In this method, we are setting the different initial guesses for the values of $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ in which all the profiles must satisfy the boundary conditions (13) and (14) asymptotically. The effect of the solid volume fraction φ , Brownian motion parameter Nb, and thermophoresis parameter Nt are analysed for three different nanofluids, namely Cu , Al_2O_3, and TiO_2 as the working fluids. The Prandtl number is taken to be $\Pr = 6.2$ and nanoparticle volume fraction is considered from 0 to 0.2 $(0 \le \varphi \le 0.2)$, where $\varphi = 0$ is corresponding to the regular fluid. The thermophysical properties of the base fluid and nanoparticles are listed in Table 1.

4. Results and discussion

Figures 1(a) and 1(b) illustrate the variations of the skin friction coefficient given by Eq. (18) with parameter φ for three different nanoparticles: Copper Cu , Aluminia Al_2O_3 and Titania TiO_2,respectively when $\Pr = 6.2$, $Le = 2$, $Nb = 0.3$ and $Nt = 0.1$. It is interesting to observe that the skin friction coefficient increase almost monotonically with increasing φ for both Blasius and Sakiadis problem. These figures show that Al_2O_3 has the lowest skin friction coefficient and the difference between the values for TiO_2 and Al_2O_3 is very small, as can be seen from Tables 2 and 3 and Figs. 1(a) and 1(b). However, this behaviour is the same with that reported by Ahmad et al. [13]. This is explained by looking at Eq. (10) where the ordinary differential equation shows that it is only sensitive to the parameter φ and not influenced by parameter Nb and Nt. Other than that, Tables 2 and 3 also shows the comparison of $\mathrm{Re}_x^{\frac{1}{2}} C_f$ with Ahmad et al. [13], which show a favourable agreement, thus give confidence that the numerical results obtained are accurate.

However, we are more interested to know the influence of the parameter φ, Nb and Nt towards heat transfer rate. The variation of Nusselt number with φ for Blasius and Sakiadis problem are presented in Figs. 2(a) – 3(b) considering various value of Nb and Nt. It is seen that Nusselt number increase when parameter φ increase but the reverse effect for parameter Nb and Nt for both problems. From the Figs. 2(a) and 2(b), Cu has the highest heat transfer rate, while TiO_2 has the lowest heat transfer rate compared to Cu due to domination of conduction mode of heat transfer. Table 1 clearly show that TiO_2 has the lowest value of thermal conductivity compared with Cu and Al_2O_3. In contrast, for the Sakiadis problem, Al_2O_3 has the highest heat transfer rate. The difference in the value of Cu and Al_2O_3 is negligible. The thermal conductivity of Al_2O_3 is approximately one tenth of Cu , as given in Table 1. However, Al_2O_3 has its own unique property which is low thermal diffusivity. Decrease in thermal diffusivity leads to higher temperature gradient and thus will increase an enhancement in heat transfers. However, Cu nanoparticle have high value of thermal diffusivity and therefore, this automatically will reduces the temperature gradient and will affect the performance of Cu -water working fluid. In summation, it is noted that the lowest heat transfer rate obtained for TiO_2 nanoparticles.

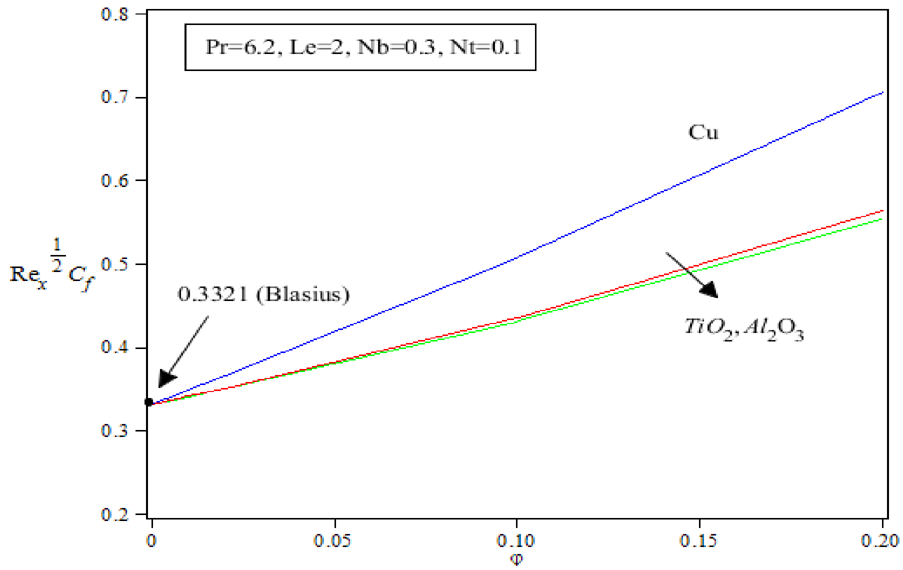
Figures 4 – 9 presents the velocity, temperature and nanoparticle volume fraction profiles for both Blasius and Sakiadis problems for some values of parameter φ $(0 \le \varphi \le 0.2)$ when $\Pr = 6.2$, $Le = 2$, $Nb = 0.3$, $Nt = 0.1$ and Cu -water as working fluid. Figs. 3a and 4a show that the momentum boundary layer increases with φ for the Blasius problem, while decrease with φ for the Sakiadis problem. However, Figs. 4 and 7 shows that the thermal boundary layer thickness increase with an increase in parameter φ because of the increase in local Nusselt number. While, the velocity, temperature and nanoparticle volume fraction profiles for both problems for different type of nanoparticles when $\varphi = 0.1$ are presented in Figs. 10 - 15. This figures show that by using different types of nanofluids, the values of the $f'(0)$, $\theta(0)$ and $\phi(0)$ change. Thus, nanofluids are capable to change the velocity, temperature and nanoparticle volume fraction profiles within the boundary layer. It can be seen that all these profiles asymptotically satisfied all boundary conditions Eq. (13) – (14). Hence, we can say that the numerical results obtained are valid.

Table 2 Value of $\mathrm{Re}_x^{\frac{1}{2}} C_f$ for the Blasius problem

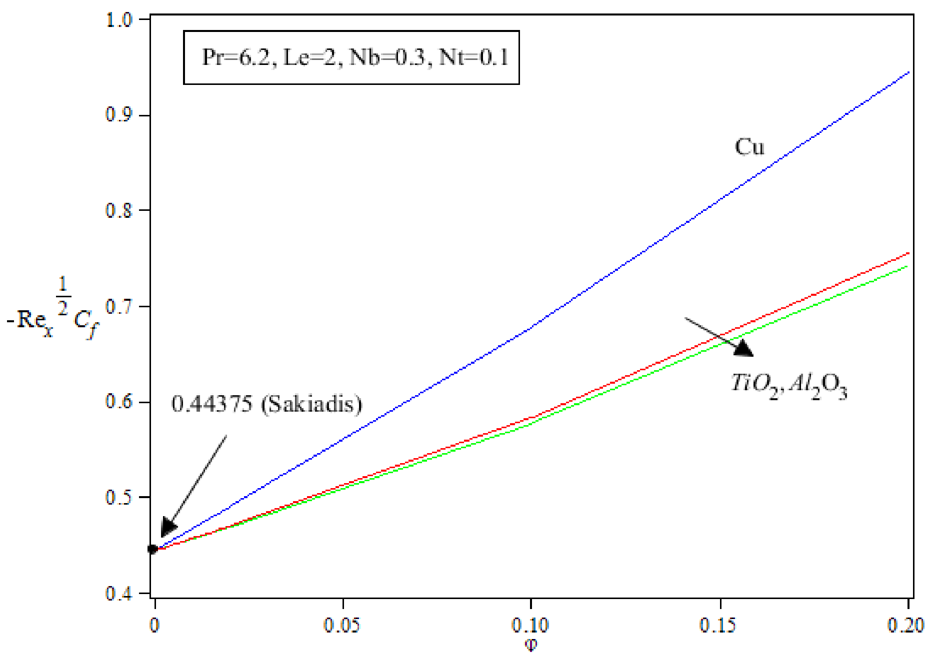
φ	Present			Ahmad et al. [13]			Blasius
	Cu -water	Al_2O_3 -water	TiO_2 -water	Cu -water	Al_2O_3 -water	TiO_2 -water	[3]
0	0.3321	0.3321	0.3321	0.3321	0.3321	0.3321	0.3321
0.002	0.3355	0.3339	0.3340	0.3355	0.3339	0.3340	
0.004	0.3390	0.3357	0.3359	0.3390	0.3357	0.3359	
0.008	0.3459	0.3394	0.3398	0.3459	0.3394	0.3398	
0.01	0.3494	0.3412	0.3417	0.3494	0.3412	0.3417	
0.012	0.3528	0.3431	0.3436	0.3528	0.3431	0.3436	
0.014	0.3563	0.3449	0.3456	0.3563	0.3449	0.3456	
0.016	0.3597	0.3468	0.3476	0.3597	0.3468	0.3476	
0.018	0.3632	0.3487	0.3495	0.3632	0.3487	0.3495	
0.02	0.3667	0.3506	0.3515	0.3667	0.3506	0.3515	
0.1	0.5076	0.4316	0.4362	0.5076	0.4316	0.4362	
0.2	0.7066	0.5545	0.5642	0.7066	0.5545	0.5642	

Table 3 Value of $\mathrm{Re}_x^{\frac{1}{2}} C_f$ for the Sakiadis problem

φ	Present			Ahmad et al. [13]			Sakiadis
	Cu -water	Al_2O_3 -water	TiO_2 -water	Cu -water	Al_2O_3 -water	TiO_2 -water	[4]
0	0.4446	0.4446	0.4446	0.4446	0.4446	0.4446	0.44375
0.002	0.4492	0.4470	0.4471	0.4492	0.4470	0.4471	
0.004	0.4538	0.4494	0.4497	0.4538	0.4494	0.4497	
0.008	0.4630	0.4544	0.4548	0.4630	0.4544	0.4548	
0.01	0.4676	0.4568	0.4574	0.4676	0.4568	0.4574	
0.012	0.4722	0.4593	0.4600	0.4722	0.4593	0.4600	
0.014	0.4768	0.4618	0.4626	0.4768	0.4618	0.4626	
0.016	0.4814	0.4643	0.4653	0.4818	0.4643	0.4653	
0.018	0.4860	0.4668	0.4679	0.4860	0.4668	0.4679	
0.02	0.4906	0.4693	0.4705	0.4906	0.4693	0.4705	
0.1	0.6788	0.5778	0.5840	0.6788	0.5778	0.5840	
0.2	0.9446	0.7428	0.7556	0.9446	0.7428	0.7556	

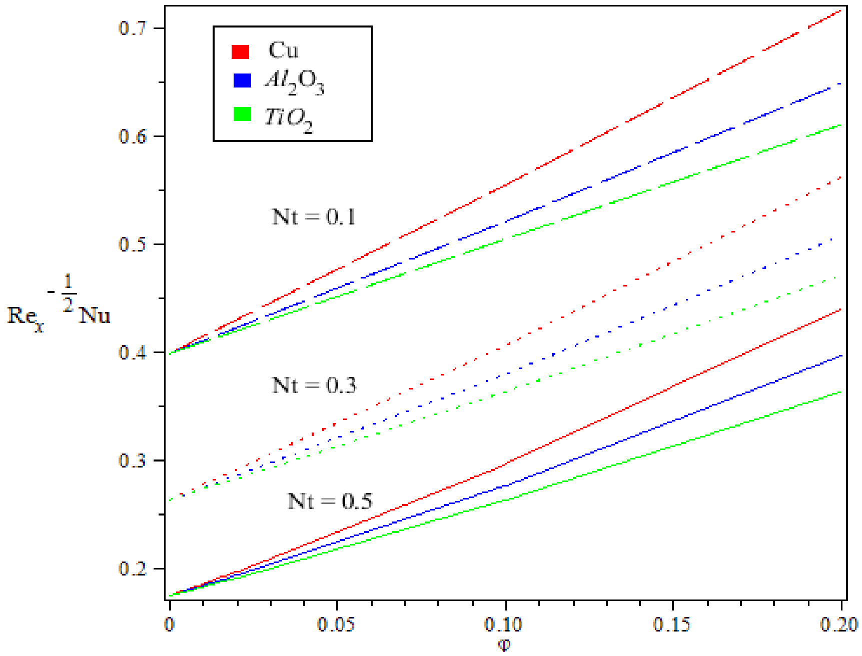


(a)

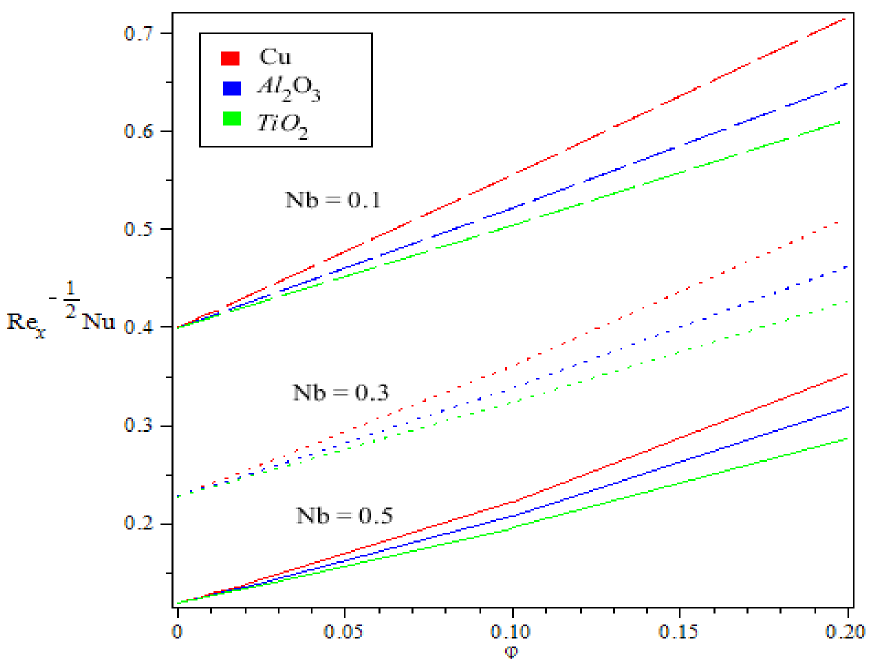


(b)

Figure 1 Variation of the skin friction coefficient with φ for the
(a) Blasius problem (b) Sakiadis problem

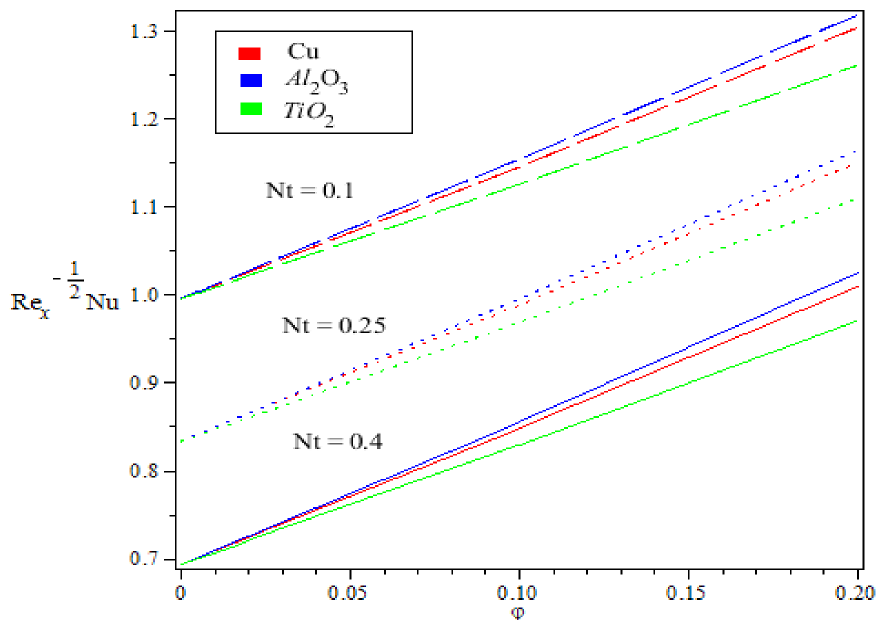


(a)

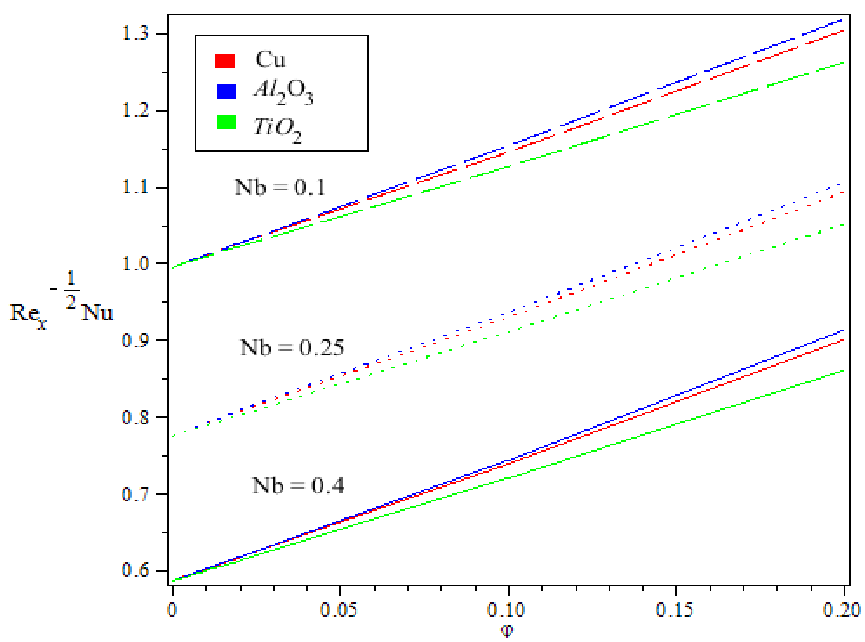


(b)

Figure 2 Variation of Nusselt number with φ for the Blasius problem when Pr=6.2 and Le=2 with different values of (a) Nt when Nb=0.1 (b) Nb when Nt=0.1



(a)



(b)

Figure 3 Variation of Nusselt number with φ for the Sakiadis problem when Pr=6.2 and Le=1 with different values of (a) Nt when Nb=0.1 (b) Nb when Nt=0.1

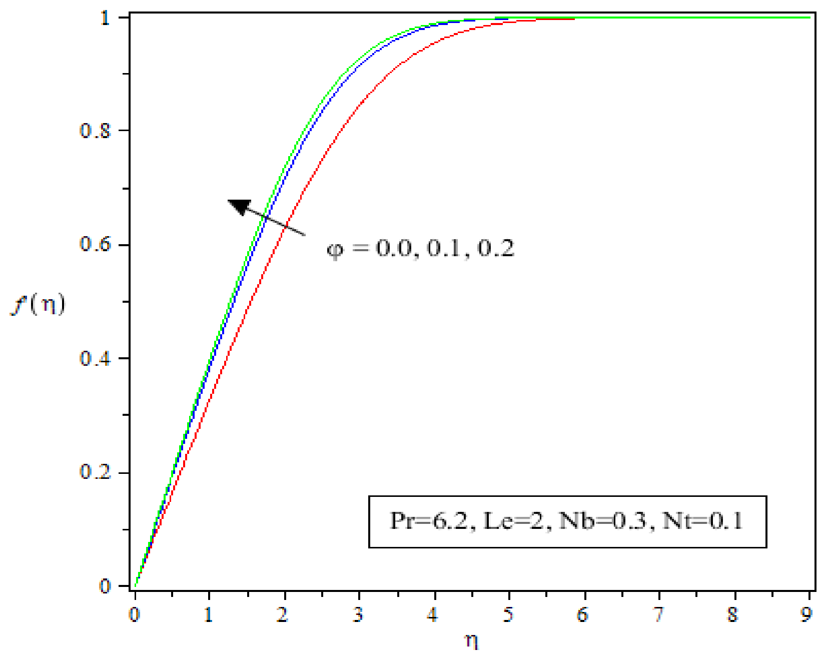


Figure 4 Velocity profiles for various φ for the Blasius problem with Cu-water as working fluid

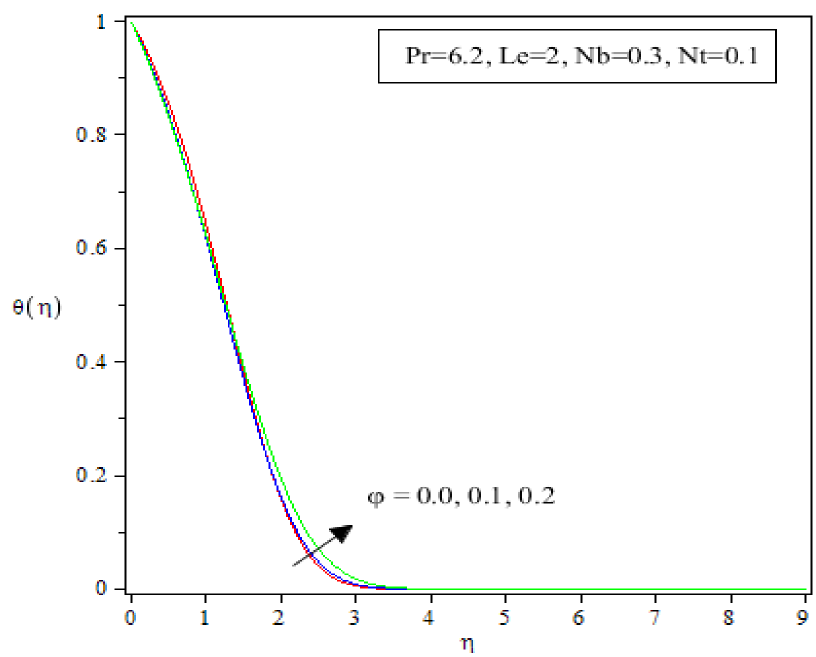


Figure 5 Temperature profiles for various φ for the Blasius problem with Cu-water as working fluid

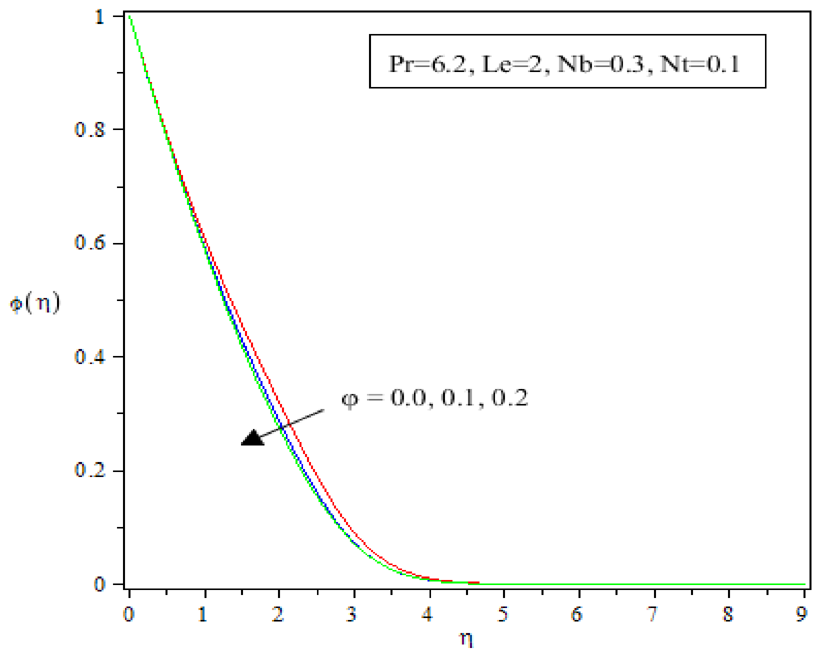


Figure 6 Nanoparticle volume fraction profiles for various φ for the Blasius problem with Cu-water as working fluid

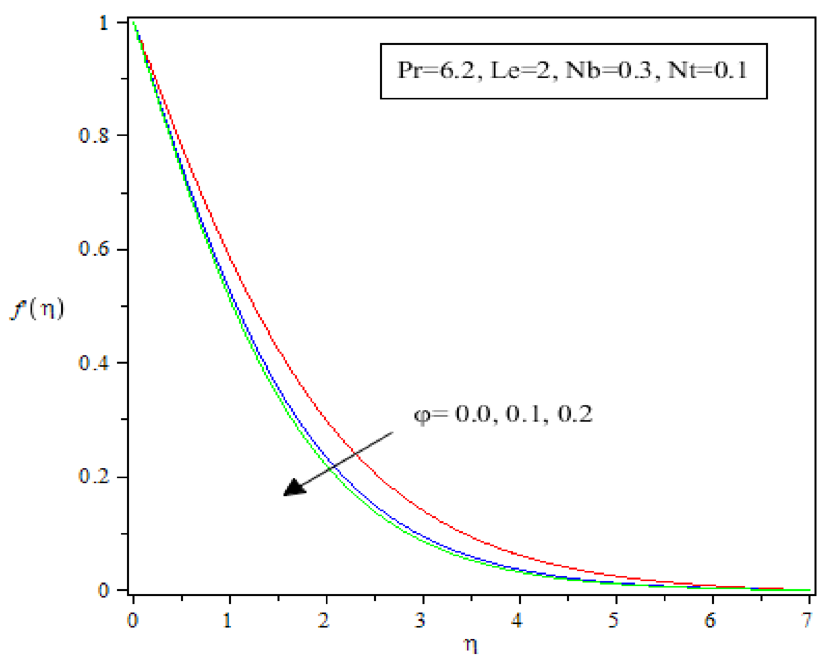


Figure 7 Velocity profiles for various φ for the Sakiadis problem with Cu-water as working fluid

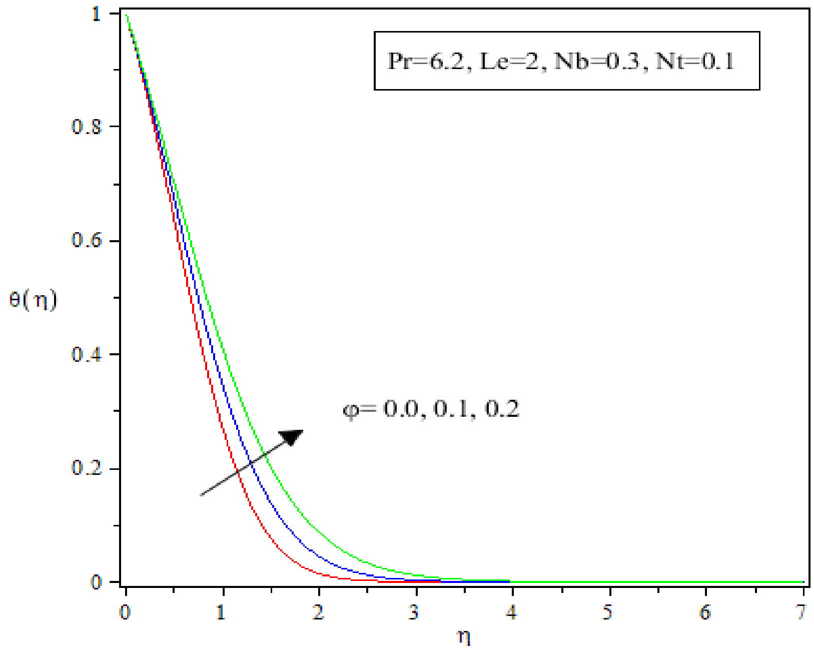


Figure 8 Temperature profiles for various φ for the Sakiadis problem with Cu-water as working fluid

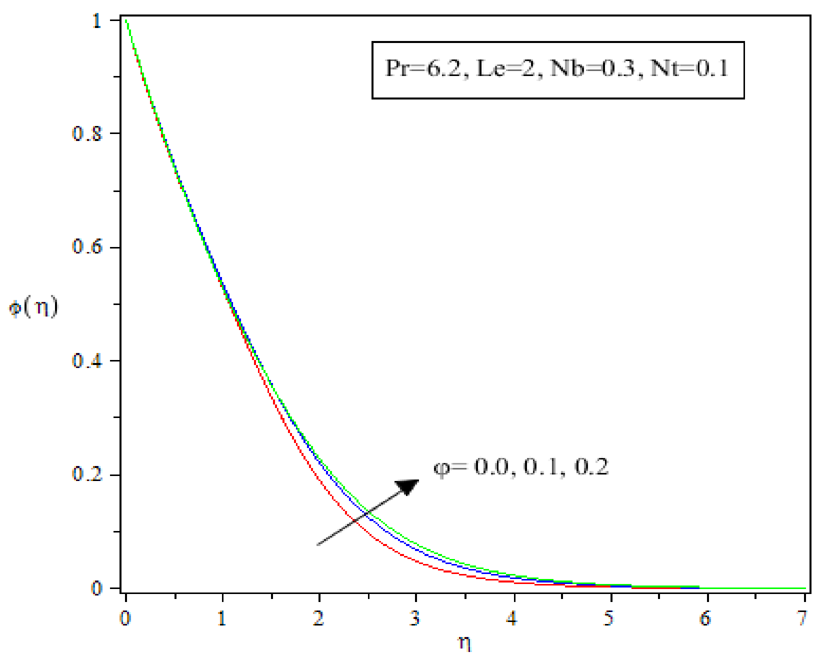


Figure 9 Nanoparticle volume fraction profiles for various φ for the Sakiadis problem with Cu-water as working fluid

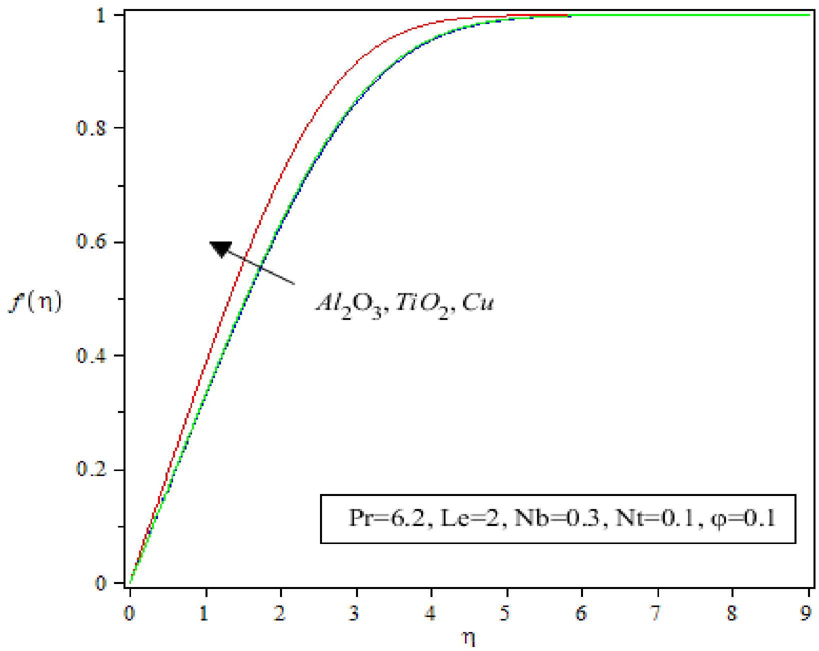


Figure 10 Velocity profiles for various type of nanoparticle for the Blasius problem

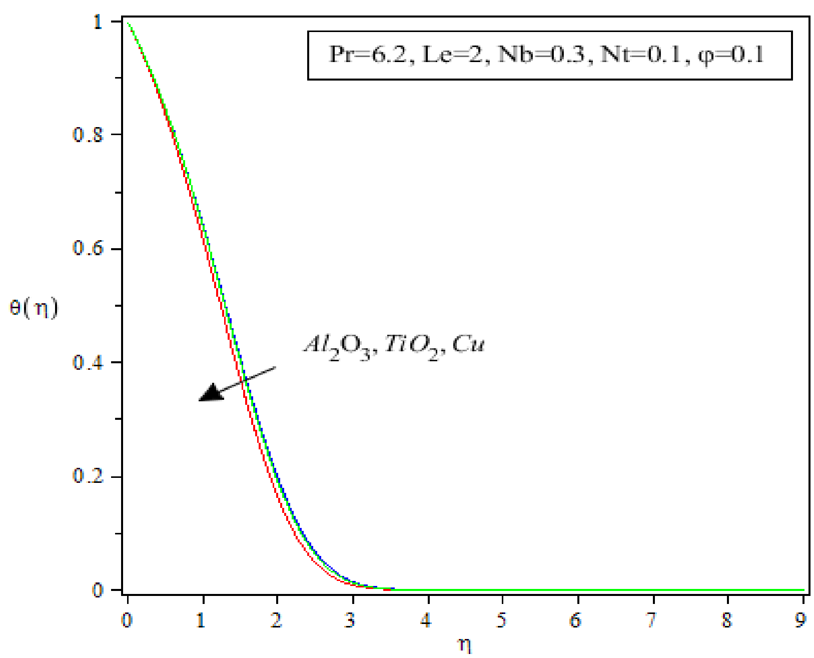


Figure 11 Temperature profiles for various type of nanoparticle for the Blasius problem

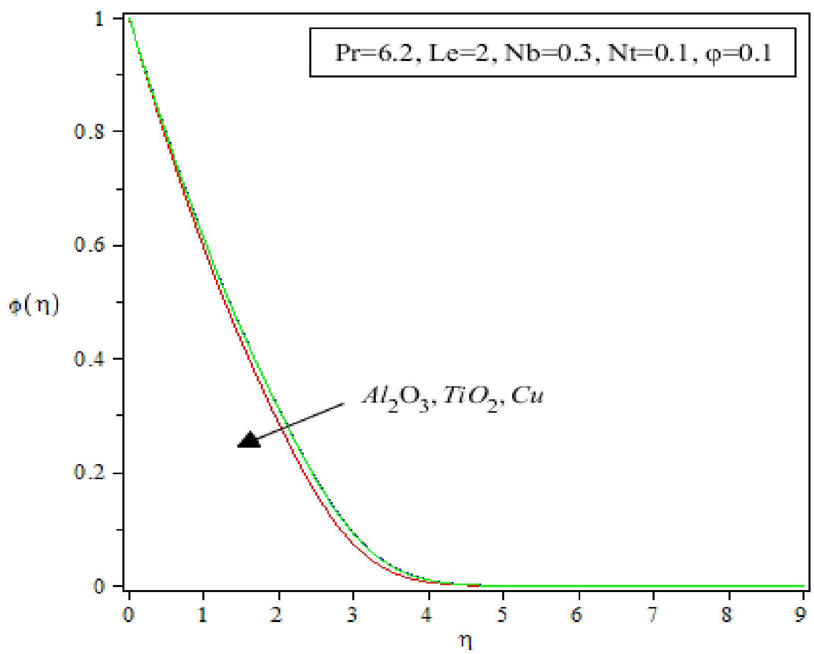


Figure 12 Nanoparticle volume fraction profiles for various type of nanoparticle for the Blasius problem

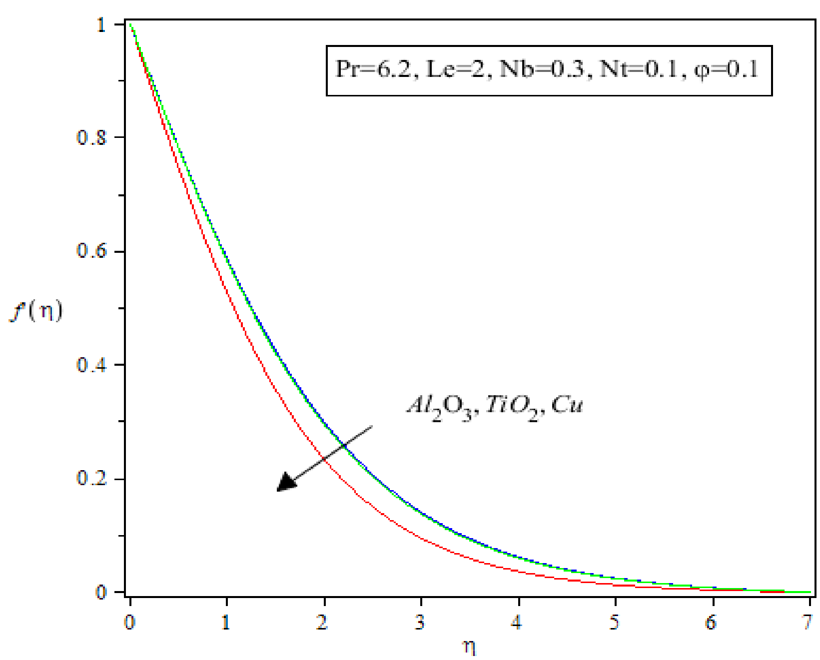


Figure 13 Velocity profiles for various type of nanoparticle for the Sakiadis problem

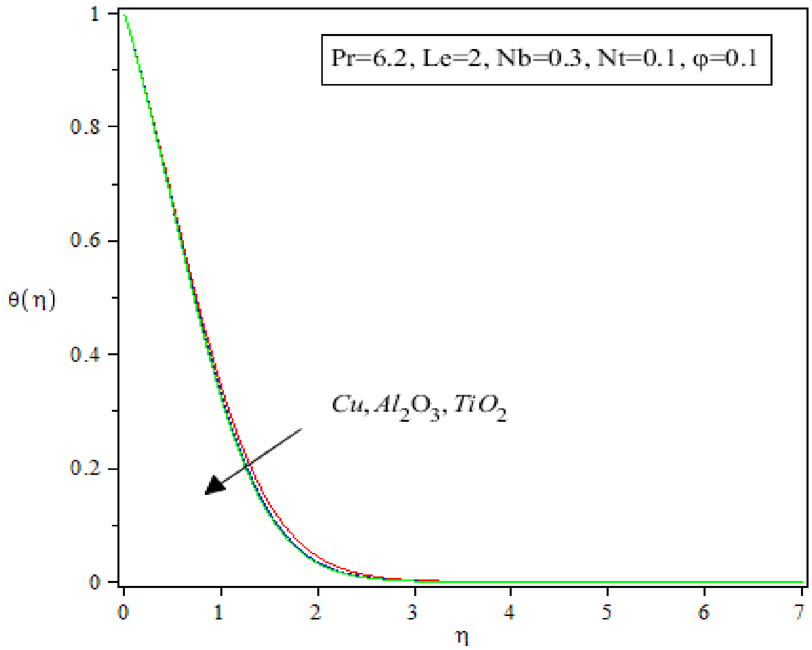


Figure 14 Temperature profiles for various type of nanoparticle for the Sakiadis problem

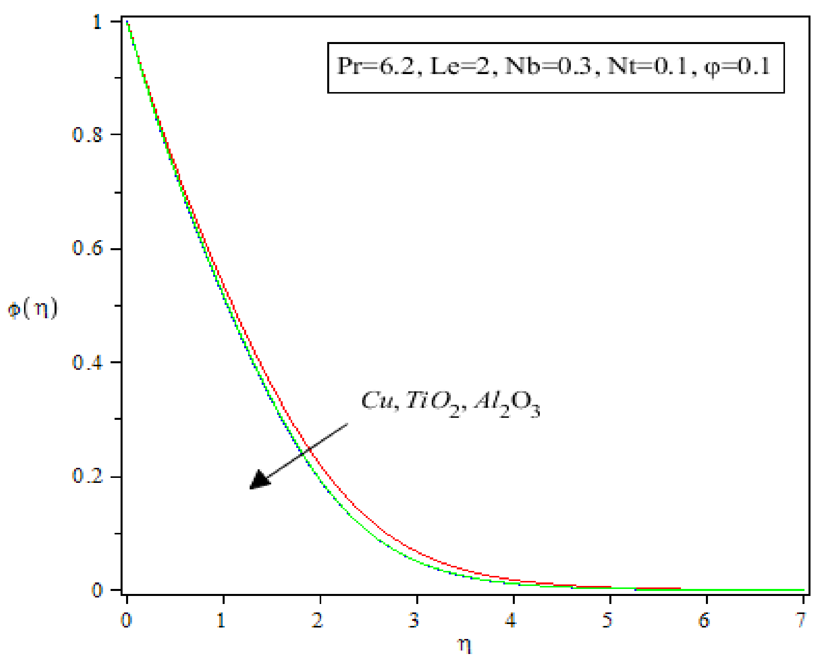


Figure 15 Nanoparticle volume fraction profiles for various type of nanoparticle for the Sakiadis problem

5. Conclusion

In this paper, we have studied theoretically and analysed the effects of Brownian motion, thermophoresis and solid volume fraction parameters on the fluid flow and heat transfer characteristics. The problem was solved using a shooting method by Maple software. Results for various parametric conditions are presented and discussed.

- the skin friction coefficients increase as the solid volume fraction parameter increases.
- the local Nusselt number increase significantly with the increase of solid volume fraction parameter but the reverse effects for Brownian motion and thermophoresis parameter.
- the presence of nanoparticles in a water based fluid results in an increase in the skin friction and nusselt number.
- The type of nanofluid is a key factor for heat transfer enhancement.
- the heat transfer rate in Cu-water nanofluid and Al_2O_3-water nanofluid is higher compared to the rate in TiO_2-water nanofluid.

References:

[1] Das SK, Choi SUS, Yu W, Pradet T. Nanofluids: Science and Technology, Wiley, New Jersey, 2008.

[2] Choi SUS, Eastman JA. Enhancing Thermal Conductivity of Fluids with Nanoparticles. ASME International Mechanical Engineering Congress & Exposition, San Francisco. CA. ANL/MSD/CP-84938. 1995;xxx: xxx-xxx.

[3] Blasius H. Grenzschichten in Flüssigkeiten mit kleiner Reibung. Z. Math. Phys. 1908;56: 1-37.

[4] Sakiadis BC. Boundary-layer behaviour on continous solid surfaces: I. Boundary-layer equations for two-dimensional and axisymmetric flow. AIChe J. 1961;7: 26-28.

[5] Cortell R. Flow and heat transfer in a moving fluid over a moving flat surface. Theory Computational Fluid Dynamic 2007;21: 435-446.

[6] Ishak A, Nazar R, Pop I. Flow and heat transfer characteristics on a moving flat plate in a parallel stream with constant surface heat flux. Heat Mass Transfer 2009;45: 563-567.

[7] Bachok N, Ishak A. The effects of suction and injection on a moving flat plate in a parallel stream with prescribed surface heat flux. WSEAS Transactions on heat and mass transfer 2010;5: 73-82.

[8] Ishak A, Yacob NA, Bachok N. Radiation effects on the thermal boundary layer flow over a moving plate with convective boundary condition. Meccanica 2011;46: 795-801.

[9] Cortell R. Radiation effects for the Blasius and Sakiadis flows with a convective surface boundary condition. Applied Mathematics and Computation 2008;206: 832-840.

[10] Buongiorno J. Convective Transport in Nanofluids. Journal of Heat Transfer 2006;128: 240 - 250.

[11] Tiwari RK, Das MK. Heat Transfer Augmentation in a Two-Sided Lid-Driven Differentially Heated Square Cavity Utilizing Nanofluids. International Journal of Heat and Mass Transfer 2007;50: 2002 - 2018.

[12] Rohni AM, Ahmad S, Pop I. Boundary layer flow over a moving surface in a nanofluid beneath a uniform free stream. International Journal of Numerical Methods for Heat & Fluid Flow 2011;21: 828-846.

[13] Ahmad S, Rohni AM, Pop I. Blasius and Sakiadis problems in nanofluids. Acta Mechanica 2011;218: 195-204.

[14] Bachok N, Ishak A, Pop I. Flow and heat transfer characteristics on a moving plate in a nanofluid. International Journal of Heat and Mass Transfer 2012;55: 642-648.

[15] Bachok N, Ishak A, Pop I. Boundary layer flow over a moving surface in a nanofluid with suction or injection. Acta Mech. Sin. 2012;28(1): 34-40.
[16] Elbashbeshy EMA, Emam TG, Abdel-wahed MS. An exact solution of boundary layer flow over a moving surface embedded into a nanofluid in the presence of magnetic field and suction/injection. Heat Mass Transfer 2014;50: 57-64.
[17] Subhashini SV, Sumathi R. Dual solutions of mixed convection flow of nanofluids over a moving vertical plate. International Journal of Heat and Mass Transfer 2014;71: 117-124.
[18] Bachok N, Ishak A, Pop I. Boundary-layer flow of nanofluids over a moving surface in a flowing fluid. International Journal of Thermal Sciences 2010;49: 1663-1668.
[19] Rosca NC, Pop I. Unsteady boundary layer flow of a nanofluid past a moving surface in an external uniform free stream using Buongiorno's model. Computers & Fluids 2014;95: 49-55.
[20] Pop I, Seddighi S, Bachok N. Ismail F. Boundary layer flow beneath a uniform free stream permeable continous moving surface in a nanofluid. Journal of Heat and Mass Transfer Research 2014;1: 55-65.
[21] Noor A, Nazar R, Jafar K, Pop I. Boundary-layer flow and heat transfer of nanofluids over a permeable moving surface in the presence of a Coflowing fluid. Advances in a Mechanical Engineering 2014;521236: 1-10.
[22] Oztop HF, Abu-Nada E. Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. Int. J. Heat Fluid Flow 2008;29: 1326-1336