

# RELEVANCE OF SPHERICAL LANGMUIR PROBE IN NON-MAXWELLIAN SPACE PLASMA

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## ABSTRACT

*This research reconnoiters the study of Spherical Langmuir Probe I-V features in Non-Maxwellian space plasma. With the help of the (volt–ampere curves) of spherical Langmuir probes, the different parameters of plasma can resolute such as plasma potential, floating potential, probe currents in different probe voltage. The variation of Electron Energy Distribution Function (EEDF) with Maxwellian and Druyvesteyn distributions was also scrutinized. General equations for current-collection and I-V curves for spherical probe in isotropic and anisotropic Non-Maxwellian plasmas are examined. The direction of energetic beam is irrelevant for the spherical probe, this is the best example of anisotropic distributin. The ion saturation region is flat because the calculation assumes an infinite plane; i.e., a perfect guarded spherical probe. This work is conducted using computational techniques to create the exact plasma conditions of the experimental testing environments. The investigations address the development of a technique to model Non-Maxwellian plasma. The low temperature component dominates the shape of the curves at low retarding potentials, while higher element dominates the more negative end of the retarding region of the I-V curves. The EEDF has been investigated with probeto overcome some margins of Druyvesteyn method and fluctuations from Maxwllian to Druyvesteyn with increasing energy of electron. For anisotropic distributions, spherical probe geometries are equally appropriate for measuring the electron temperature and density or for determining the distribution function in the presence of Non-Maxwellian space plasma.*

**Keywords:** Langmuir probe, I-V Characteristic, Non-Maxwellian plasma, Anisotropic distributions

## 1. Introduction

An electrostatic probe was first used to extent the potential distribution in gas discharges on the ground by J.J. Thomson. The theory was later developed by Langmuir and his collaborators [1]. The technique, with further developments, has been extensively applied to the study of gas discharges and also to the study of the ionosphere. A Langmuir probe refers to an electrode engrossed in charged particle plasma, whose current-voltage characteristics can be measured [2]. From the  $I$ - $V$  characteristics, one can estimate the temperature and number density of thermal electrons as bulk parameters. The procedure has been used to measure thermal plasma populations on spacecraft in the ionosphere, although the conditions are more complex on a fast moving platform. The first in situ measurement of electron temperature in the ionosphere was thru by Langmuir probe in 1947 [3].

For determining the basic characteristics of thermal plasma in the ionosphere, Langmuir probe is a simple and straight utensil and has been frequently installed on sounding rockets and satellites for more than five eras. It is possible to estimate not only the number density and temperature of electrons but also the energy distribution function and ion density of the ionospheric plasma. In the lower ionosphere, because thermal population is the most important essential as plasma, the temperature and density of the plasma are important parameters for understanding the general characteristics of the ionosphere, and have been extensively measured since the early stages of satellite observations.

There is no common theory of Langmuir probes which is applicable to all measurement conditions, because it be contingent on the probe size and geometry, plasma density and temperature, platform velocity, and other factors. By considering the relationship between the probe dimensions and the Debye length of the plasma, the actual design of the probe is usually determined. In overall, two estimates are used to express the current on the probe in the plasma: 1) orbital motion limited (OML) and 2) sheath area limited (SAL). OML theory can be approved when the probe radius is smaller than the thickness of the sheath surrounding the probe, while it must be equal to or longer than the sheath thickness in the case of SAL theory [5].

The number of electrons which are episode perpendicular to a given plane per unit time due to thermal motion is given by [5],

$$N = \int_0^{\infty} v_x dn_e(v_x) \quad (1)$$

Where,  $x$  is taken in the direction perpendicular to the plane and  $dn_e(v_x)$  is the number of electrons whose velocity is between  $v_x$  and  $v_x + dv_x$ . If the velocity of the electrons obeys a Maxwellian distribution,  $dn_e(v_x)$  is expressed by

$$dn_e(v_x) = N_e \left( \frac{m_e}{2\pi k T_e} \right)^{1/2} e^{-\frac{m_e v_x^2}{2k T_e}} dv_x \quad (2)$$

Where  $N_e$  and  $T_e$  are the electron number density and electron temperature, respectively,  $m_e$  is the electron mass, and  $k$  is the Boltzmann constant.

Then, Eq. (1) can be written as

$$N = N_e \left( \frac{m_e}{2\pi k T_e} \right)^{1/2} \int_0^{\infty} v_x e^{-\frac{m_e v_x^2}{2kT_e}} dv_x = N_e \left( \frac{kT_e}{2\pi m_e} \right)^{1/2} \quad (3)$$

The electron current incident on the probe at zero potential with respect to the surrounding plasma is called the random probe current and is expressed by [5],

$$I_e = \frac{1}{4} N_e e \left( \frac{8kT_e}{\pi m_e} \right)^{1/2} A \quad (4)$$

where 'A' is the surface area of the probe.

Langmuir probes could have different electrode shapes, such as cylindrical, spherical, and planar probes. The cylindrical probe is a straight piece of wire typically made from tantalum or similar materials such as molybdenum, tungsten and graphite are also popular choices, which are chosen for their high melting points and mechanical strength. In contrast, the planar probe is a flat conductor that is typically single-sided, with the rest of the probe being insulated with a simple glass sleeve or coaxial cables built into the sleeve. The Spherical probe has the spherical bob at the top classically made from tantalum, molybdenum, tungsten or graphite [6].

Maxwellian plasma may occur disorder near a spacecraft due to photoemission, interactions between the spacecraft and thermospheric gases, or electron emissions from other devices on the spacecraft. Significant non-Maxwellian plasma distributions may also occur in nature as a mixture of ionospheric and magnetospheric plasmas or secondaries produced by photoionization in the thermosphere or auroral drizzle and presence of dusty plasma in ionosphere.

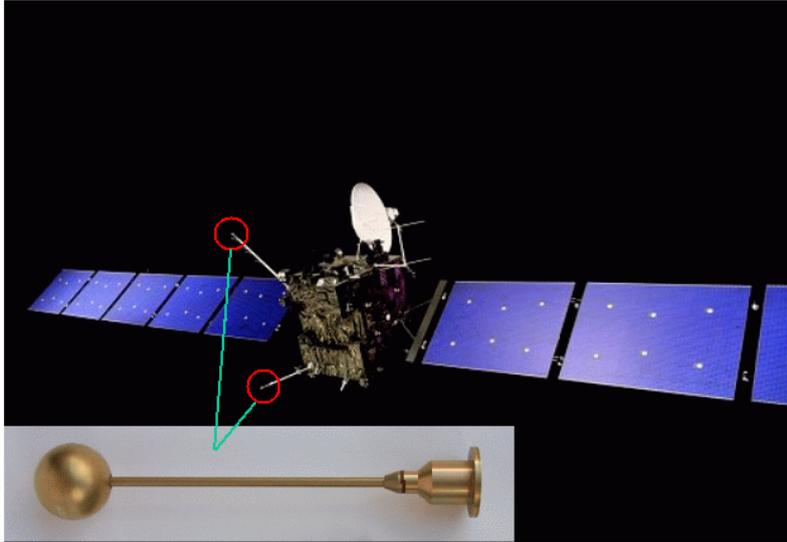


Figure 1: ESA's comet hunter Rosetta, with the two Langmuir probes from IRF Uppsala at the end of the booms protruding from the spacecraft. Each probe is a sphere of 50 mm diameter.

The geophysical sources although present much of the time, usually have a slight influence on the  $I$ - $V$  curves, except perhaps in regions of very low ionospheric density. The non-geophysical components can have high enough densities to significantly distort the  $I$ - $V$  curves and affect the determination of  $T_e$  by twisting the electron retarding regions of the  $I$ - $V$  curves. They may also affect the measurements of total ionospheric density,  $N_e$  and  $N_i$ . In these cases, the ionospheric parameters (of the low energy electrons) can only be obtained by identifying the non-ionospheric contributions to the  $I$ - $V$  curves and detracting them out, or by fitting the  $I$ - $V$  curve with theoretical expressions which include the non-ionospheric components.

## 2. Plasma and other Non-Maxwellian Distributions

A Langmuir probe is habitually employed under the assumption that the electron velocity distribution is Maxwellian, which makes the calculations simple enough to handle routinely. Moreover, in non-Maxwellian electron velocity distributions, electrons are not in energy equilibrium with ions or neutral particles, and the electron temperatures are much higher than the ion and neutral temperatures [6]. Henceforward, conventional analysis and the use of a Maxwellian EEDF are not operative methods for the study of non-equilibrium plasma phenomena [6]. The EEDF in low-pressure discharges is generally non-Maxwellian, besides the electron temperature is usually thought of as an effective electron temperature corresponding to the mean electron energy determined from the EEDF.

When inferring plasma parameters from the probe characteristics and assuming a departure of the actual EEDF from a Maxwellian distribution only for the small number of electrons with energies higher than the energy of the inelastic threshold while; Non-Maxwellian effects can be ignored [6]. This is not permanently the case, as the EEDF in low-pressure discharges is not Maxwellian even in the low energy range [7]. There are two methods for obtaining the

plasma parameters from Langmuir probe measurements. The first of these methods is based on EEDF integrals [8], and the second method hires a fluid model for the modified ion flux [9]. The approach taken in this dissertation uses the EEDF integrals [10], where the EEDF is measured by a double differentiation of the  $I$ - $V$  characteristics according to the Druyvesteyn formula, which is considered more precise for non-equilibrium distributions [11]. In argon discharges, the EEDF shows a two-temperature Maxwellian structure at low pressures and becomes Druyvesteyn-like at higher pressures. Hence, to reproduce the  $I$ - $V$  characteristics, it is important to implement the two-temperature distribution, which could potentially be used as an alternative to the Druyvesteyn distribution when fitting experimental data [13].

While non-ionospheric (non-Maxwellian) electrons are present in detectable quantities, the standard Langmuir equations may lead to significant errors in the measurements of  $N_e$  and  $T_e$ . To handle these circumstances, we inspect the current equations for arbitrary electron energy distributions for the spherical Langmuir probe. This equation is integrals over the velocity distribution functions (VDF) for multicomponent populations of ions and electrons [13]. The discussion begins with formulas for general anisotropic VDFs, and then considers the general isotropic case, and finally, the anisotropic case for a Maxwell Boltzmann distribution. The special effects of these nonionospheric populations are demonstrated by calculating  $I$ - $V$  curves for a two temperature distribution as an example of the isotropic case, and an energetic electron beam with a superposed Maxwell–Boltzmann distribution as an illustration of the anisotropic case.

### **3. Two Temperature Maxwellian Method**

The two-temperature Maxwellian (bi-Maxwellian) is an anisotropic distribution model provisional on physical orientation. This method has an unstable and non-equilibrium velocity distribution where there is temperature anisotropy, meaning that the temperature characterizing the movement of the particles in one direction is different from that in another direction [14]. The coordinate system usually used consists of the parallel and perpendicular velocities with respect to a background magnetic field. A stable two-dimensional distribution is independent of the direction [14].

### **4. Significance of Non-Maxwellian Distributions**

Computational simulation concerning non-equilibrium distributions have been conducted in a vacuum test facility. For these argon was used, and a Langmuir probe was used to determine the EEDF. The results were used to compare non-Maxwellian to Maxwellian distributions for non-equilibrium plasma applications [15]. Results from the Langmuir probe investigate are presented for the Druyvesteyn method for the non-Maxwellian distribution. As the distribution functions obtained from a single  $I$ - $V$  curve were not clear enough to make a comparison with Maxwellian and non-Maxwellian distributions, the EEDF was obtained to preserve significant features in the curve [15].

## 5. Spherical Probe General Formulas for Non-Maxwellian Distributions

Spherical Langmuir probe is a very simple device, it may have a rather complicated theory. This is because probe acts as a boundary and the equations governing the plasma motion change its characteristics. Due to the quasi-neutral behavior of plasma a sheath is formed around the probe, which can sustain a large electric field. The characteristics of the Langmuir probe can be easily understood by plotting the  $I$ - $V$  curve. The general form of the velocity distribution function for the  $s^{\text{th}}$  specie ( $s$  is an electron or an ion specie) divided into distinct populations  $j$  is a sum over the  $j$  populations is given by [16],

$$F_s(u, v, w) = \sum_j n_s^j F_s^j(u, v, w) \quad (5)$$

where  $n_s^j$  is the number density of population  $j$  for specie  $s$ ,  $F_s^j$  is the individual VDF, and  $u, v, w$  are velocities in the  $x, y, z$  directions. The sum of the number densities over the  $j$  populations is the total number density of specie  $s$ ,  $\sum_j n_s^j = n_s$  and the VDFs are normalized to unity,

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dudvdw F_s^j(u, v, w) = 1 \quad (6)$$

Two classes of distributions: isotropic and anisotropic are considered. For the isotropic VDF, the general form is

$$F_s^j(u, v, w)_{\text{isotropic}} = G_s^j(u^2 + v^2 + w^2) \quad (7)$$

Where  $G$  is a function of the sum of velocities squared or of the energy. The most applicable form for a collision dominated plasma is the Maxwell distribution function [16],

$$G_{\text{Maxwell}}(u^2 + v^2 + w^2) = \left( \frac{m}{2\pi kT} \right)^{3/2} \exp\left( -\frac{m(u^2 + v^2 + w^2)}{kT} \right) \quad (8)$$

Where  $m$  and  $T$  are the mass and temperature of the  $j^{\text{th}}$  component of the  $s^{\text{th}}$  specie.

The anisotropic distribution has a preferred direction in space and could result, for example, from a beam of electrons induced by an acceleration process, or from rapid motion of the probe through the plasma, or from an anisotropy in the temperature of the electrons or ions [16]. We will employ a specific form of the anisotropic velocity distribution function, which applies to many space plasma situations:

$$F_s^j(u, v, w)_{\text{anisotropic}} = F_s^j[a(u - u_0), b(v - v_0), c(w - w_0)]_{\text{isotropic}} \quad (9)$$

where  $a$ ,  $b$ , and  $c$  are constants that could represent temperature anisotropy, and  $u_0, v_0, w_0$  represent a drift velocity of the plasma relative to the probe.

To abridge the formulas assuming large symmetrical sheaths and orbital-motion-limited collection for the ionosphere conditions as long as the relevant probe dimension is smaller than a few cm. For the spherical Langmuir probe, there is no preferred direction and the current is an integral over  $4\pi$  directions on the sphere. The general rotation transformation of velocities from the fixed  $x,y,z$  velocities,  $u, v, w$  to velocities on the sphere at the position given by the spherical polar angles,  $\theta, \phi$  with  $u$  the radial velocity and  $v$  and  $w$  the tangential velocities, is given by [16],

$$u = u \cos \theta + \sin \theta (v \cos \phi + w \sin \phi)$$

$$v = -u \sin \theta + \cos \theta (v \cos \phi + w \sin \phi) \quad (10)$$

$$w = -v \sin \phi + w \cos \phi$$

Where  $\theta, \phi$  are the polar and azimuthal angles, respectively. The inverse transformation is the transpose of Eq. (9). Then the spherical probe current in the limit of a large sheath (orbital-motion-limited case) is given by [16],

$$i_{\text{sphere}} = \frac{A}{4} \sum_{s,j} n_s^j q_s^j \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi \times \int_{0, \sqrt{-2qV/m}}^\infty u du \left( u^2 + \frac{2qV}{m} \right) \quad (11)$$

$$F_s^j [a(u \cos \theta - u_0), b(u \sin \theta \cos \phi - v_0), c(u \sin \theta \sin \phi - w_0)]$$

where  $A$  is the Spherical Langmuir probe surface area, and  $F^j$  is general anisotropic 3D velocity distribution function given by Eq. (11) for retarded electrons, the first derivative is

$$\frac{di_{\text{sphere}}}{deV} = \frac{A}{4} \sum_j n_e^j e \frac{2}{m_e} \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi \times \int_{0, \sqrt{-2eV/m_e}}^\infty u du F^j \quad (12)$$

and the second derivative is

$$\frac{d^2 i_{\text{sphere}}}{deV^2} = \frac{A}{4} \sum_j n_e^j e \frac{2}{m_e^2} \int_0^\pi \sin \theta d\theta \times \int_0^{2\pi} d\phi F^j \Big|_{u=\sqrt{-2eV/m_e}} \quad (13)$$

Thus, the second derivative of the spherical probe current is the average over all orientations of the anisotropic distribution function, or it yields the isotropic part of the distribution.

## 6. Electron Energy Distribution Function

The electron energy distribution function (EEDF) plays an essential role in plasma modeling. It has important consequences in calculating the rate coefficients of electron induced processes, such as ionization and excitation [17]. Various approaches can be used to describe the EEDF, such as Maxwellian, Druyvesteyn, or using a solution of the Boltzmann equation. The electron energy distribution function in low pressure discharges may often be approximated by either Maxwellian or Druyvesteyn distribution. Physically, Maxwellian distribution function is realized when the electron collision frequency is velocity independent, while Druyvesteyn distribution prevails when the mean free path is velocity independent [17]. The EEDF can be determined simply by the second derivative of the probe current with respect to the probe bias. In 1930, the more commonly used ‘‘Druyvesteyn Relation’’ was derived for spherical probes.

$$I_e(V_{probe}) = -\frac{A_p e}{2} \sqrt{\frac{2}{m}} \int_{E=V_{probe}}^{\infty} f(E) \sqrt{E} \left(1 - \frac{eV_{probe}}{E}\right) dE \quad (14)$$

$$f(E)_{E=-eV_{probe}} = -\frac{4}{A_p e^2} \sqrt{\frac{m_e V_{probe}}{2e}} \frac{d^2 I_e}{dV_{probe}^2} \quad (15)$$

Where  $f(E)$  is the electron energy distribution function,  $A_p$  is the surface area of the probe (typically approximation for a spherical probe by  $4\pi r_p^2$ ),  $e$  is the electron charge,  $m_e$  is the electron mass,  $V_{probe}$  is the probe voltage and  $I_e$  is the electron current through the probe circuit. This was only derived for the spherical geometry.

## 7. Results and Discussions

The Langmuir probe procedure for accurate measurement of plasma parameters has been around for eight decades, deriving the parameters with accuracy from the data acquired by a Langmuir probe immersed in space plasma is still a inspiring task. In the present research work theoretical study of Spherical Langmuir Probe in Non-Maxwellian plasma are considered. This work is typically based on the  $I$ - $V$  characteristic of Spherical Langmuir Probe. The main propose of this work is to study the Current-Voltage characteristic of spherical Langmuir probe. The methodology of our study includes the theoretical derivation of the relationships between various parameters and then plotting the parameters in appropriate range from a mathematical software MATLAB.

### 7.1 The variation of Spherical Langmuir Probe I-V curves for Thermal Electrons and Energetic Beam

The  $I$ - $V$  characteristic is a essential part of dagnosis of plasma parameters, simulation using a two-temperature distribution. So, it is depicted in the following figure. For this purpose, it is assumed that for density,  $N_e=10^{16} \text{m}^{-3}$ , radius of spherical probe ( $r_p=2.5 \text{mm}$ ). These figures

illustrate the effect of  $T_e$  on the width of the electron retradation regions. As the distributions transition from one point to another, the graph appears to look different than the others.

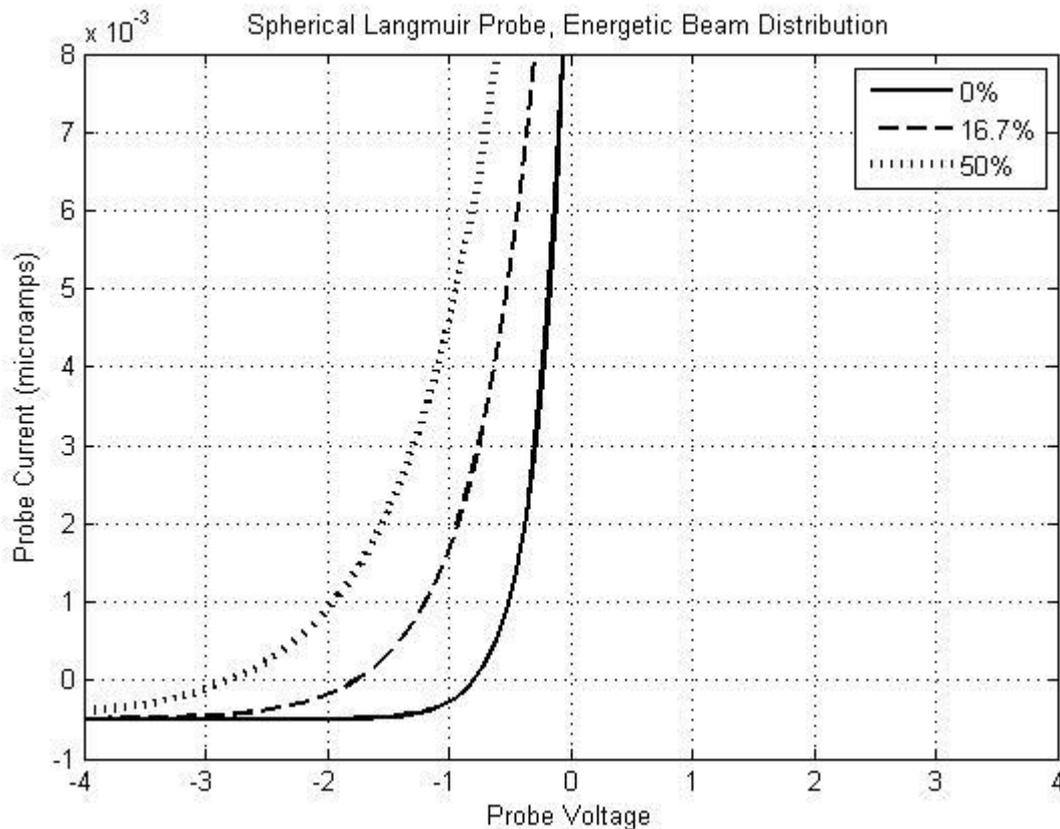


Figure 2: The Spherical Langmuir Probe I-V curves for the energetic beam with a 3000K energy spread and a 6000K thermal component for a bi-Maxwellian plasma

The curves are categorized with the percentage of the total density represented by the beam. Energetic beam components is less sensitive to the presence of a beam in spherical Langmuir probe it is because of thermal electron are collected from  $4\pi$  radians. Here, Fig.2 depicts theoretical I-V curves for combinations 3000K thermal electrons and a 1 eV energetic beam having a 6000K thermal spread, assumed total density is,  $N_e=10^{16}\text{m}^{-3}$ , The direction of beam is irrelevant for the spherical probe, this is the best example of anisotropic distributin. The ion saturation region is flat because the calculation assumes an infinite plane; i.e., a perfect guarded spherical probe. This floating potential is typically negative because mobile electrons tend to strike the probe more frequently than positive ions. The knee occurs when the probe has been saturated with electrons, causing additional electrons to be repelled.

## 7.2 The variation of Spherical Langmuir Probe I-V curve for two temperature distribution

Here, Fig. 3 indications spherical probe curves for mixtures of electron populations having temperatures of 3000K and 9000K. The curves are labeled with the percentage of the total density represented by the higher temperature components.

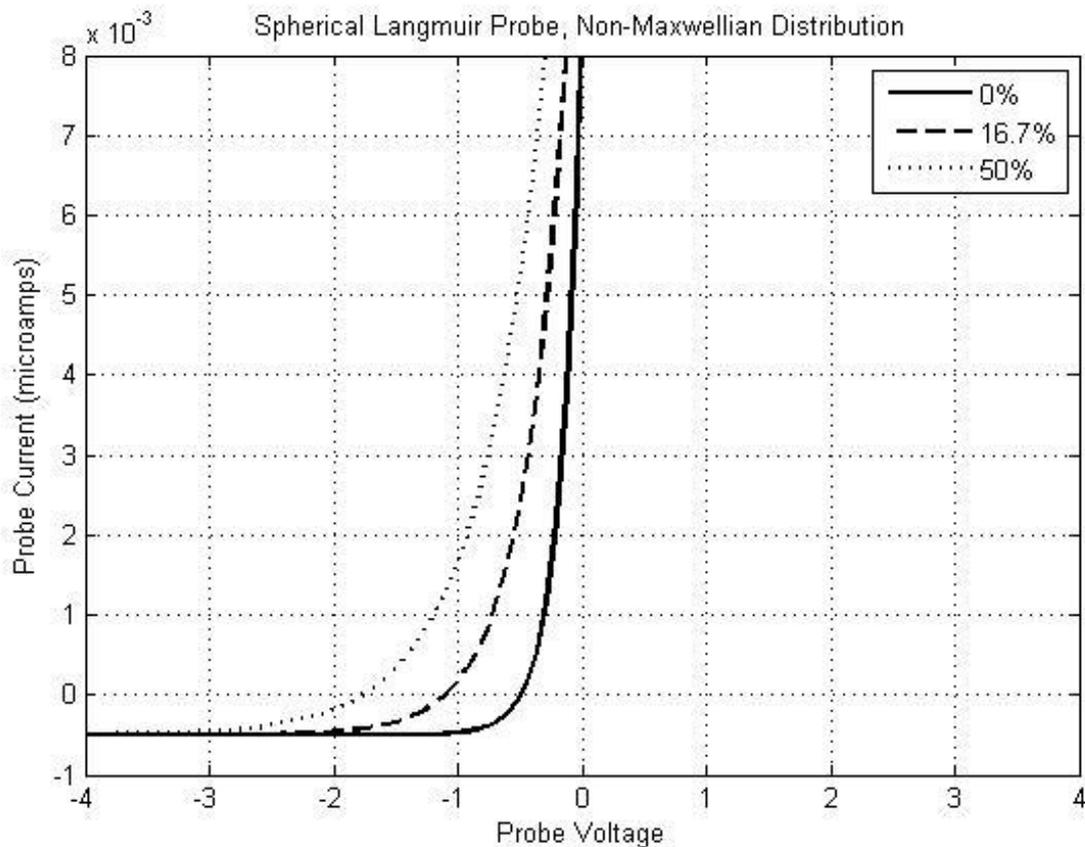


Figure 3: The Spherical Langmuir Probe I-V curves computed for Two Temperature Distribution

The low temperature component dominates the shape of the curves at low retarding potentials, while higher component dominates the more negative end of the retarding region of the  $I$ - $V$  curves. Plasma Potential  $V_s$  is 0.1 volt. For probe potential is greater than plasma potential ( $V_p > V_s$ ),  $I_e$  increases slowly as the collection area grows due to an increase in sheath thickness. In the Transition Region, the ion current is negligible, and the electrons are partially repelled by the negative potential  $V_p - V_s$ . The probe voltage is given with respect to the facility ground, and the plasma is at the plasma potential. When the probe voltage is well below the plasma potential, only the highest energy electrons reach the probe. As the probe voltage approaches the plasma potential, additional lower energy electrons are collected. A low probe voltage with respect to ground corresponds to high electron energy with respect to the plasma. Therefore, a change of variables is necessary to express the EED with respect to the plasma potential. This also changes the reference voltage of the probe from the facility ground to the plasma potential.

### 7.3 Maxwellian and Druyvesteyn Distributions

The electron energy distribution function (EEDF) is proportional to the second derivative of the  $I$ - $V$  curve. It gives a measure of the number density of the electrons as a function of the electron energy. In Fig 4,  $I$ - $V$  characteristics of the simulated data are compared. The

comparison of the  $I-V$  curves highlights some differences, such as discrepancies at high voltages. At negative voltages, the current is low and these data are in good agreement, with some minor differences. These slight differences at negative voltages appear to be due to the change in distributions; in this case, the distribution becomes Druyvesteyn. This is anticipated, because the velocity distribution of electrons in weakly ionized plasmas often deviate from Maxwellian to Druyvesteyn, where the collision frequency is velocity dependent but the mean free path length of electrons is constant. This also confirms that the deviation from a Maxwellian distribution can be large enough that using a Maxwellian function for all plasma modeling is not a justified approximation.

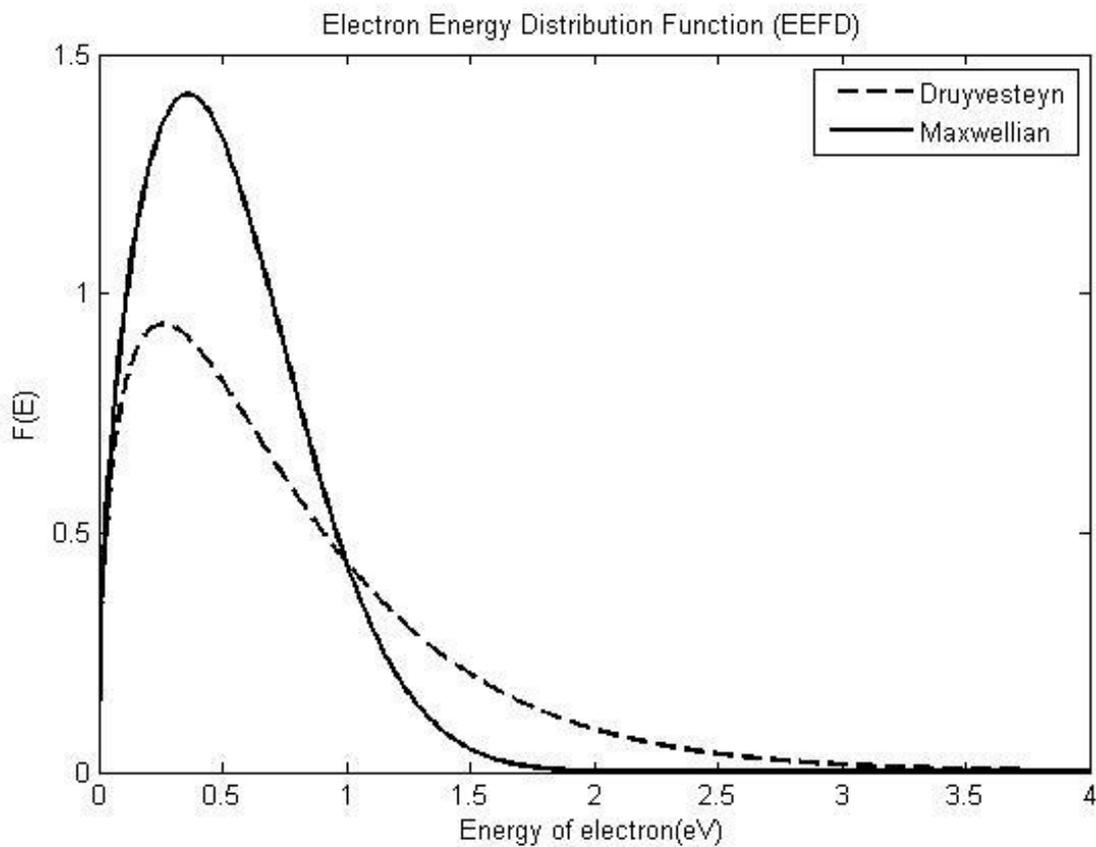


Figure 4: Comparison of Maxwellian and Druyvesteyn Distributions

Comparison of Maxwellian and Druyvesteyn Distributions demonstrates that as the electron temperature increases width of the curve also increases and vice-versa. For the smaller the value of temperature energy distribution raising sharply after the certain value of energy it decreases slightly. The simulation of the Non-Maxwellian distribution shown in Fig. 4 matches up very well in shape but loses the density profile for temperatures out to  $T_e$ . For energies, greater than  $T_e$ , the overall shape of the distribution is maintained, but with a slightly elevated tail temperature is obtained.

## 8. Conclusion

Langmuir probes are broadly used for plasma diagnostic. Their small size and relatively meek theory of operation make them vital tools. The  $I$ - $V$  characteristic of the Spherical Langmuir Probe in the Non-Maxwellian Space plasma at different temperature range are plotted in graph by using the mathematical software by putting the seemingly ranges for various parameters. All elementary processes were extensively deliberated and most of information has been presented. And the effect of temperature  $T_e$  on the width of the electron retardation regions are clearly studied in this research. Thus, measured plasma parameters, and the relationship between the densities of the plasma on probe current, effect of temperature on  $I$ - $V$  curve, are of essential significance for investigation on laboratory plasma, ionosphere terrestrial plasma and industrial application. The electron temperature of the plasma effect the Non-Maxwellian electron energy distribution function (EEDF). Maxwellian EEDF function curve demonstrates that at low energy (less than  $T_e$ ), the calculated distribution presents a significantly underestimated density profile. At higher energy range, the shape of the distribution mends and the tail trend with energy is maintained and declines exponentially.

This research show that EEDF is not essentially Maxwellian everywhere in the plasma. Considerable differences in electron density and temperature are observed, and these values differ dramatically in many cases. According to these results, the EEDF is best described by the Druyvesteyn distribution because its validity is independent of the sheath size and its determination of the EEDF provides a more detailed description of the plasma. The knowledge of the real EEDF is of great importance in understanding the underlying physics of processes occurring at the Non-Maxwellian Space Plasma diagnostic. Therefore, for anisotropic distributions, spherical probe geometries are equally well-matched for measuring the electron temperature and density or for determining the distribution function in the manifestation of non-Maxwellian space plasma.

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