Explicit Logarithmic Finite Difference Schemes For Numerical Solution of Burgers Equation

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Abstract

In this paper, we consider the numerical solutions of Burgers equation by using explicit logarithmic finite difference schemes. The obtained numerical results are compared with the exact solutions and the numerical solutions obtained by some other methods. In order to show the accuracy of the results L_2 *and* L_∞ *error normsa are used.*

Keywords: Burgers Equation, Expilicit Logarithmic Finite Difference Method

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1. Introduction

We consider the Burgers Equation which is the one dimensional nonlinear partial differential equation

$$u_t + uu_x = \nu u_{xx}, \qquad a < x < b, \quad t > 0$$

with initial condition

$$u(x,0) = f(x), \qquad a < x < b$$

and boundary conditions

$$u(a,t) = g_1(t), \quad u(b,t) = g_2(t), \quad t > 0$$

where $\nu > 0$ is a parameter indicating the kinematics viscosity of the fluid, $f(x)$, $g_1(t)$ and $g_2(t)$ are known fonctions.

The Burgers equation was first seen in a paper given by Bateman[3] and later Burgers[4] used the equation as a model of turbulence. In literature, numerical solutions of Burgers equation have been studied by many researchers[1,2,7-13].

In this study, the numerical solutions of Burgers equation are obtained by using explicit logarithmic finite difference schemes. Three model problemsare used to verify the performance of the methods.

2. Model Problems and Numerical Method

2.1.Model Problems

2.1.1. Problem 1.

We consider Burgers equation (1) with the initial condition

$$u(x,0)=\sin(\pi,x),\ 0<x<1 \tag{2}$$

and boundary conditions

$$u(0,t)=u(1,t)=0, \qquad t>0. \tag{3}$$

The exact solution of Burgers equation (1) with (2) and (3) conditions is [5];

$$u(x,t)=\frac{2\pi v\sum_{n=1}^{\infty} a_n \exp\left(-n^2\pi^2 vt\right) n\sin(n\pi x)}{a_0+\sum_{n=1}^{\infty} a_n \exp\left(-n^2\pi^2 vt\right)\cos\left(n\pi x\right)}$$

where

$$a_0=\int_0^1 \exp\left[-(2\pi v)^{-1}\left(1-\cos\left(\pi x\right)\right)\right]dx \ a_n=2\int_0^1 \exp\left[-(2\pi v)^{-1}\left(1-\cos\left(\pi x\right)\right)\right]\cos(n\pi x)dx,$$

$n=1,2,3,...$

2.1.2. Problem 2.

We consider Burgers equation (1) with the initial condition

$$u(x,0)=4x\left(1-x\right),\ 0<x<1 \tag{4}$$

and boundary conditions

$$u(0,t)=u(1,t)=0, \qquad t>0. \tag{5}$$

The exact solution of Burgers equation (1) with (4) and (5) conditions is [5];

$$u(x,t)=\frac{2\pi v\sum_{n=1}^{\infty}a_n\exp\left(-n^2\pi^2 vt\right)n\sin(n\pi x)}{a_0+\sum_{n=1}^{\infty}a_n\exp\left(-n^2\pi^2 vt\right)\cos\left(n\pi x\right)}$$

where

$$a_0=\int_0^1\exp\left[-x^2(3v)^{-1}(3-2x)\right]dx$$

$$a_n=2\int_0^1\exp\left[-x^2(3v)^{-1}(3-2x)\right]\cos(n\pi x)dx\,,\quad n=1,2,3,\ldots$$

2.1.3. Problem 3.

We consider Burgers equation (1) with the initial condition

$$u(x,1)=\frac{x}{1+\exp\left[\frac{1}{4v}\left(x^2-\frac{1}{4}\right)\right]},\qquad a<x<b \tag{6}$$

and boundary conditions

$$u(a,\ t)\ =\ u(b,\ t)\ =\ 0\,,\qquad t\ >\ 0 \tag{7}$$

The exact solution of Burgers equation (1) with (6) and (7) conditions is [9,10];

$$u(x,t)=\frac{x/t}{1+\left[t/\exp\left(1/8v\right)\right]^{1/2}\exp\left(x^2/4vt\right)}.$$

2.2.Numerical Method

We represent the finite difference approximation of $u(x,t)$ at the mesh point (x_i,t_n) by u_i^n in which $x_i=ih(i=0,1,\ldots,N)$, $t_n=t_0+nk(n=0,1,2,\ldots)$, $h=\frac{b-a}{N}$ is the mesh size in x direction and k is the time step.

We follow the procedure of ref. [6] to explain the numerical method. If we suppose that $F\left(u\right)$ is any continuous differential function and multiplying equation (1) by $\frac{\partial F}{\partial u}$ the following equation is obtained:

$$\frac{\partial F}{\partial u}\frac{\partial u}{\partial t}=F'\left(u\right)\left(-u\frac{\partial u}{\partial x}+v\frac{\partial^2 u}{\partial x^2}\right)$$

and

$$\frac{\partial F}{\partial t} = F'(u)\left(-u\frac{\partial u}{\partial x} + v\frac{\partial^2 u}{\partial x^2}\right). \qquad (8)$$

Using the forward difference approximation for $\frac{\partial F}{\partial t}$ the finite difference representation of equation (8) is obtained as:

$$F\left(u_i^{n+1}\right) = F\left(u_i^n\right) + kF'\left(u_i^n\right)\left[-\left(u\frac{\partial u}{\partial x}\right)_i^n + v\left(\frac{\partial^2 u}{\partial x^2}\right)_i^n\right]$$

in which k is the time step. Let $F(u) = e^u$ then the expilicit logarithmic finite difference scheme for equation (1) is obtained as:

$$u_i^{n+1} = u_i^n + \ln\left\{1 - k\left(u\frac{\partial u}{\partial x}\right)_i^n + vk\left(\frac{\partial^2 u}{\partial x^2}\right)_i^n\right\} \qquad (9)$$

If we use central difference approximation in place of $\frac{\partial^2 u}{\partial x^2}$ in equation (9)

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_i^n \cong \frac{u_{i-1}^n - 2u_i^n + u_{i+1}^n}{h^2}, \quad 1 \le \mathrm{i} \le \mathrm{N}\text{-}1$$

and then apply the following linearization techniques in place of the non-linear term $u\frac{\partial u}{\partial x}$

FD-I. $\left(u\frac{\partial u}{\partial x}\right)_i^n \cong \left(u_i^n\right)\left(\frac{u_{i+1}^n - u_{i-1}^n}{2h}\right), \quad 1 \le \mathrm{i} \le \mathrm{N}\text{-}1$

FD-II. $\left(u\frac{\partial u}{\partial x}\right)_i^n \cong \left(\frac{u_{i+1}^n + u_i^n + u_{i-1}^n}{3}\right)\left(\frac{u_{i+1}^n - u_{i-1}^n}{2h}\right), \quad 1 \le \mathrm{i} \le \mathrm{N}\text{-}1$

respectively, we obtained the following explicit logarithmic finite difference methods (E-LFDM)

E-LFDM-I. $u_i^{n+1} = u_i^n + \ln\left\{1 - \frac{k}{2h}u_i^n\left(u_{i+1}^n - u_{i-1}^n\right) + rv\left(u_{i+1}^n - 2u_i^n + u_{i-1}^n\right)\right\}$

E-LFDM-II.

$$u_i^{n+1} = u_i^n + \ln\left\{1 - \frac{k}{6h}\left(u_{i+1}^n + u_i^n + u_{i-1}^n\right)\left(u_{i+1}^n - u_{i-1}^n\right) + rv\left(u_{i+1}^n - 2u_i^n + u_{i-1}^n\right)\right\},$$

respectively, where $r = \frac{k}{h^2}, \quad 1 \le \mathrm{i} \le \mathrm{N}\text{-}1$.

3. Numerical Results and Discussion

Numerical solutions of model problems are obtained by explicit logarithmic finite difference schemes. To show the accuracy of the results, L_2 and L_∞ error norms:

$$L_2 = \left\| u - U_N \right\|_2 = \sqrt{h \sum_{j=0}^{N} \left| u_j - \left(U_N\right)_j \right|^2},$$

$$L_\infty = \left\| u - U_N \right\|_\infty = \max_j \left| u_j - \left(U_N\right)_j \right|$$

are used, in which u and U_N represent exact and computed numerical solutions, respectively.

The obtained numerical results for Problem 1 are represented in Table 1-3 and Figure 1. Table 1 present the numerical solutions obtained by E-LFDM-I for $v=1, k=10^{-5}$ and different values of h at $t=0.1$. Table 2 present the numerical solutions obtained by E-LFDM-II for $v=1, k=10^{-5}$ and different values of h at $t=0.1$. It can be seen from Table 1 and Table 2 that the values of L_2 and L_∞ decrease with decrease of h. The obtained numerical results are compared with the exact solutions and the numerical solutions obtained by some other methods[7,8,9,11,13] for $v=0.1, h=0.0125$ and $k=10^{-4}$ at times $t=0.4, 0.6, 0.8, 1$ in Table 3. The comparison showed that the present methods give better results than others. Figures 1 shows behavior of the numerical solutions for $v=0.1$ with $h=0.025$, $k=10^{-5}$ at times $t=0.1, 0.5, 1, 2, 3$ for E-LFDM-I.

Table 4-6 represent the obtained numerical results for Problem 2. Table 4 present the numerical solutions obtained by E-LFDM-I for $v=1, k=10^{-5}$ and different values of h at $t=0.1$. Table 5 present the numerical solutions obtained by E-LFDM-II for $v=1, k=10^{-5}$ and different values of h at $t=0.1$. It is seem from Table 1 and Table 2 that the values of L_2 and L_∞ decrease with decrease of h. The obtained numerical results are compared with the exact solutions and the numerical solutions obtained by some other methods[7,8,9,11,13] for $v=0.1, h=0.0125$ and $k=10^{-4}$ at times $t=0.4, 0.6, 0.8, 1$ in Table 6. The comparison showed that the present methods give better results than others.

Table 7 and Table 8 present numerical solutions obtained by E-LFDM-I and E-LFDM-II at different times for $v=0.005,\ 0.01$, $h=0.0125$ and $k=10^{-5}$ for Problem 1 and Problem 2, respectively.

The obtained numerical results for Problem 3 are represented in Table 9 and Figure 2. The numerical solutions obtained by present methods for $a=0$, $b=8$, $v=0.5$, $h=0.0125$ and $k=10^{-4}$ at times $t=1.5, 3, 4.5$ are presented in Table 9. Figures 2 shows behavior of the numerical solutions for $a=0$, $b=1.2$, $v=0.005$ with $h=0.01$, $k=10^{-4}$ at times $t=1, 2, 3, 4$ for E-LFDM-I. Numerical results obtained by present two methods are very close to each other so the figure for numerical solutions from E-LFDM-II is not drawn.

Table 1. Comparison of the numerical solutions obtained by E-LFDM-I for $v=1, k=10^{-5}$ and different values of h at $t=0.1$ with exact solutions for Problem 1

x	$h=0.05$	$h=0.025$	$h=0.0125$	$h=0.01$	Exact
0.1	0.109719	0.109577	0.109542	0.109538	0.109538
0.2	0.210149	0.209870	0.209800	0.209792	0.209792
0.3	0.292415	0.292010	0.291909	0.291896	0.291896
0.4	0.348578	0.348068	0.347941	0.347925	0.347924
0.5	0.372320	0.371742	0.371598	0.371581	0.371577

0.6	0.359809	0.359216	0.359069	0.359051	0.359046
0.7	0.310601	0.310062	0.309928	0.309911	0.309905
0.8	0.228352	0.227939	0.227836	0.227824	0.227817
0.9	0.120978	0.120754	0.120698	0.120691	0.120687
$L_2 \times 10^3$	0.533375	0.118971	0.015669	0.003837	
$L_\infty \times 10^3$	0.763022	0.171028	0.023216	0.006581	

Table 2. Comparison of the numerical solutions obtained by E-LFDM-II for $\nu = 1, k = 10^{-5}$ and different values of h at $t = 0.1$ with exact solutions for Problem 1

x	$h = 0.05$	$h = 0.025$	$h = 0.0125$	$h = 0.01$	Exact
0.1	0.109758	0.109587	0.109544	0.109539	0.109538
0.2	0.210214	0.209886	0.209804	0.209794	0.209792
0.3	0.292482	0.292026	0.291913	0.291899	0.291896
0.4	0.348621	0.348079	0.347943	0.347927	0.347924
0.5	0.372319	0.371742	0.371598	0.371581	0.371577
0.6	0.359760	0.359204	0.359066	0.359049	0.359046
0.7	0.310518	0.310042	0.309922	0.309908	0.309905
0.8	0.228266	0.227918	0.227831	0.227820	0.227817
0.9	0.120924	0.120740	0.120695	0.120689	0.120687
$L_2 \times 10^3$	0.524262	0.116596	0.014823	0.002661	
$L_\infty \times 10^3$	0.741868	0.164718	0.020578	0.003257	

Table 3. Comparison theobtained numerical results for Problem 1with the exact solutions and the numerical solutions obtained by some other methods[7,8,9,11,13] for $\nu = 0.1, h = 0.0125$ and $k = 10^{-4}$ at different times

x	t	RHC[7]	RPA[8]	[11]	NM[13]	I-EFDM[9]	FI-EFDM[9]	E-LFDM-I	E-LFDM-II	Exact
0.25	0.4	0.317062	0.308776	0.31215	0.30415	0.308936	0.308962	0.308877	0.308906	0.308894
	0.6	0.248472	0.240654	0.24360	0.23629	0.240775	0.240795	0.240730	0.240750	0.240739
	0.8	0.202953	0.195579	0.19815	0.19150	0.195709	0.195725	0.195674	0.195688	0.195676
	1.0	0.169527	0.162513	0.16473	0.15861	0.162599	0.162612	0.162570	0.162578	0.162565
0.50	0.4	0.583408	0.569527	0.57293	0.56711	0.569727	0.569762	0.569652	0.569687	0.569632
	0.6	0.461714	0.447117	0.40588	0.44360	0.447307	0.447337	0.447236	0.447254	0.447206
	0.8	0.373800	0.359161	0.36286	0.35486	0.359343	0.359368	0.359278	0.359282	0.359236
	1.0	0.306184	0.291843	0.29532	0.28710	0.292026	0.292046	0.291964	0.291959	0.291916
0.75	0.4	0.638847	0.625341	0.63038	0.61874	0.625659	0.625676	0.625656	0.625519	0.625438
	0.6	0.506429	0.487089	0.49268	0.47855	0.487495	0.487513	0.487432	0.487307	0.487215
	0.8	0.393565	0.373827	0.37912	0.36467	0.374187	0.374203	0.374104	0.374003	0.373922
	1.0	0.305862	0.029726	0.03038	0.27860	0.287700	0.287714	0.287618	0.287542	0.287474

Table 4. Comparison of the numerical solutions obtained by E-LFDM-I for $v=1, k=10^{-5}$ and different values of h at $t=0.1$ with exact solutions for Problem 2

x	$h=0.05$	$h=0.025$	$h=0.0125$	$h=0.01$	Exact
0.1	0.113078	0.112932	0.112896	0.112892	0.112892
0.2	0.216618	0.216332	0.216260	0.216251	0.216252
0.3	0.301498	0.301082	0.300978	0.300966	0.300966
0.4	0.359535	0.359011	0.358880	0.358864	0.358863
0.5	0.384187	0.383592	0.383444	0.383426	0.383422
0.6	0.371446	0.370834	0.370682	0.370663	0.370658
0.7	0.320787	0.320229	0.320089	0.320073	0.320066
0.8	0.235926	0.235498	0.235391	0.235378	0.235371
0.9	0.125021	0.124788	0.124730	0.124723	0.124718
$L_2 \times 10^3$	0.550267	0.122888	0.016284	0.004151	
$L_\infty \times 10^3$	0.787827	0.176631	0.024176	0.007117	

Table 5. Comparison of the numerical solutions obtained by E-LFDM-II for $v=1, k=10^{-5}$ and different values of h at $t=0.1$ with exact solutions for Problem 2

x	$h=0.05$	$h=0.025$	$h=0.0125$	$h=0.01$	Exact
0.1	0.113120	0.112943	0.112899	0.112893	0.112892
0.2	0.216688	0.216349	0.216264	0.216254	0.216252
0.3	0.301571	0.301100	0.300983	0.300969	0.300966
0.4	0.359582	0.359023	0.358883	0.358866	0.358863
0.5	0.384187	0.383592	0.383444	0.383426	0.383422
0.6	0.371393	0.370821	0.370678	0.370661	0.370658
0.7	0.320698	0.320207	0.320084	0.320069	0.320066
0.8	0.235834	0.235475	0.235385	0.235374	0.235371
0.9	0.124963	0.124774	0.124727	0.124721	0.124718
$L_2 \times 10^3$	0.540670	0.120375	0.015356	0.002836	
$L_\infty \times 10^3$	0.764293	0.169825	0.021208	0.003387	

Table 6. Comparison the obtained numerical results for Problem 2 with the exact solutions and the numerical solutions obtained by some other methods[7,8,9,11,13 for $v = 0.1, h = 0.0125$ and $k = 10^{-4}$ at different times

x	t	$k = 10^{-5}$			$k = 10^{-4}$					
		RHC[7]	RPA[8]	[11]	NM[13]	I-EFDM[9]	FI-EFDM[9]	E-LFDM-I	E-LFDM-II	Exact
0.25	0.4	0.306529	0.317399	0.32091	0.31247	0.317567	0.317595	0.317502	0.317536	0.317523
	0.6	0.236051	0.246058	0.24910	0.24148	0.246175	0.246196	0.246127	0.246150	0.246138
	0.8	0.190181	0.199437	0.20211	0.19524	0.199589	0.199606	0.199552	0.199568	0.199555
	1.0	0.156646	0.165529	0.16782	0.16153	0.165633	0.165647	0.165602	0.165612	0.165599
0.50	0.4	0.565994	0.584429	0.58788	0.58176	0.584627	0.584664	0.584551	0.584592	0.584537
	0.6	0.438926	0.457888	0.46174	0.45414	0.458077	0.458110	0.458004	0.458026	0.457976
	0.8	0.348328	0.367320	0.37111	0.36283	0.367507	0.367533	0.367439	0.367446	0.367398
	1.0	0.280038	0.298271	0.30183	0.29336	0.298455	0.298476	0.298391	0.298388	0.298343
0.75	0.4	0.626990	0.645527	0.65054	0.63858	0.645850	0.645866	0.645848	0.645703	0.645616
	0.6	0.477908	0.502564	0.50825	0.49362	0.502969	0.502987	0.502904	0.502774	0.502676
	0.8	0.360630	0.385232	0.39068	0.37570	0.385613	0.385630	0.385527	0.385422	0.385336
	1.0	0.272623	0.295779	0.30057	0.28663	0.296092	0.296106	0.296006	0.295928	0.295857

Table 7. Comparison of the numerical solutions with the exact solutions of Problem 1 and Problem 2 for $v = 0.005$, $h = 0.0125$ and $k = 10^{-5}$

x	t	Problem 1			Problem 2		
		E-LFDM-I	E-LFDM-II	Exact	E-LFDM-I	E-LFDM-II	Exact
0.25	3	0.075227	0.075229	0.075227	0.076392	0.076398	0.076391
	5	0.046964	0.046964	0.046963	0.047415	0.047417	0.047415
	10	0.024217	0.024217	0.024217	0.024336	0.024337	0.024336
	15	0.016308	0.016308	0.016308	0.016362	0.016362	0.016362
0.50	3	0.150409	0.150413	0.150408	0.152681	0.152692	0.152679
	5	0.093920	0.093922	0.093920	0.094815	0.094819	0.094814
	10	0.048422	0.048422	0.048421	0.048661	0.048662	0.048660
	15	0.032442	0.032441	0.032439	0.032553	0.032553	0.032550
0.75	3	0.225499	0.225505	0.225498	0.228770	0.228785	0.228768
	5	0.140837	0.140838	0.140832	0.142160	0.142165	0.142154
	10	0.071173	0.071160	0.071134	0.071557	0.071544	0.071517
	15	0.044174	0.044155	0.044133	0.044370	0.044352	0.044328

Table 8. Comparison of the numerical solutions with the exact solutions of Problem 1 and Problem 2 for $v = 0.01$, $h = 0.0125$ and $k = 10^{-5}$

x	t	Problem 1			Problem 2		
		E-LFDM-I	E-LFDM-II	Exact	E-LFDM-I	E-LFDM-II	Exact
0.25	0.10	0.566349	0.566391	0.566328	0.607367	0.607426	0.607363
	0.15	0.512170	0.512216	0.512148	0.549427	0.549499	0.549421
	0.20	0.466605	0.466650	0.466583	0.499835	0.499913	0.499828
	0.25	0.428015	0.428057	0.427995	0.457421	0.457500	0.457413
0.50	0.10	0.947458	0.947493	0.947414	0.956027	0.956053	0.956007
	0.15	0.900161	0.900217	0.900098	0.914457	0.914503	0.914426
	0.20	0.848434	0.848503	0.848365	0.867172	0.867234	0.867136
	0.25	0.796829	0.796902	0.796762	0.818373	0.818445	0.818337
0.75	0.10	0.860121	0.860018	0.860134	0.886734	0.886657	0.886704
	0.15	0.922827	0.922706	0.922756	0.938505	0.938422	0.938410
	0.20	0.962071	0.961984	0.961891	0.969880	0.969824	0.969732
	0.25	0.974938	0.974912	0.974689	0.979643	0.979629	0.979467

Table 9. Comparison of the numerical solutions with the exact solutions of Problem 3 $a = 0$, $b = 8$, $v = 0.5$, $h = 0.0125$ and $k = 10^{-4}$ at different times

	$t = 1.5$			$t = 3.0$			$t = 4.5$		
x	E-LFDM-I	E-LFDM-II	Exact	E-LFDM-I	E-LFDM-II	Exact	E-LFDM-I	E-LFDM-II	Exact
0.5	0.153273	0.153278	0.153273	0.064262	0.064264	0.064262	0.037989	0.037990	0.037989
1.0	0.265773	0.265777	0.265771	0.118804	0.118807	0.118804	0.071869	0.071870	0.071869
1.5	0.304127	0.304128	0.304125	0.155087	0.155090	0.155087	0.097930	0.097932	0.097931
2.0	0.261419	0.261419	0.261422	0.167623	0.167625	0.167623	0.113386	0.113389	0.113387
2.5	0.172163	0.172164	0.172169	0.156294	0.156295	0.156296	0.116983	0.116985	0.116984
3.0	0.088065	0.088067	0.088070	0.127378	0.127379	0.127382	0.109489	0.109490	0.109491
3.5	0.035819	0.035820	0.035820	0.091320	0.091321	0.091324	0.093682	0.093684	0.093685
4.0	0.011859	0.011859	0.011859	0.057971	0.057972	0.057975	0.073602	0.073603	0.073605
4.5	0.003247	0.003247	0.003246	0.032842	0.032843	0.032844	0.053297	0.053298	0.053300
5.0	0.000741	0.000741	0.000741	0.016734	0.016735	0.016735	0.035713	0.035714	0.035717
$L_2 \times 10^3$	0.006373	0.007707		0.017987	0.017903		0.408777	0.408686	
$L_\infty \times 10^3$	0.006599	0.006304		0.038152	0.038152		0.743130	0.743127	

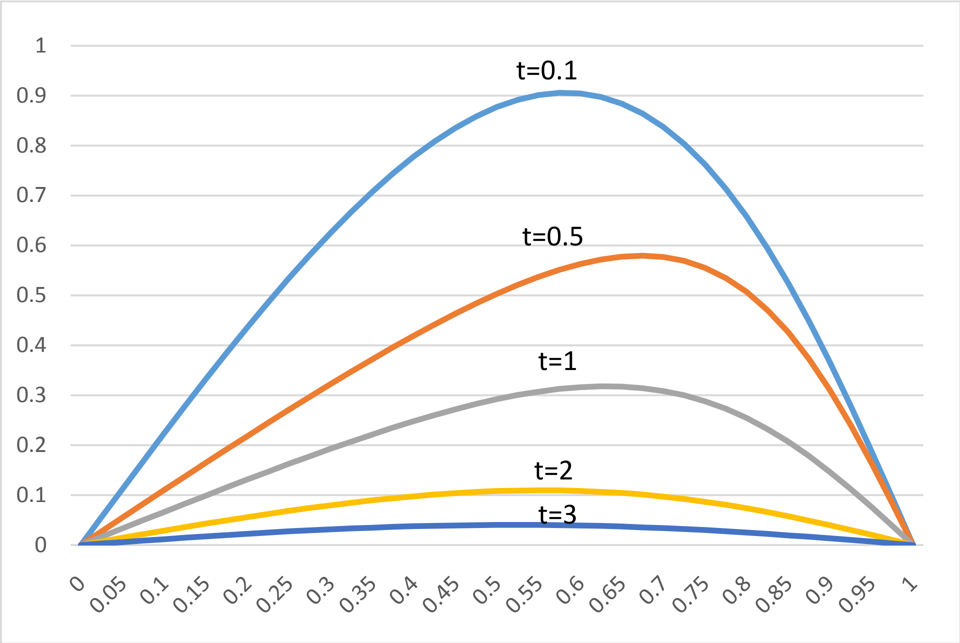

Figure 1. Numerical solutions of Problem 1 for $\nu = 0.1$ with $h = 0.025$, $k = 10^{-5}$ at times $t = 0.1, 0.5, 1, 2, 3$ for E-LFDM-I.

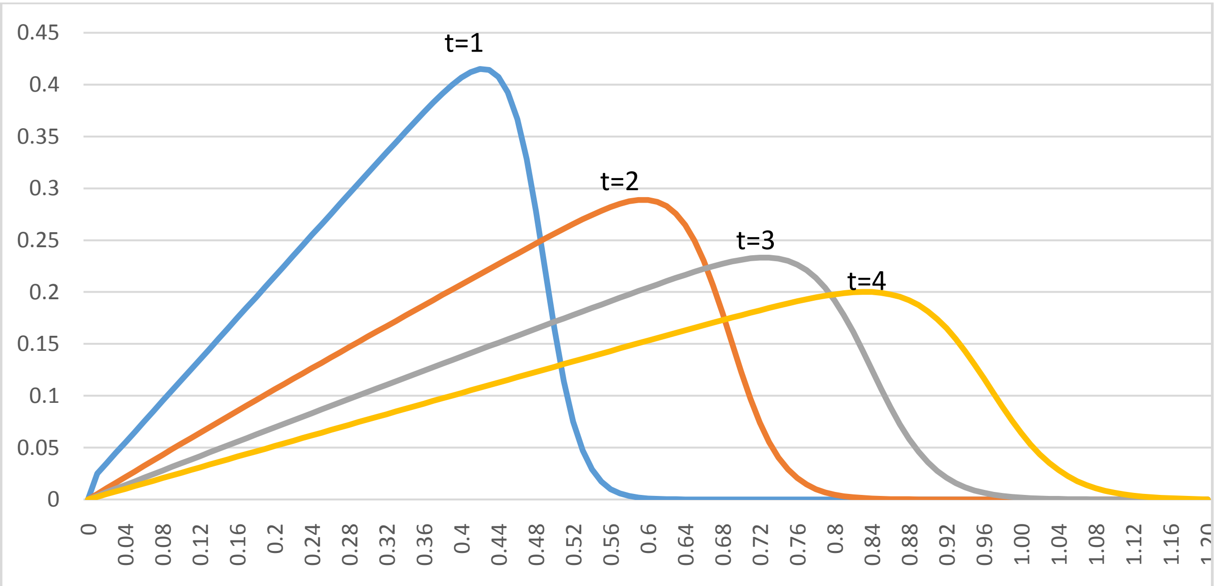

Figure 2. Numerical solutions of Problem 3 for $\nu = 0.005$ with $a = 0$, $b = 1.2$, $h = 0.01$, $k = 10^{-4}$ at times $t = 1, 2, 3, 4$ for E-LFDM-I.

4. Conclusion

In this study,we obtained the numerical solutions of Burgers equation by using explicit logarithmic finite difference schemes and we used three model problems to verify the performance of the methods. Comparisons were made with the some other methods in the literature. L_2 and L_∞ error norms have been calculated and given.The obtained results showedthat the presentschemes are successful for solving the Burgers' equation andprovides high accuracy.

References

[1] Aksan, E. N.,Özdeş, A.(2004).A numerical solution of Burgers' equation. Applied Mathematics and Computation, 156, 395-402.

[2] Aksan, E. N., (2005). A numerical solution of Burgers' equationby finite element method constructed on the method of discretization in time. Applied Mathematics and Computation, 170, 895-904.

[3]Bateman, H.(1915). Some recent researches in motion of fluids. Monthly Weather Review, 43, 163-170.

[4] Burgers, J. M.(1939).Mathematical examples illustrating relations occuring in the theory of turbulent fluid motion. Transactions of the Royal Netherlands Academy of Science (Amsterdam), 17, 1-153.

[5] Cole, J.D. (1951). On a quasi-linear parabolic equation occurring in aerodynamics, Quart. Appl. Math. 9, 225–236.

[6]El-Azab, M. S. , El-Kalla, I. L. and El Morsy, S. A. (2014). Composite Finite Difference Scheme Applied to a Couple of Nonlinear Evolution Equations. Electronic Journal of Mathematical Analysis and Applications,Vol. 2(2) , 254-263.

[7]Gülsu, M., Öziş, T.(2005). Numerical solution of Burgers' equation with restrictive Taylor approximation. Applied Mathematics and Computation, 171, 1192-1200.

[8] Gülsu, M.(2006). A finite difference approach for solution of Burgers' equation. Appl. Math. Comput. 175, 1245-1255.

[9] İnan, B., Bahadır, A. R. (2013). Numerical solution of the one-dimensional Burgers' equation: Implicit and fully implicit exponential finite difference methods. PRAMANA journal of physics, Vol. 81, No. 4, 547-556.

[10] İnan, B., Bahadır, A. R. (2013). An explicit exponential finite difference method for the Burgers' equation. *European International Journal of Science and Technology*, Vol. 2, No. 10, 61-72.

[11] Kutluay, S., Esen, A. & Dag, I.(2004).Numerical solutions of the Burgers' equation by the least-squares quadratic B-spline finite element method. Journal of Computational and Applied Mathematics, 167, 21-33.

[12] Öziş, T., Aksan, E. N. and Özdeş, A. (2003). A finite element approach for solution of Burgers' equation. Applied Mathematics and Computation, 139, 417–428

[13] Salkuyeh, D. K. , Sharafeh, F. S.(2009). On the numerical solution of the Burgers's equation, Int. J. Comput. Math. 86, 1334-1344